On-Line PI Controller Tuning Using Closed-Loop Setpoint Response

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Abstract: The proposed method is similar to the Ziegler-Nichols (1942) tuning method, but it is faster to use and does not require the system to approach instability with sustained oscillations. The method requires one closed-loop step setpoint response experiment using a proportional only controller with gain K_{c0} . Based on simulations for a range of first-order with delay processes, simple correlations have been derived to give PI controller settings similar to those of the SIMC tuning rules (Skogestad, 2003). The controller gain (K_c/K_{c0}) is only a function of the overshoot observed in the setpoint experiment whereas the controller integral time (τ_1) is mainly a function of the time to reach the peak (t_p). Importantly, the method includes a detuning factor F that allows the user to adjust the final closed-loop response time and robustness. The proposed tuning method, originally derived for first-order with delay processes, has been tested on a wide range of other processes typical for process control applications and the results are comparable with the SIMC tunings using the open-loop model.

Keywords: PI controller, step test, closed-loop response, IMC, overshoot

1. INTRODUCTION

The proportional integral (PI) controller is widely used in the process industries due to its simplicity, robustness and wide ranges of applicability in the regulatory control layer. On the basis of a survey of more than 11 000 controllers in process industries, Desborough and Miller (2002) have reported that more than 97% of regulatory controllers utilise the PID algorithm. A recent survey (Kano and Ogawa; 2009) from Japan shows that the ratio of applications of PID control, conventional advanced control (feedforward, ratio, valve position control, etc.) and model predictive control is about 100:10:1. In addition, the vast majority of the PID controllers do not use derivative action. Even though the PI controller only has two adjustable parameters, they are often poorly tuned. One reason is that quite tedious plant tests may be needed to obtain improved controller setting. The objective of this paper is to derive a method which is simpler to use than the present ones.

To obtain the information required for tuning the controller one may use open-loop or closed-loop plant tests. Most tuning approaches are based on open-loop plant information; typically the plant's gain (k), time constant (τ) and time delay (θ). One popular approach is direct synthesis (Seborg et al., 2004) which includes the IMC-PID tuning method of Rivera et al. (1986). The original direct synthesis approaches give very good performance for setpoint changes but give sluggish responses to input (load) disturbances for lag-dominant (including integrating) processes with τ/θ larger than about 10. To improve load disturbance rejection, Skogestad (2003) proposed the modified SIMC method where the integral time is reduced for processes with a large value of the time constant τ . The SIMC rule has one tuning parameter, the closed-loop time constant τ_c , and for "fast and robust" control is recommended to choose $\tau_c = \theta$, where θ is the (effective) time delay. However, these approaches require that one first obtains an open-loop model of the process. There are two problems here. First, an open-loop experiment, for example a step test, is normally needed to get the required process data. This may be time consuming and may upset the process and even lead to process runaway. Second, approximations are involved in obtaining the process parameters (e.g., k, τ and θ) from the data.

The main alternative is to use closed-loop experiments. One approach is the classical method of Ziegler-Nichols (1942) which requires very little information about the process. However, there are several disadvantages. First, the system needs to be brought its limit of instability and a number of trials may be needed to bring the system to this point. To avoid this problem one may induce sustained oscillation with an on-off controller using the relay method of Åström and Hägglund, (1984). However, this requires that the feature of switching to on/off-control has been installed in the system. Another disadvantage is that the Ziegler-Nichols (1942) tunings do not work well on all processes. It is well known that the recommended settings are quite aggressive for lagdominant (integrating) processes (Tyreus and Luyben, 1992) and quite slow for delay-dominant process (Skogestad, 2003). A third disadvantage is of the Ziegler-Nichols (1942) method is that it can only be used on processes for which the phase lag exceeds -180 degrees at high frequencies. For example, it does not work on a simple second-order process.

Therefore, there is need of an alternative closed-loop approach for plant testing and controller tuning which avoids the instability concern during the closed-loop experiment, reduces the number of trails, and works for a wide range of processes. The proposed new method satisfies the above concerns: In summary, the proposed method is simpler in use than existing approaches and allows the process to be kept under closed-loop control.

