Explicit MPC with output feedback using self-optimizing control

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Outline

1. Optimal operation paradigms
2. Self optimizing control
3. Explicit MPC
4. Link between the two
5. Output feedback
6. Extension to noisy measurements
7. Examples

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Implementation of optimal operation using off-line computations

Paradigm 1

On-line optimizing control where measurements are primarily used to update the model. With arrival of new measurements, the optimization problem is resolved for the inputs.

Paradigm 2

Pre-computed solutions based on off-line optimization. Typically, the measurements are used to (indirectly) update the inputs using feedback control schemes. Focus of this work.

Implementation of optimal operation using off-line computations

Paradigm 1

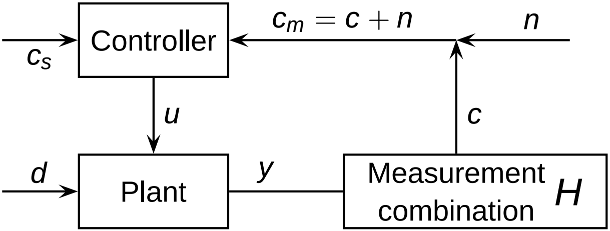
On-line optimizing control where measurements are primarily used to update the model. With arrival of new measurements, the optimization problem is resolved for the inputs.

Example: Classical (implicit) MPC.

Paradigm 2

Pre-computed solutions based on off-line optimization. Typically, the measurements are used to (indirectly) update the inputs using feedback control schemes. Focus of this work.

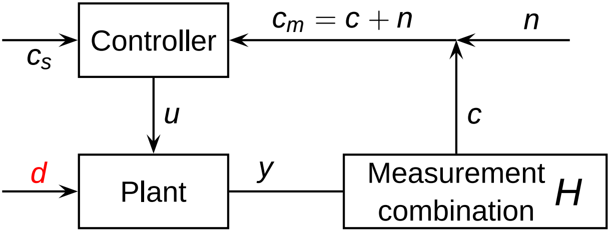
Examples: Explicit MPC and self-optimizing control.



Self-optimizing control

Choice of H such that acceptable operation is achieved with constant setpoints (c_s constant).

What variables should we control?



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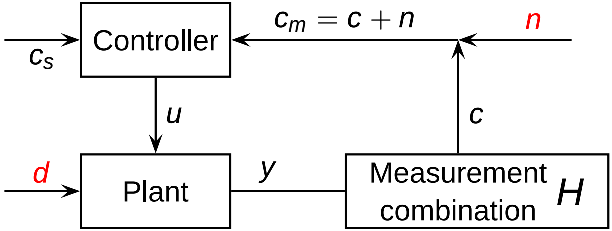
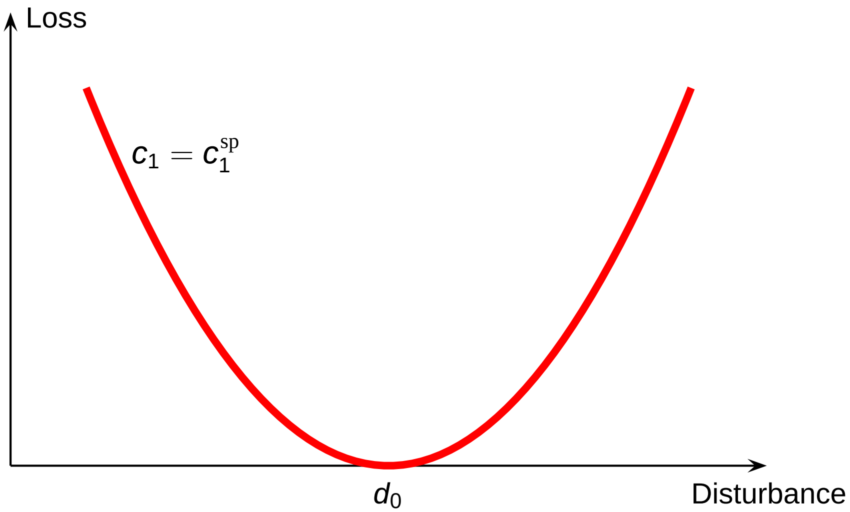


Self-optimizing control

Choice of H such that acceptable operation is achieved with constant setpoints (c_s constant).

