MIQP formulation for Controlled Variable Selection in Self Optimizing Control

Ramprasad Yelchuru*, Sigurd Skogestad*, Henrik Manum*

**Department of Chemical Engineering*
Norwegian Science and Technological University, Trondheim 7032.

Abstract In order to facilitate the optimal operation in the presence of process disturbances, the optimal selection of controlled variables plays a vital role. In this paper, we present a Mixed Integer Quadratic Programming methodology to select controlled variables $c=Hy$ as the optimal combinations of fewer/all measurements of the process. The proposed method is evaluated on a toy test problem and on a binary distillation column case study with 41 trays.

Key words: Optimal operation, selection of controlled variables, measurement combination, plantwide control, Mixed Integer Quadratic Programming

1. INTRODUCTION

To facilitate the optimal operation in the presence of disturbances, the optimal control structure selection is important. The decision on which variables should be controlled, which variables should be measured, which input variables should be manipulated and which links should be made between them is called control structure selection. Usually, control structure decisions are based on the intuition of process engineers or on heuristic methods. This does not guarantee optimality and makes it difficult to analyze and improve the proposals.

This paper considers the selection of controlled variables (CVs) associated with the unconstrained degrees of freedom. We assume that the CVs c s are selected as a individual measurements or combinations of subset or all available measurements y . This may be written as

$c=Hy$ where $ny \geq nc$;

ny : number of measurements;

nc: number of CVs = number of unconstrained DOFs = nu ;

where the objective is to find a good choice for the matrix H. In general, we also include inputs (MVs) in the available measurement set y.

Skogestad and coworkers have proposed to use the steady state process model to find "self-optimizing" controlled variable as combinations of measurments. The objective is to find H such that when the CVs are kept at constant set points, the operation gives acceptable steady state loss from the optimal operation in the presence of disturbances.

The theory for self-optimizing control (SOC) is well developed for quadratic optimization problems with linear models. This may seem restrictive, but any unconstrained optimization problem may locally be approximated suitably by this method. The "exact local method" of Halvorsen et al. (2003) handles both disturbances and measurement noise. The problems of finding CVs as optimal variable combinations ($c=Hy$, where H is a full matrix) are originally believed to be difficult to solve numerically (Halvorsen, 2003), but recently it has been shown that SOC problem may be reformulated as a quadratic optimization problem with linear constraints (Alstad et al., 2009). The problem of selecting individual measurements as controlled variables (so H contains nc number of columns with a single 1 and rest of the columns are zero, mathematically $HH^T = I$) is more difficult. The maximum gain rule (Halvorsen et al., 2003) may be useful for prescreening but it is not exact. Even though these methods simplify the loss evaluation for a single alternative, it requires evaluation of every feasible alternative to find the optimal solution. As the number of alternatives increase rapidly with the process dimensions, resorting to exhaustive search methods to find the optimal solution is computationally intractable. Kariwala and Cao (2009) have derived effective branch and bound methods for the exact local method. These branch and bound methods require monotonicity property in the objective function. Furthermore, branch and bound methods are quite complex and they require derivation of good upper and lower bounds. This motivates the need to develop simple and efficient methods to find the optimal solution.

We consider three interesting problems related to finding H:

1. Selection of best individual measurements as CVs (select $n = nc$ measurements)
2. Selection of CVs as combination of all (ny) measurements.
3. Selection of CVs as combination of best subset of n measurements. Where $n \in \{nu, ny\}$

We consider the solution of these problems when applied to the exact local method formulation of Halvorsen et al. (2003). Problem 2 is the easiest one, Problems 1 and 3 involve structural decisions (discrete variables) and are therefore more difficult to solve. Nevertheless, from a practical point of view Problems 1 and 3 are important as

it is not wise to use more measurements than necessary to get an acceptable loss.

To solve Problem 1, Cao and Kariwala (2008) has developed bidirectional branch and bound methods to find the best individual measurements as CVs using minimum singular value criterion. To solve Problem 2, Alstad et al. (2009) has reformulated the self optimizing control problem as a constrained quadratic optimization problem. To solve Problem 3, Kariwala and Cao (2009) developed partial bidirectional branch and bound (PB^3) methods to find best subset of measurements. The methods developed by Kariwala and Cao (2009) exploit the monotonic property of objective function in SOC problem and these methods are of limited/no use if the objective functions are not monotonic.