An obvious alternative to the proposed method is a two-step procedure where one first identifies an open-loop model from the closed-loop setpoint experiment, and then obtains the PI or PID controller using standard tuning rules (e.g., the SIMC rules of Skogestad, 2003). This approach was used by Yuwana and Seborg (1982). We found that this two-step approach gives result comparable or slightly inferior (Shamsuzzoha and Skogestad, 2010) to the more direct approach proposed in this paper by using the SIMC method. In addition, the proposed approach avoids the extra step of obtaining the process parameters (k, τ , θ) and is therefore simpler to use.

2. SIMC PI TUNING RULES

In process control, a first-order process with time delay is a common representation of the process dynamics:

$$g(s) = \frac{k e^{-\theta s}}{\tau s + 1}$$
(1)

Here k is the process gain, τ the dominant lag time constant and θ the effective time delay. Most processes in the chemical industries can be satisfactorily controlled using a PI controller:

$$c(s) = K_c \left(1 + \frac{1}{\tau_1 s} \right)$$
⁽²⁾

The PI controller has two adjustable parameters, the proportional gain K_c and the integral time τ_I . The ratio $K_I=K_c/\tau_I$ is known as the integral gain.

The SIMC tuning rule is widely used in the process industry and for the process in Eq. (1) is given as:

$$K_{c} = \frac{\tau}{k(\tau_{c} + \theta)}$$
(3)

$$\tau_{\rm I} = \min\left\{\tau, 4(\tau_{\rm c} + \theta)\right\} \tag{4}$$

Note that the original IMC tuning rule (Rivera et al., 1986) always uses $\tau_I = \tau$, but the SIMC rule increases the integral contribution for close-to integrating processes (with τ large) to avoid poor performance (slow settling) to load disturbance. There is one adjustable tuning parameter, the closed-loop time constant (τ_c), which is selected to give the desired trade-off between performance and robustness. Initially, this study is based on the "fast and robust" setting $\tau_c = \theta$, which gives a good trade-off between performance and robustness. In terms of robustness, this choice gives a gain margin is about 3 and a sensitivity peak (M_s-value) of about 1.6. On dimensionless form, the SIMC tuning rules with $\tau_c = \theta$ become

$$K_{c}' = kK_{c} = 0.5 \frac{\tau}{\rho}$$
(5)

$$K_{I} = \frac{kK_{c}}{\tau_{I}/\theta} = \max\left(0.5, \frac{1}{16}\frac{\tau}{\theta}\right)$$
(6)

The dimensionless gains K_c and K_I are plotted as a function of τ/θ in Fig. 1. We note that the integral term (K_I) is most important for delay dominant processes ($\tau/\theta < 1$), while the proportional term K_c is most significant for processes with a smaller time delay. These insights are useful for the next step when we want to derive tuning rules based on the closed-loop setpoint response.

3. CLOSED-LOOP EXPERIMENT

As mentioned earlier, the objective is to base the controller tuning on closed-loop data. The simplest closed-loop experiment is probably a setpoint step response (Fig. 2) where one maintains full control of the process, including the change in the output variable. The simplest to observe is the time t_p to reach the (first) overshoot and its magnitude, and this information is therefore the basis for the proposed method.

We propose the following procedure:

1. Switch the controller to P-only mode (for example, increase the integral time τ_I to its maximum value or set the integral gain K_I to zero). In an industrial system, with bumpless transfer, the switch should not upset the process.

2. Make a setpoint change that gives an overshoot between 0.10 (10%) and 0.60 (60%); about 0.30 (30%) is a good value. Record the controller gain K_{c0} used in the experiment. Most likely, unless the original controller was quite tightly tuned, one will need to increase the controller gain to get a sufficiently large overshoot.

Note that small overshoots (less than 0.10) are not considered because it is difficult in practice to obtain from experimental data accurate values of the overshoot and peak time if the overshoot is too small. Also, large overshoots (larger than about 0.6) give a long settling time and require more excessive input changes. For these reasons we recommend using an "intermediate" overshoot of about 0.3 (30%) for the closed-loop setpoint experiment.

3. From the closed-loop setpoint response experiment, obtain the following values (see Fig. 2):

- Fractional overshoot, $(\Delta y_p \Delta y_{\infty}) / \Delta y_{\infty}$
- Time from setpoint change to reach peak output (overshoot), t_p

• Relative steady state output change, $b = \Delta y_{\infty} / \Delta y_s$.