- Optimal c_s is invariant with respect to disturbances d
- Insensitive to measurement errors n

What variables should we control?



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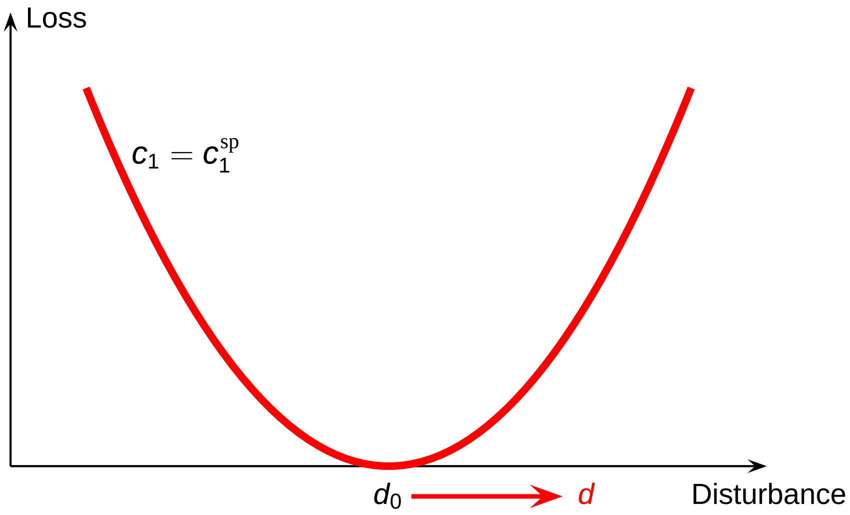

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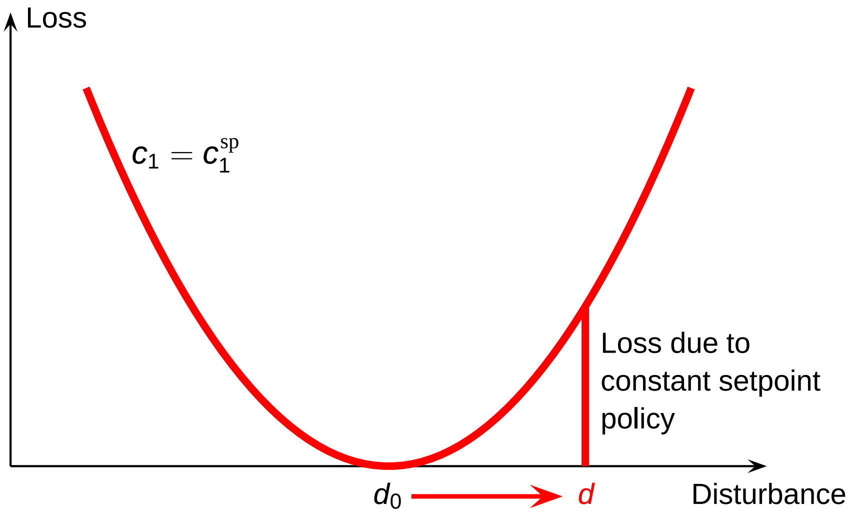

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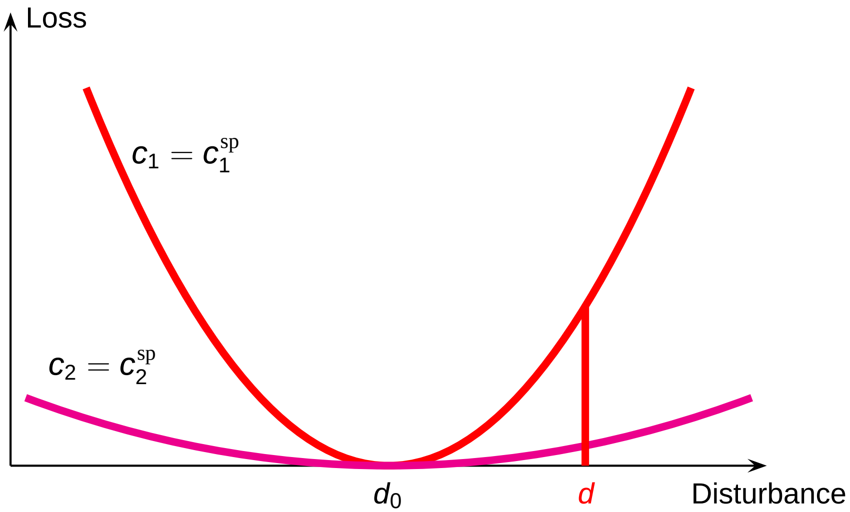