In this paper we propose a different method to solve Problems 1 and 3 by reformulating the exact local method problem formulation as a Mixed Integer Quadratic Programming (MIQP) problem. The MIQP formulation is simple and can easily be extended to other cost functions. The developed methods are evaluated on a toy problem and on a binary distillation column with 41 trays. The developed MIQP methods for SOC are generic and can easily be evaluated for any system.

2.EXACT LOCAL METHOD FORMULATION

We here review the "exact local method" formulation from Halvorsen et al. (2003) and its optimal solution from Alstad et al. (2009). We want to operate the plant close to optimal steady state operation, by using available degrees of freedom $\mathrm{u}_{\mathrm{all}} = \{\mathrm{u}_{\mathrm{ac}}\} \bigcup \{\mathrm{u}\}$. The steady state cost function $J(u_{all},d)$ is minimized for any given disturbance d. The possible process parameter variations are also included as disturbances. Few of the available degrees of freedom u_{ac} are used to implement optimally "active constraints", so that u contains only the remaining unconstrained steady state degrees of freedom.

The "reduced space" unconstrainted optimization problem then becomes

$$\min_{u} J(u,d) \tag{1}$$

In this work we want to find a set of nc = nu controlled variables c, or more specifically optimal measurement combinations

$$c = Hy \tag{2}$$

such that a constant set point policy (where u is adjusted to keep c constant) yields optimal operation (equation (1)), at least locally. With a given d , solving equation (1) for u gives $J_{opt}(d)$, $u_{opt}(d)$ and $y_{opt}(d)$. In practice, presence of implementations errors and changing disturbances makes it impossible to have $u = u_{opt}(d)$ and results in deviation from optimal operation and this deviation is quantified as loss. The resulting loss (L) is defined as the difference between the cost J, when using a non-optimal input u , and $J_{opt}(d)$ as in Skogestad and Postlethwaite (2005):

$$\mathrm{L} = \mathrm{J(u,d)} - \mathrm{J_{opt}(d)} \tag{3}$$

The local second-order accurate Taylor series expansion of the cost function around the nominal point ($u^*; d^*$) can be written as

$$J(u,d) = J(u^*,d^*) + \begin{bmatrix} J_u & J_d \end{bmatrix}^T \begin{bmatrix} \Delta u \\ \Delta d \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \Delta u \\ \Delta d \end{bmatrix}^T \begin{bmatrix} J_{uu} & J_{ud} \\ J_{ud}^T & J_{dd} \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta d \end{bmatrix} \tag{4}$$

where $\Delta u = (u - u^*)$ and $\Delta d = (d - d^*)$. nu and nd are sizes of Δu and Δd. For a given disturbance ($\Delta d = 0$), the second-order accurate expansion of the loss function around the optimum ($J_u = 0$) becomes

$$L = \frac{1}{2}(u - u^{opt})^T J_{uu} (u - u^{opt}) = \frac{1}{2} z^T z = \frac{1}{2}\|z\|^2 \tag{5}$$

where $z \triangleq J_{uu}^{1/2}(u - u^{opt})$

In this paper, we consider a constant set point policy for the controlled variables which are chosen as linear combinations of the measurements as in equation (2).

The constant set point policy implies that u is adjusted to give $c_s = c + n$ where n is the implementation error for c. Here we assume implementation error is caused by the measurement error i.e. $n = H^* ny$. Now we want to express the loss variables z in terms of d and ny when we use a constant set point policy.