Here the output variable changes are:

 Δy_s : Setpoint change

 Δy_p : Peak output change (at time t_p)

 Δy_{∞} : Steady-state output change after setpoint step test

To find Δy_{∞} one needs to wait for the response to settle, which may take some time if the overshoot is relatively large (typically, 0.3 or larger). In such cases, one may stop the experiment when the setpoint response reaches its first minimum and record the corresponding output, Δy_u .

$$\Delta y_{\infty} = 0.45(\Delta y_{\rm p} + \Delta y_{\rm u}) \tag{7}$$

4. CORRELATION BETWEEN SETPOINT RESPONSE AND SIMC-SETTINGS

The objective of this paper is to provide a more direct approach similar to the Ziegler-Nichols (1942) closed-loop method. Thus, the goal is to derive a correlation, preferably as simple as possible, between the setpoint response data (Fig. 2) and the SIMC PI-settings in Eq. (3) and (4), initially with the choice $\tau_c=0$. For this purpose, we considered 15 first-order with delay models $g(s)=ke^{-0s}/(\tau s+1)$ that cover a wide range of processes; from delay-dominant to lag-dominant (integrating):

 $\tau/\theta = 0.1, 0.2, 0.4, 0.8, 1.0, 1.5, 2.0, 2.5, 3.0, 7.5, 10.0, 20.0, 50.0, 100$

Since we can always scale time with respect to the time delay (θ) and since the closed-loop response depends on the product of the process and controller gains (kK_c) we have without loss of generality used in all simulations k=1 and $\theta=1$.

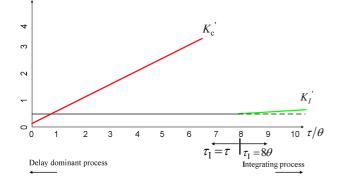


Fig. 1. Scaled proportional and integral gain for SIMC tuning rule.

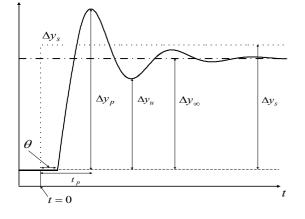


Fig. 2. Closed-loop step setpoint response with P-only control.

For each of the 15 process models (values of τ/θ), we obtained the SIMC PI-settings (K_c and τ_1) using Eqs. (3) and (4) with the choice $\tau_c=\theta$. Furthermore, for each of the 15 processes we generated 6 closed-loop step setpoint responses using P-controllers that give different fractional overshoots.

Overshoot= 0.10, 0.20, 0.30, 0.40, 0.50 and 0.60

In total, we then have 90 setpoint responses, and for each of these we record four data: the P-controller gain K_{c0} used in the experiment, the fractional overshoot, the time to reach the overshoot (t_p), and the relative steady-state change, $b = \Delta y_{\omega}/\Delta y_{s}$.

Controller gain (K_c). We first seek a relationship between the above four data and the corresponding SIMC-controller gain K_c. Indeed, as illustrated in Fig. 3, where we plot kK_c (SIMC PI-controller for the corresponding process) as a function of kK_{c0} for the 90 setpoint experiments, the ratio K_c/K_{c0} is approximately constant for a fixed value of the overshoot, independent of the value of τ/θ . Thus, we can write

$$\frac{K_{c}}{K_{c0}} = A \tag{8}$$

where the ratio A is a function of the overshoot only. In Fig. 4 we plot the value of A, which is obtained as the best fit of the slopes of the lines in Fig. 3, as a function of the overshoot. The following equation (solid line in Fig. 4) fits the data in Fig. 3 well,

 $A = [1.152(\text{overshoot})^2 - 1.607(\text{overshoot}) + 1.0]$ (9)

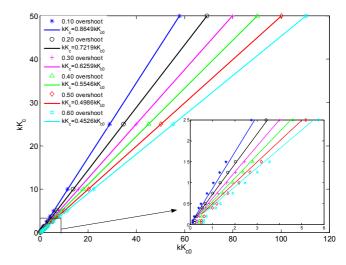


Fig. 3. Relationship between P-controller gain kK_{c0} used in setpoint experiment and corresponding SIMC controller gain kK_{c} .

Actually, a closer look at Fig. 3 reveals that a constant slope, use of Eq.(8) and (9), only fits the data well for K_c '=k K_c greater than about 0.5. Fortunately, a good fit of the controller gain K_c is not so important for delay-dominant processes ($\tau/\theta < 1$) where K_c '<0.5, because we recall from the discussion of the SIMC rules (Fig. 1) that the integral gain K_I is more important for such processes. This is discussed in more detail below.