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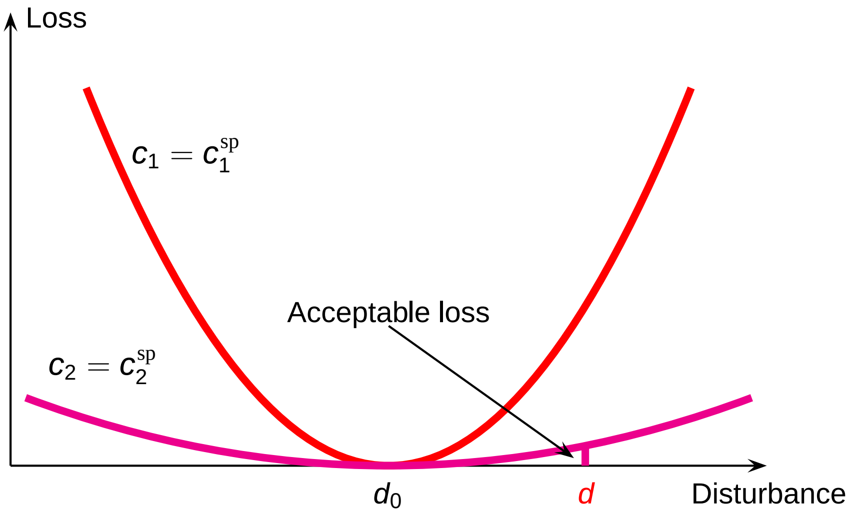

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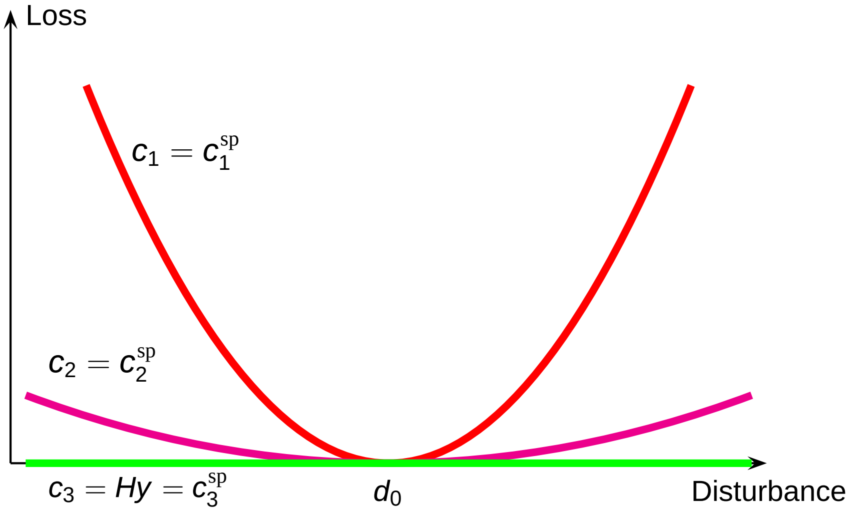


Nullspace method for QP problems

The nullspace method is restated for QP's

Theorem (Nullspace method for QP)

- *Consider the quadratic problem*

$$\min_u J = \begin{bmatrix} u & d \end{bmatrix} \begin{bmatrix} J_{uu} & J_{ud} \\ J_{ud}^{\mathrm{T}} & J_{dd} \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix} \tag{1}$$

  If there exists $n_y \geq n_u + n_d$ independent measurements, then the optimal solution to (1) *has the property that there exists variable combinations $c = Hy$ that are invariant to the disturbances d.*