The linearized (local) model in terms of the deviation variables is written as

$$\Delta y = G^y \Delta u + G_d^y \Delta d \tag{6}$$

$$\Delta c = G \Delta u + G_d \Delta d \tag{7}$$

where $G = HG^y$ and $G_d = HG_d^y$

For a constant set point policy ($\Delta c_s = 0$) (Halvorsen et. al. 2003)

$$\Delta u^{opt} = -J_{uu}^{-1} J_{ud} \Delta d$$

$$\Delta y^{opt} = -(G^y J_{uu}^{-1} J_{ud} - G_d^y)\Delta d = F \Delta d \tag{8}$$

The F in equation (8) is the disturbance sensitivity matrix from disturbances d to measurements y at optimal operating points and can be evaluated directly from optimal process operating data. For illustration, select the process operating data close to optimal operation for the possible process disturbances Δd and for these disturbances Δy^{opt} are known and disturbance sensitivity matrix F can be calculated directly. And this obviates the need to calculate G^y , G_d^y and J_{uu} , J_{ud} . The magnitudes of the disturbances d and measurement error ny are quantified by the diagonal scaling matrices W_d and W_n^y respectively. And we write

$$\Delta d = W_d d' \tag{9}$$

$$n^y = W_{n^y} n^{y'} \tag{10}$$

and by introducing the magnitudes of Δd and ny, the loss variables z in equation (3) can be written as

$$z = M_d d' + M_{n^y} n^{y'} \tag{11}$$

where $$M_d = -J_{uu}^{1/2}(HG^y)^{-1}HFW_d \tag{12}$$

$$M_n = -J_{uu}^{1/2}(HG^y)^{-1}HW_{n^y} \tag{13}$$

$$Y = [(G^y J_{uu}^{-1} J_{ud} - G_d^y)W_d \quad W_n]_{ny\times(ny+nd)} \tag{14}$$

Using the equations (12), (13), (14) and (5) the loss can be rewritten as

$$L = \frac{1}{2}\left\| (J_{uu}^{1/2}(HG^y)^{-1}HY)\begin{bmatrix} d' \\ n^{y'} \end{bmatrix}\right\|^2 \tag{15}$$

The loss in equation (15) can be minimized with H as the decision variable. Similar to Halvorsen et.al. 2003 the norm of d', $n^{y'}$ is chosen to be constrained by $\left\|\begin{bmatrix} d' \\ n^{y'} \end{bmatrix}\right\| \le 1$, and the opitmization problem is formulated to minimize the worst case loss and average loss as in Kariwala and Cao (2008).

$$\min_H \frac{1}{2}\bar{\sigma}(J_{uu}^{1/2}(HG^y)^{-1}HY)^2 \tag{16}$$

$$\min_H \frac{1}{6(ny+nd)}\left\|(J_{uu}^{1/2}(HG^y)^{-1}HY)\right\|_F^2 \tag{17}$$

For these SOC problems Kariwala et.al. (2008) proved that the combination matrix H that minimizes the average loss in equation (17) is super optimal and in the sense that the same H minimizes the worst case loss in equation (16). Hence solving the optimization problem in equation (17) is considered in the rest of the paper. The scaling factor $\frac{1}{6(ny+nd)}$ does not have any effect on the solution of the equation (17) and hence it is omitted in the problem formulation.

Lemma 1: The problem in equation (17) may seem non-convex, but it can be reformulated as a constrained quadratic programming problem (Alstad et al., 2009).

$$\begin{aligned} \min_H \quad & \|HY\|_F^2 \\ st. \quad & HG^y = J_{uu}^{1/2} \end{aligned} \tag{18}$$

Proof: From the original problem in equation (17) the optimal solution H is non-unique. If H is a solution then $H_1 = DH$ is also a solution as $(J_{uu}^{-1/2}(HG_y)^{-1})(HY) = (J_{uu}^{-1/2}(H_1G_y)^{-1})(H_1Y)$ for any non-singular matrix D of nu x nu size. This means the objective function is unaffected by the choice of D. One implication is that HG_y can be chosen freely. We can thus make H unique by adding a constraint, for example $HG^y = J_{uu}^{1/2}$. More importantly this simplifies the optimization problem in equation (17) to optimization problem shown in equation (18). **End proof**

The problem in equation (18) is a constrained quadratic programming problem in measurement combination matrix H. We further reformulate the problem in (18) by vectorizing the decision matrix H to a vector x as described in Alstad et al., (2009).