Integral time (τ_1). Next, we want to find a simple correlation for the integral time. Since the SIMC tuning formula in Eq. (4) uses the minimum of two values, it seems reasonable to look for a similar relationship, that is, to find one value ($\tau_{11} = \tau$) for processes with a relatively large delay, and another value ($\tau_{12} = 8\theta$) for processes with a relatively small delay including integrating processes.

(1) Process with relatively large delay. For processes with a relatively large delay $\tau/\theta < 8$ or $\theta > \tau/8$, the SIMC-rule is to use $\tau_I = \tau$. Inserting $\tau = \tau_I$ into the SIMC rule for K_c in Eq. (5) and solving for τ_I gives:

(10)

 $\tau_{11}=2kK_c\theta$

As just mentioned, for processes with a relatively large delay it is the integral gain $K_I = K_c/\tau_I$ that matters most (Fig. 1) and to avoid that any error in K_c originating from our correlation Eq.(8) propagates into K_I , we should in Eq. (10) use $K_c = K_{c0}A$, where A is given as a function of the overshoot in Eq. (9). In (10), we also need the value of the process gain k, and to this effect write

$$kK_{c} = kK_{c0}K_{c}/K_{c0}$$
(11)

Here, the value of the loop gain kK_{c0} for the P-control setpoint experiment is given from the value of b:

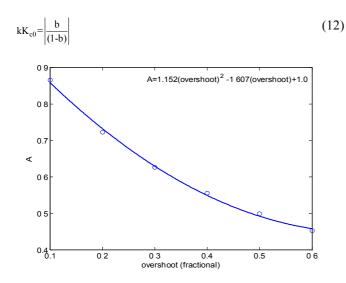


Fig. 4. Variation of A with overshoot using data (slopes) from Fig. 3.

To prove this, the closed-loop setpoint response is $\Delta y/\Delta y_s = gc/(1+gc)$ and with a P-controller with gain K_{c0} , the steadystate value is $\Delta y_{\alpha}/\Delta y_s = kK_{c0}/(1+kK_{c0})=b$ and we derive Eq.(12). The absolute value is included to avoid problems if b>1, as may occur for an unstable process or because of inaccurate data.

In summary, we have derived following expression for τ_I for a delay-dominant process:

$$\tau_{11} = 2A \left| \frac{b}{(1-b)} \right| \theta \tag{13}$$

One could obtain the **effective time delay** θ directly from the closed-loop setpoint response, but this is generally not easy. Fortunately, as shown in Fig. (5), there is a reasonably good correlation between θ and the setpoint peak time t_p which is easier to observe. For processes with a relatively large time delay ($\tau/\theta < 8$), the ratio θ/t_p varies between 0.27 (for $\tau/\theta = 8$ with overshoot=0.1) and 0.5 (for $\tau/\theta=0.1$ with all overshoots). For the intermediate overshoot of 0.3, the ratio θ/t_p varies between 0.32 and 0.50. A conservative choice would be to use $\theta=0.5t_p$ because a large value increases the integral time. However, to improve performance for processes with smaller time delays, we propose to use $\theta=0.43t_p$ which is only 14% lower than 0.50 (the worst case).

In summary, we have for process with a relatively large time delay:

$$\tau_{11} = 0.86 \text{ A} \left| \frac{\text{b}}{(1-\text{b})} \right| t_p \tag{14}$$

(2) Process with relatively small delay. For a lag-dominant (including integrating) process with $\tau/\theta>8$ the SIMC rule gives

$$\tau_{12}=8\theta \tag{15}$$

For $\tau/\theta > 8$ we see from Fig. (5) that the ratio θ/t_p varies between 0.25 (for $\tau/\theta=100$ with overshoot=0.1) and 0.36 (for $\tau/\theta=8$ with overshoot 0.6). We select to use the average value $\theta= 0.305t_p$ which is only 15% lower than 0.36 (the worst case). Also note that for the intermediate overshoot of 0.3, the ratio θ/t_p varies between 0.30 and 0.32. In summary, we have for a lag-dominant process

$$\tau_{12} = 2.44 t_p$$
 (16)

Conclusion. The integral time τ_I is obtained as the minimum of the above two values:

$$\mathbf{r}_{\mathrm{I}} = \min\left(0.86\mathrm{A} \left| \frac{\mathrm{b}}{(\mathrm{I} \cdot \mathrm{b})} \right| \mathbf{t}_{\mathrm{p}}, 2.44\mathbf{t}_{\mathrm{p}} \right)$$
(17)

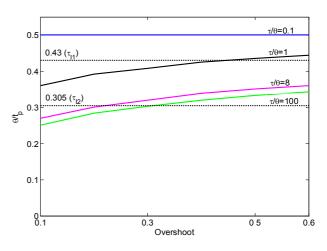


Fig. 5. Ratio between delay and setpoint overshoot peak time (θ/t_p) for P-only control of first-order with delay processes (solid lines); Dotted lines: values used in final correlations.