- *H may be found from $HF = 0$, where $F = \frac{\partial y^{opt}}{\partial d^{\mathrm{T}}}$*

Explicit MPC

- The "classical" MPC problem can, by substitution, be written as a quadratic problem:

$$\min_U J(U, x(t)) = \begin{bmatrix} U^{\mathrm{T}} & x(t)^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} H & F \\ H & Y \end{bmatrix} \begin{bmatrix} U \\ x(t) \end{bmatrix}$$

$$\text{s.t. } GU \leq W + Ex(t)$$

- The initial state $x(t)$ is considered to be a parameter and a parametric program is solved.
- The solution of the parametric program gives regions in the state space.
- Given an algorithm for deciding the current region (i), one implements a continuous piece-wise affine control law

$$u = F^i x + g^i.$$

Link between explicit MPC and self-optimizing control

Let

$$d = x_0 \quad \text{and} \quad y = \begin{bmatrix} u \\ x \end{bmatrix}$$

The optimal combination

$$c = Hy$$

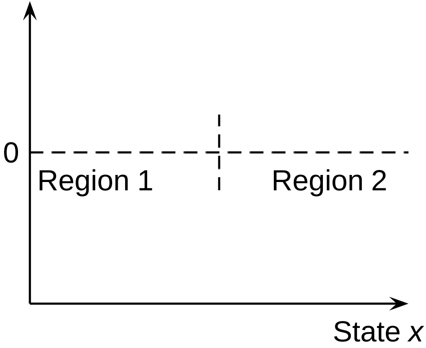
can be written as the feedback law

$$c = u - (Kx + g)$$

and H (or K) can be obtained from nullspace method

When to switch region?

- The invariants can be used to track region changes
- By monitoring neighboring regions we switch regions when $c_i - c_j$ changes sign



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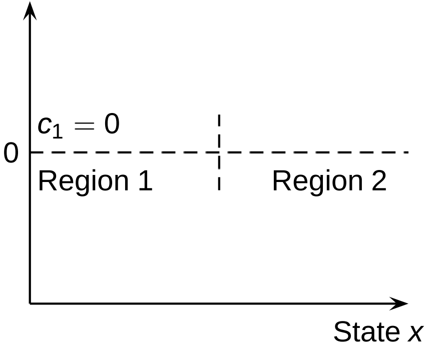

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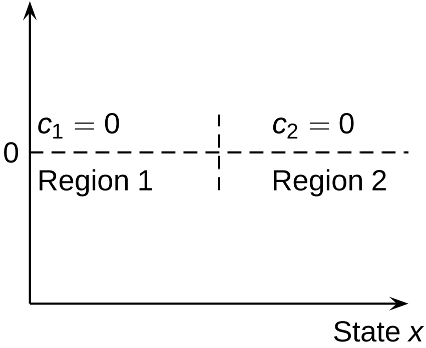

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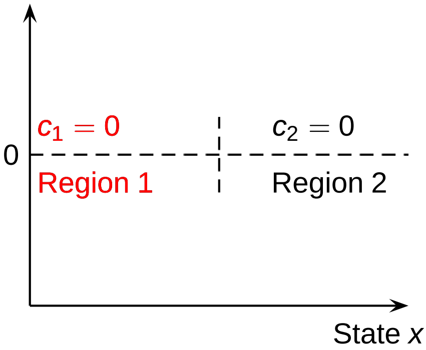

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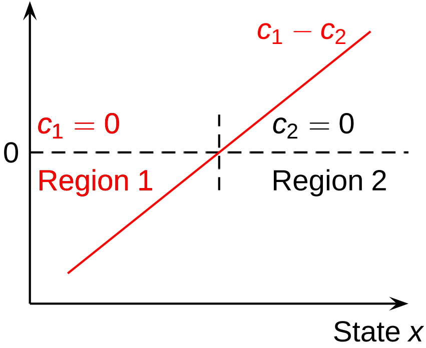