First X is introduced as $X \triangleq H^T$. The matrices X and $J_{uu}^{1/2}$ are split into vectors as $X = [x_1\, x_2 \cdots x_{nu}]; J_{uu}^{1/2} = [J_1\, J_2 \cdots J_{nu}];$ and we further introduce the long vectors as below

$$X_\delta = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{nu} \end{bmatrix}_{(nu*ny)\times 1} ; J_\delta = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_{nu} \end{bmatrix}_{(nu*ny)\times 1}$$

and the large matrices

$$G_\delta^T = \begin{bmatrix} G^{y^T} & 0 & 0 & \cdots \\ 0 & G^{y^T} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ 0 & 0 & \cdots & G^{y^T} \end{bmatrix}_{(nu*nu)\times(ny*nu)} ; Y_\delta = \begin{bmatrix} Y & 0 & 0 & \cdots \\ 0 & Y & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ 0 & 0 & \cdots & Y \end{bmatrix}_{(nu*ny)\times(nu*(nu+nd))}$$

As

$$\|HY\|_F^2 = \left[(HY)^T(HY)\right] = \left\| \begin{bmatrix} x_1^T Y \\ x_2^T Y \\ \vdots \\ x_{nu}^T Y \end{bmatrix} \right\|_F^2 = \left\| x_1^T Y\; x_2^T Y \cdots x_{nu}^T Y \right\|_F^2$$

$$= \left\| X_\delta{}^T Y_\delta \right\|_F^2 = \left\| Y_\delta^T X_\delta \right\|_F^2 = X_\delta{}^T Y_\delta Y_\delta^T X_\delta$$

and as J_{uu} is symmetric positive definite matrix, $J_{uu}^{1/2}$ is also symmetric positive definite $HG_y = G_y^T H^T = G_y^T X = J_{uu}^{1/2}$ and as $G^{y^T} X = \left[G^{y^T} x_1\; G^{y^T} x_2 \cdots G^{y^T} x_n \right] = [J_1\, J_2 \cdots J_{nu}]$ the constraint can be written as

$$\begin{bmatrix} G^{y^T} x_1 \\ G^{y^T} x_2 \\ \vdots \\ G^{y^T} x_{nu} \end{bmatrix}_{(nu*nu)\times 1} = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_n \end{bmatrix}_{(nu*nu)\times 1} \rightarrow G_\delta{}^T X_\delta = J_\delta .$$

In summary, the optimization problem (18) for finding the optimal H can be written as a constrained quadratic programming problem in the variables X_δ as follows.

$$\begin{aligned} \min_{XN} \quad & X_\delta{}^T Y_\delta Y_\delta{}^T X_\delta \\ st. \quad & G_\delta{}^T X_\delta = J_\delta \end{aligned} \tag{19}$$

Note here that X_δ is a stacked vector of all the columns in X or H^T.

3. MIQP FORMULATION

The mixed integer quadratic programming (MIQP) approach provides a different method to solve Problems 1 and 3 described in introduction. Note here that Problem 1 and Problem 2 may be considered as special cases of Problem 3. The main advantages with the MIQP formulation are that these are simple, easily extendable and exact.

We start from the formulation given in (19) to find the optimal loss for the exact local method. Then we address this best measurement subset selection problem by formulating the problem in equation (19) as a Mixed Integer Quadratic Programming (MIQP) problem as described below. Let $\sigma_1, \sigma_2, \cdots \sigma_{ny} \in \{0,1\}$ be binary variables and let rest of the variables be the same as in equation (19). For the chosen measurement subset in the *ny* measurements, the decision variables associated to that binary variables are chosen to be bounded in a range of $-M$ to M. And these bounds are formulated as big-M constraints. Thus the MIQP problem with big-M constraints can be written as in equation (20).

$$\begin{aligned} \min_{x_{aug}} \quad & \left(x_{aug}^T F x_{aug}\right) \\ st. \quad & G_{new}^{y^T} x_{aug} = JN \\ & P x_{aug} = n \\ & \left\{ \begin{bmatrix} -M & 0 & 0 & \cdots \\ 0 & -M & 0 & \cdots \\ \vdots & \vdots & \ddots & \ddots \\ 0 & 0 & \cdots & -M \end{bmatrix} \sigma_i \le \begin{bmatrix} x_i \\ x_{ny+i} \\ \vdots \\ x_{(nu-1)*ny+i} \end{bmatrix} \le \begin{bmatrix} M & 0 & 0 & \cdots \\ 0 & M & 0 & \cdots \\ \vdots & \vdots & \ddots & \ddots \\ 0 & 0 & \cdots & M \end{bmatrix} \sigma_i \right\} for\, i = 1,2,\cdots,ny \\ & \sigma_i \in \{0,1\} \end{aligned} \tag{20}$$

where $x_{aug} = \begin{bmatrix} X_\delta \\ \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_{ny} \end{bmatrix}_{(nu*ny+ny)\times 1}$ and