5. ANALYSIS AND SIMULATION

Closed-loop simulations have been conducted for 7 different processes and the proposed tuning procedure provides in all cases acceptable controller settings with respect to both performance and robustness. For each process, PI-settings are obtained based on step response experiments with three different overshoot (around 0.1, 0.3 and 0.6) and are compared with the SIMC settings.

The closed-loop performance is evaluated by introducing a unit step change in both the set-point and load disturbance i.e, $(y_s=1 \text{ and } d=1)$. To evaluate the robustness, the maximum sensitivity, M_s , defined as $M_s=\max|1/[1+gc(i\omega)]|$, is used. Since M_s is the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point (-1, 0), a small M_s -value indicates that the control system has a large stability margin.

The results for the 7 example processes, which include the different types of the process mainly stable, integrating and unstable plant dynamics, are listed in Table 1.

All results are without detuning (F=1). The complete simulation results with additional examples are available in a technical report (Shamsuzzoha and Skogestad, 2010). As expected, when the method is tested on first-order plus delay processes, similar to those used to develop the method, the responses are similar to the SIMC-responses, independent of the value of the overshoot. Typical cases are E1, E2 (pure time delay) and E3 (integrating with delay); see Figs. 6-8.

For models that are *not* first-order plus delay (typical cases are E4, E5 and E6, see Fig. 9 for E6 only), the agreement with the SIMC-method is best for the intermediate overshoot (around 0.3). A small overshoot (around 0.1) typically give

"slower" and more robust PI-settings, whereas a large overshoot (around 0.6) gives more aggressive PI-settings. In some sense this is good, because it means that a more "careful" step response results in more "careful" tunings. Also note that the user always has the option to use the detuning factor F to correct the final tunings. Case E7 (Fig. 10) illustrates that the method works well for a simple unstable process with delay.

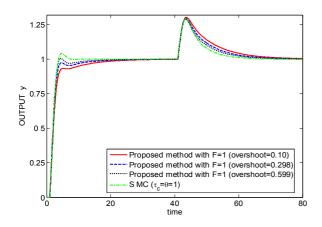


Fig. 6. Responses for PI-control of $g = e^{-s}/(5s+1)$ (E1).

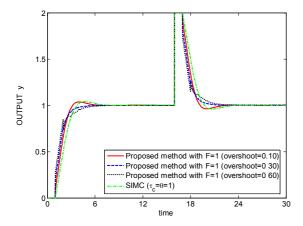


Fig. 7 Responses for PI-control of $g = e^{-s}$ (E2).

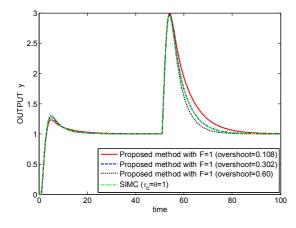


Fig. 8. Responses for PI-control of $g = e^{-s}/s$ (E3).

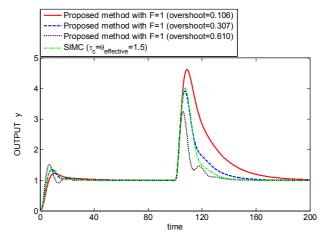


Fig. 9. Responses for PI-control of $g = 1/[s(s+1)^2]$ (E6).

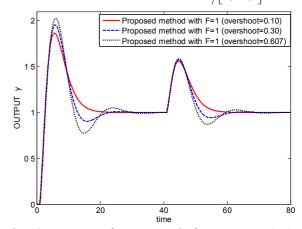


Fig. 10. Responses for PI-control of $g = e^{-s}/(5s-1)$ (E7).