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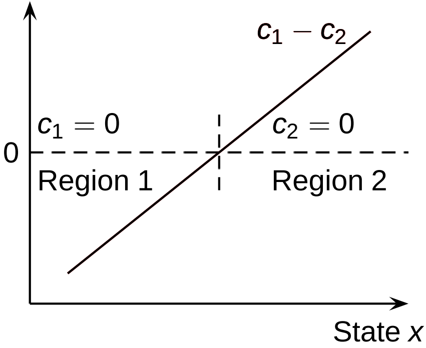

Example 1: Output feedback

Process

- $y(t) = \frac{2}{s^2+3s+2}$
- Input constraint: $|u(t)| \leq 2$
- Sample the system and get two-state discrete model
- Quadratic objective function

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Alternative 2

- $y = (y_k, y_{k+1}, u_k, u_{k+1})$
- Write
  $$y = G^y \begin{bmatrix} u_k \\ u_{k+1} \end{bmatrix} + G^y_d x_k$$
- Sensitivity
  $$F = -(G^y J_{uu}^{-1} J_{ud} - G^y_d)$$
- Find H such that $HF = 0$

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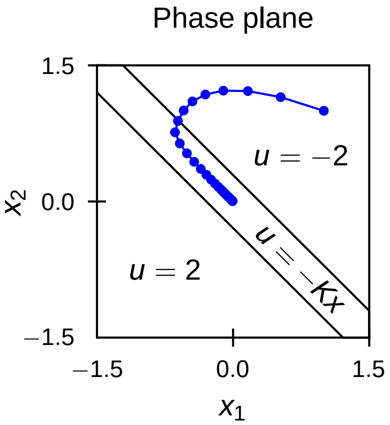
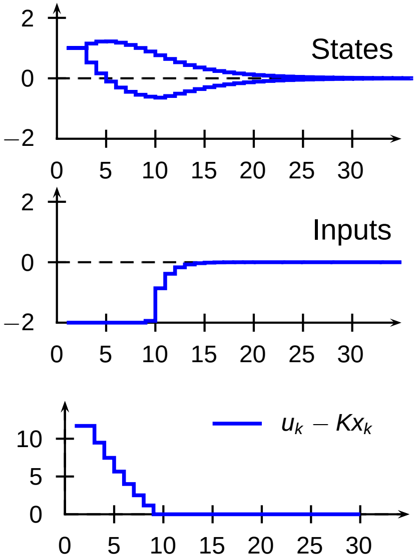
- Sensitivity
  $$F = -(G^y J_{uu}^{-1} J_{ud} - G_d^y)$$
- Find H such that $HF = 0$

$$\Downarrow$$

$$u_k = -(-16.7y_k + 13.7y_{k-1})$$





State space partition and simulation from $x_0 = (1, 1)$

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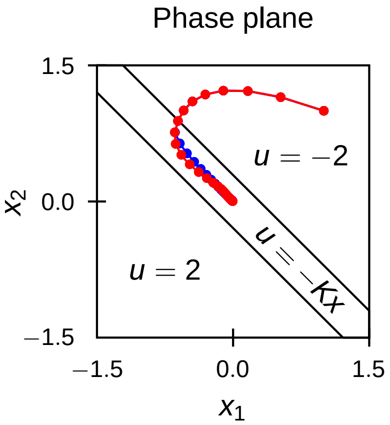