$F = [Y_\delta Y_\delta{}^T \;\; zeros(ny, ny)]; G_{new}^{y^T} = [G_\delta{}^T \;\; zeros(nu*ny, ny)];$

$P = [zeros(1, nu*ny) \;\; ones(1, ny)]$

and n is the measurement subset size.

In MIQP formulations, selections of a higher value for M in big-M constraints guarantee optimal solution, when bounds on decision variables are unknown. But higher M requires increased computational time in finding the optimal solution. Hence to find the suitable M value in finding optimal solution in an acceptable computational time, the constrained QP problem in (19) with *ny* measurements is solved. Based on the solution of equation (19) M is chosen as 2 times the maximum absolute value of the solution. Then MIQP problem in equation (20) is solved for different values of *n* from *nu* to *ny*. Later, the optimal measurement subset size *n* can be selected for the concerned process.

Lemma 2: The best individual measurements in exact local method (Problem 1) can be obtained from the MIQP problem formulation (equation (20)) solution for measurement subset size equal to *nc*.

Proof: As mentioned in the proof of Lemma 1, If H is a solution then $H_1 = DH$ is also a solution for any non-singular matrix D of size *nuxnu* as $(J_{uu}^{-1/2}(HG_y)^{-1})(HY) = (J_{uu}^{-1/2}(H_1 G_y)^{-1})(H_1 Y)$. Hence the objective function is unaffected by the choice of D.

Let H_{nc}. *be* the optimal solution to this MIQP problem (equation 20) for best nc measurements combination matrix. Now by choosing $D = H_{nc}^{-1}$ and we find the best indiviual measurements H_{im}.(Solution to Problem 1) **End proof**

Application to toy test problem. To illustrate the problem formulation, consider the toy problem of Halvorsen et a.l. (2003) which has two inputs $u = \begin{pmatrix} u_1 & u_2 \end{pmatrix}^T$, one disturbance d and two output measurements $x = \begin{pmatrix} x_1 & x_2 \end{pmatrix}^T$. The cost function is

$$J = (x_1 - x_2)^2 + (x_1 - d)^2$$

where the outputs depended linearly on u , d as

$$x = G^x u + G_d^x d \text{ with } G^x = \begin{bmatrix} 11 & 10 \\ 10 & 9 \end{bmatrix}; G_d^x = \begin{bmatrix} 10 \\ 10 \end{bmatrix};$$

At the optimal point we have $x_1 = x_2 = d$ and $J_{opt}(d) = 0$.

Both the inputs and outputs are included in the candidate set of measurements y. For the example, the steady gain matrix from y to u (G^y), steady disturbance gain matrix from y to d (G_d^y), hessian of cost function with u , d J_{uu}, J_{ud} and disturbance, noise weight matrices W_d, W_n used

ar

$$C = \begin{bmatrix} y_1 \\ y_2 \\ u_1 \\ u_2 \end{bmatrix}; G^y = \begin{bmatrix} 11 & 10 \\ 10 & 9 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}; G_d{}^y = \begin{bmatrix} 10 \\ 10 \\ 0 \\ 0 \end{bmatrix};$$

$$J_{uu} = \begin{bmatrix} 244 & 222 \\ 222 & 202 \end{bmatrix}; J_{ud} = \begin{bmatrix} 198 \\ 180 \end{bmatrix}; W_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; W_n = 0.01 * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The resulting optimal sensitivity matrix is computed as follows

$$Y = [(G^y J_{uu}^{-1} J_{ud} - G_d^y) W_d \quad W_n]_{ny \times (ny+nd)}$$

These matrices are used to get the stacked vector X_δ, J_δ, G_δ^T and Y_δ and the associated matrices in MIQP formulation in equation (20) are