6. CONCLUSION

A simple and new approach for PI controller tuning has been developed. It is based on a single closed-loop setpoint step experiment using a P-controller with gain K_{c0} . The PI-controller settings are then obtained directly from following three data from the setpoint experiment:

- Overshoot, $(\Delta y_p \Delta y_\infty) / \Delta y_\infty$
- Time to reach overshoot (first peak), t_p
- Relative steady state output change, $b = \Delta y_{\infty} / \Delta y_s$.

If one does not want to wait for the system to reach steady state, one can use the estimate $\Delta y_{\infty} = 0.45(\Delta y_p + \Delta y_u)$.

The proposed tuning formulas for the proposed "Setpoint Overshoot Method" method are: $K_{-}=K_{-}\circ A/F$

$$\tau_{\rm I} = \min\left(0.86 \,\mathrm{A} \left| \frac{\mathrm{b}}{(1 - \mathrm{b})} \right| t_{\rm p}, 2.44 t_{\rm p} \mathrm{F} \right)$$

where, $A = \begin{bmatrix} 1.152(\text{overshoot})^2 - 1.607(\text{overshoot}) + 1.0 \end{bmatrix}$

The factor F is a tuning parameter and F=1 gives the "fast and robust" SIMC settings corresponding to $\tau_c=\theta$. To detune the response and get more robustness one selects F>1, but in special cases one may select F<1 to speed up the closed-loop response.

The Setpoint Overshoot Method works well for a wide variety of the processes typical for process control, including the standard first-order plus delay processes as well as integrating, high-order, inverse response, unstable and oscillating process.

We believe that the proposed method is the simplest and easiest approach for PI controller tuning available and should be well suited for use in process industries.

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Case	Process model	k _{c0}	overshoot	t _p	b	k _c	$ au_{\mathrm{I}}$	M _s
E1	$\frac{e^{-s}}{(5s+1)}$	2.75	0.10	3.60	0.733	2.338	7.240	1.50
		4.0	0.298	3.049	0.80	2.494	6.538	1.56
	(33+1)	5.75	0.599	2.705	0.852	2.592	6.030	1.60
		SIMC	-	-	-	2.50	5.0	1.59
E2	e^{-s}	0.10	0.10	2.0*	0.091	0.085	0.146	1.60
		0.30	0.30	2.0	0.231	0.187	0.321	1.53
		0.60	0.60	2.0	0.270	0.465	0.375	1.59
		SIMC	-	-	-	$k_{c}/\tau_{I}=0.50$		1.59
E3	$\frac{e^{-s}}{s}$	0.59	0.108	3.976	1.0	0.495	9.702	1.67
		0.80	0.302	3.282	1.0	0.496	8.008	1.70
	3	1.10	0.60	2.909	1.0	0.496	7.098	1.72
		SIMC	-	-	-	0.50	8.0	1.70
E4	$(s^{2}+2s+9)(-2s+1)(s+1)e^{-2s}$	0.07	0.112	18.132	0.387	0.058	8.198	1.46
	$\frac{(s^2+2s+9)(-2s+1)(s+1)e^{-2s}}{(s^2+0.5s+1)(5s+1)^2}$	0.12	0.301	15.043	0.519	0.074	8.667	1.61
		0.18	0.583	13.71	0.618	0.082	8.684	1.70
		SIMC	-	-	-	-	-	-
E5	1	5.0	0.127	0.710	0.833	4.074	1.732	1.33
	$\frac{1}{(s+1)(0.2s+1)}$	15.0	0.322	0.393	0.937	9.031	0.958	1.74
		40.0	0.508	0.230	0.976	19.23	0.561	2.62
		SIMC	-	-	-	5.5	0.80	1.56
E6	$\frac{1}{s(s+1)^2}$	0.32	0.106	8.985	1.0	0.270	21.923	1.51
		0.58	0.307	6.188	1.0	0.357	15.10	1.75
		1.15	0.610	4.492	1.0	0.516	10.961	2.30
		SIMC	-	-	-	0.330	12.0	1.76
E7	$\frac{e^{-s}}{(5s-1)}$	3.10	0.10	4.647	1.476	2.636	10.54	2.12
		4.0	0.30	3.671	1.333	2.487	7.852	2.33
		5.30	0.607	3.164	1.233	2.379	6.475	2.67
		SIMC	-	-	-	-	-	-

Table 1: PI controller setting for proposed method and comparison with SIMC method	d ($\tau_c = \theta_{effective}$)
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* For the pure time delay case (E4) use the end time of the peak (or add a small time constant to get t_p in simulation).

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