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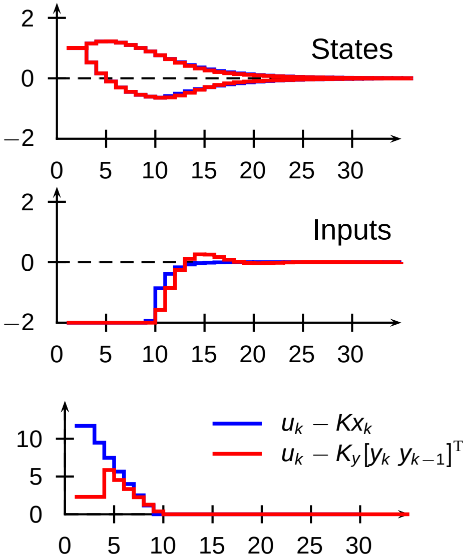
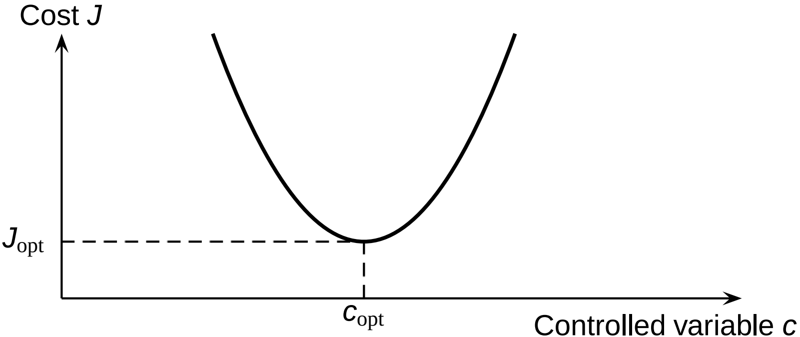
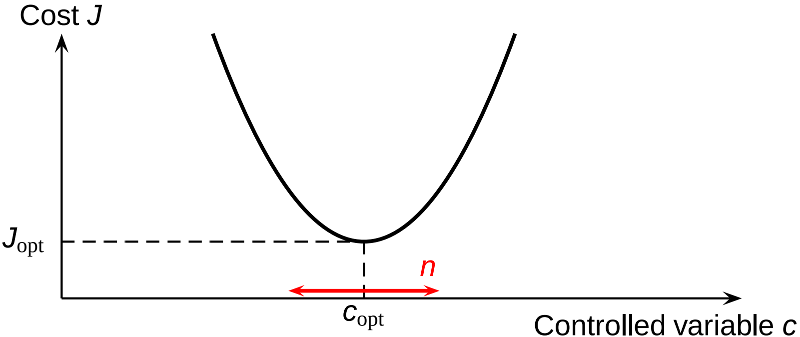


Noisy measurements





- Implementation error: $c = c_{opt} + n$.

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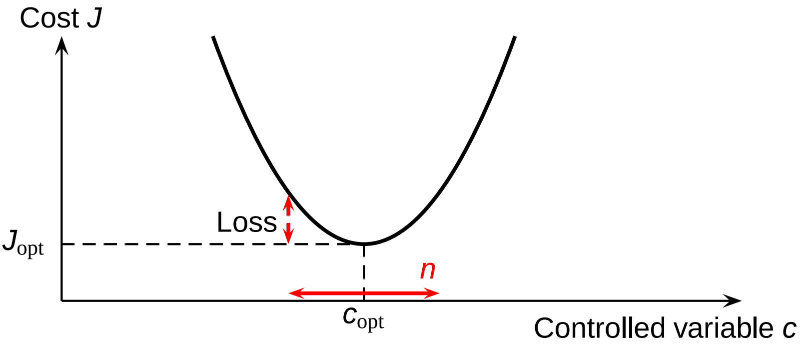

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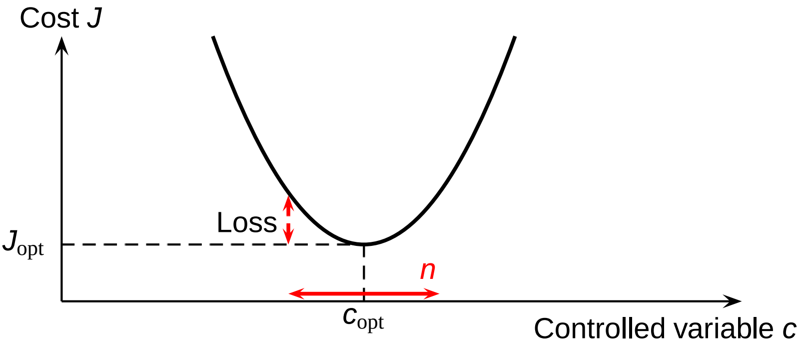

- Implementation error: $c = c_{\text{opt}} + n$.