$$F = \begin{bmatrix} 2 & 2 & -18 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & -18 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -18 & -18 & 162 & -162 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 18 & 18 & -162 & 162 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & -18 & 18 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & -18 & 18 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -18 & -18 & 162 & -162 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 18 & 18 & -162 & 162 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{12 \times 12}$$

$$G_{new}^T = \begin{bmatrix} 11 & 10 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 9 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 11 & 10 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 9 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}_{4 \times 12}$$

$$J_n = \begin{bmatrix} 11.6 \\ 10.5 \\ 10.5 \\ 9.6 \end{bmatrix}_{4 \times 1}$$

M=2

$$P = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1]_{1 \times 12}$$

4. RESULTS

4.1 Toy problem

The minimized loss function with the number of measurements used as CVs (i.e. the measurement combinations) is shown in Figure 1. From Figure 1, the loss is minimized as we use more number of measurements to find the CVs as the combinations of measurements. And the reduction in loss is very small when we increase the measurement subset size from 3 to 4.

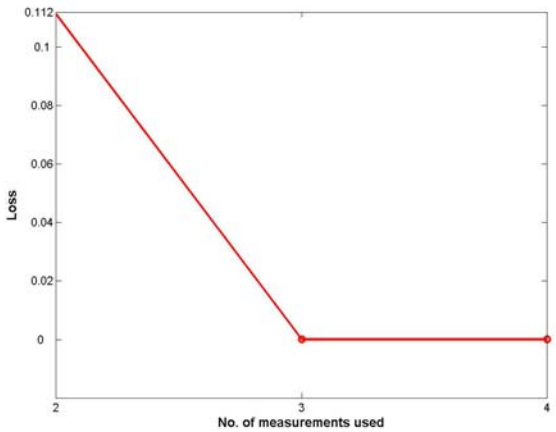

Figure 1. Optimal average loss with best measurement combinations vs no. of measurements used.

Based on the Figure 1, we can conclude that using CVs as combinations of 3 measurement subset is optimal for this toy problem.

4.2 Binary distillation column Problem

The binary distillation column and the associated data are taken from Skogestad (1997). The distillation column in LV-configuration with 41 stages is used. The 41 stage temperatures are taken as candidate measurements. Note that we do not include the inputs in the candidate measurements for this case study. We formulated the MIQP problem for the distillation column with 41 trays to find the 2 CVs as the combinations of 41 tray temperatures. An MIQP is set up for this distillation column with an M value of 1 in big-M constraints in equation (20). We solved the MIQP to find the CVs as the combinations of best measurement subset size from 2 to 41. The CPLX solver in Tomlab environment is used to solve the MIQP problem. The same problem is solved by downwards branch and bound, partial bidirectional branch bound methods of Kariwala and Cao (2009). The computational times (CPU time) taken by MIQP, Downward BAB, PB^3 method and exhaustive search method are shown in Figure 3. Note that exhaustive search is not performed and an estimate of CPU time assuming 0.01 s for each evaluation is plotted. From Figure 3, it can be seen that the MIQP finds optimal solution in 6 orders faster than exhaustive search methods in computational (CPU) time. MIQP method runs relatively quickly for measurement subset size from 25 to 41, but it took fairly longer time for subset sizes from 10 to 19. As these subset sizes (10 to 19) have very high number of possibilities ($41C_{10}$ to $41C_{19}$), the longer time taken by MIQP method is justifiable. But on an avearge basis MIQP methods are slower by 1 order to PB^3 and 0.5 orders slower than Downwards BAB methods. In conclusion, even though the MIQP methods are not computationally attractive to that of Downwards BAB and PB^3 methods; the variation in the computational time by order of 1 is acceptable as these optimal CVs selection problems are performed offline. Despite these, MIQP method is valuable as the method is

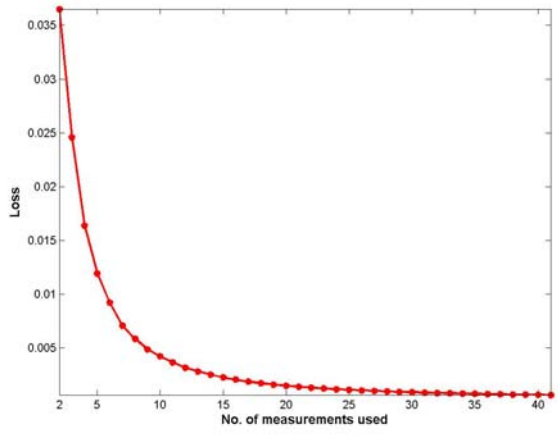

Figure 2. Optimal average loss using MIQP method with best measurement combinations vs no. of measurements used.

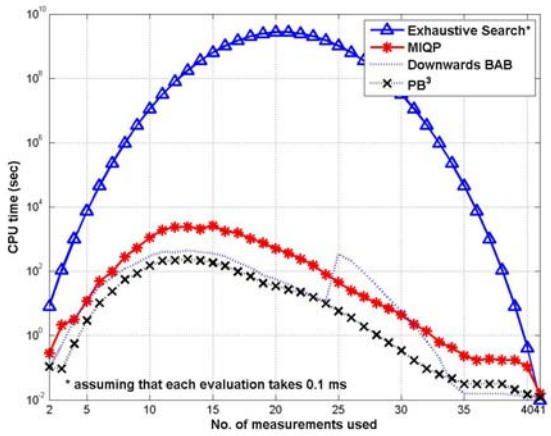

Figure 3. Comparsion of computation times

simple and can easily be extended to any quadratic cost functions to find optimal CVs in SOC framework. The minimized loss function with the number of measurements used for CVs (i.e. the measurement combinations) is shown in Figure 2. From Figure 2, it can be seen that the loss decreases rapidly when the number of measurements increased from 2 to 14, and from 14 very slowly. Based on the Figure 2, we can conclude that using CVs as combinations of 14 measurements subset is optimal for this 41 stage binary distillation column problem. MIQP formulations are easy than the BAB methods and structural constraints such as selection of certain number of measurements from top section, selection of certain number of measurement from bottom section can be done easily.

6. CONCLUSIONS

Optimal CV selection as measurement combinations to minimize the loss from the optimal operation is solved. The CV selection problem in self optimizing control framework is reformulated as a QP and CVs selection as combinations of measurement subsets is formulated as an MIQP problem. The developed MIQP based methods are easier compared to the bidirectional branch and bound methods reported in literature to find the CVs as combinations of measurement subsets. And MIQP methods cover wider spectrum of quadratic based objective functions whereas bidirectional branch and bound methods are limited to objective functions with monotonic properties. MIQP based methods takes longer time than bidirectional branch and bound methods, but this is acceptable as the optimal CV selection problem is done offline. MIQP problem formulations are easily extendable for optimal measurement subset selection for systems with few structural constraints.

REFERENCES

S. Skogestad, Plantwide control: the search for the self-optimizing control structure. Journal of Process Control, 10(5), 487 - 507.

I. J. Halvorsen, S. Skogestad, J.C. Morud, and V. Alstad., Optimal selection of controlled variables. Industrial Engineering and Chemistry Research, 42, 14, 3273 – 3284, 2003

V. Kariwala and Y.Cao., Bidirectional branch and bound for controlled variable selection. Part II: Exact local method for self-optimizing control, Computers and Chemical Engineering, 33, 8, 1402 – 1414, 2009

S. Skogestad and I. Postlethwaite., Multivariable Feedback Control: Analysis and Design. John Wiley & Sons, Chichester, UK, 2nd edition, 2005.

V. Alstad, S. Skogestad, Eduardo S. Hori, Optimal measurement combinations as controlled variables, Journal of Process Control, 19, 138 – 148, 2009.

Y. Cao and V. Kariwala, Bidirectional branch and bound for controlled variable selection Part I. Principles and minimum singular value criterion, Computers and Chemical Engineering, 32, 2306 – 2319, 2008.

S. Skogestad. Dynamics and control of distillation columns – A tutorial introduction. Trans. IChemE Part A, 75:539-562, 1997

TOMLAB v7.1 - The *TOMLAB* Optimization Environment in Matlab (1999)