Want to find invariants c to both disturbances and noise

Explicit formula for optimal H for $n \neq 0$

Loss = $J(u,d) - J_{\text{opt}}(d)$. Keep $c = Hy$ constant, where $y = G^y u + G_d^y d + n^y$

Theorem (Explicit formula for optimal H (Alstad et al, 2008))

Define $\tilde{F} = [FW_d \quad W_{n^y}]$. *Then*

$$H_{\text{opt}}^{\mathrm{T}} = (\tilde{F}\tilde{F}^{\mathrm{T}})^{-1} G^y \left((G^y)^{\mathrm{T}} (\tilde{F}\tilde{F}^{\mathrm{T}})^{-1} G^y \right)^{-1} J_{uu}^{1/2}$$

Here F is the optimal sensitivity matrix $F = \frac{\partial y_{\text{opt}}}{\partial d}$

Example 2: Output feedback with noise

Process

$$x_{k+1} = \begin{bmatrix} 0.73 & -0.09 \\ 0.17 & 0.99 \end{bmatrix} x_k + \begin{bmatrix} 0.060 \\ 0.006 \end{bmatrix} u_k + w_k$$

$$y_k = \begin{bmatrix} 0 & 1.41 \end{bmatrix} x_k + v_k$$

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Control

Alternative 1 $u_k = -Kx_k$ + Kalman filter

Alternative 2 $u_k = -K_y(y_k, y_{k-1}, y_{k-N})$ from "noisy nullspace method"

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Alternative 2

R, Q, W_{n^y}, W_d $\Rightarrow$ K_y

Example 2: Output feedback with noise

Simulated costs ($J = \frac{1}{N}\sum_{i=1}^{N} x_i^{\mathrm{T}} C^{\mathrm{T}} Q^y C x_i + u_i^{\mathrm{T}} R u_i$):

Control equation	
$u_k = -[6.08\ 6.07]x_k$ (perfect measurement)	
$u_k = -[6.08\ 6.07]\hat{x}_k$ (+ Kalman filter)*	
$u_k = -(3.25y_k)$	
$u_k = -(1.54y_k + 0.5y_{k-1})$	
$u_k = -(0.78y_k + 0.44y_{k-1} - 0.03y_{k-2})$	
$u_k = -(0.39y_k + 0.28y_{k-1} + 0.12y_{k-2} - 0.09y_{k-3})$	

*: Optimal for white noise signals

Example 2: Output feedback with noise

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Control equation	J_1
$u_k = -[6.08\ 6.07]x_k$ (perfect measurement)	2.86
$u_k = -[6.08\ 6.07]\hat{x}_k$ (+ Kalman filter)*	**3.40**
$u_k = -(3.25y_k)$	5.27
$u_k = -(1.54y_k + 0.5y_{k-1})$	3.88
$u_k = -(0.78y_k + 0.44y_{k-1} - 0.03y_{k-2})$	3.88
$u_k = -(0.39y_k + 0.28y_{k-1} + 0.12y_{k-2} - 0.09y_{k-3})$	4.11

J_1 Process noise at all time instants

*: Optimal for white noise signals

Example 2: Output feedback with noise

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Control equation	J_1	J_2
$u_k = -[6.08\ 6.07]x_k$ (perfect measurement)	2.86	0.284
$u_k = -[6.08\ 6.07]\hat{x}_k$ (+ Kalman filter)*	**3.40**	0.400
$u_k = -(3.25 y_k)$	5.27	0.569
$u_k = -(1.54 y_k + 0.5 y_{k-1})$	3.88	0.401
$u_k = -(0.78 y_k + 0.44 y_{k-1} - 0.03 y_{k-2})$	3.88	**0.394**
$u_k = -(0.39 y_k + 0.28 y_{k-1} + 0.12 y_{k-2} - 0.09 y_{k-3})$	4.11	0.416

J_1 Process noise at all time instants

J_2 Process noise at every 10$^{\text{th}}$ instant

*: Optimal for white noise signals

Current and future work

- Include measurement error in explicit MPC (with region switching)
- Explicit expressions for fixed low-order controllers, e.g. MIMO-PID

Conclusion

- MPC: Quadratic optimization problem
- Self-optimizing control: Exact results for QP's, both noise-free and with noisy measurements
- Link: $c = u - Kx$
- New results:
  - c's for region switching
  - Output feedback $c = u - K^y y$
  - Optimal invariants for *noisy* measurements