A New Approach to Explicit MPC Using Self-Optimizing Control

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Outline

1. Optimal operation paradigms
2. Self optimizing control
3. Explicit MPC
4. Link between the two
5. Examples
6. Extensions

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Implementation of optimal operation using off-line computations [Narasimhan and Skogestad(2007)]

Paradigm 1

On-line optimizing control where measurements are primarily used to update the model. With arrival of new measurements, the optimization problem is resolved for the inputs.

Paradigm 2

Pre-computed solutions based on off-line optimization. Typically, the measurements are used to (indirectly) update the inputs using feedback control schemes. Focus of this work.

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On-line optimizing control where measurements are primarily used to update the model. With arrival of new measurements, the optimization problem is resolved for the inputs.

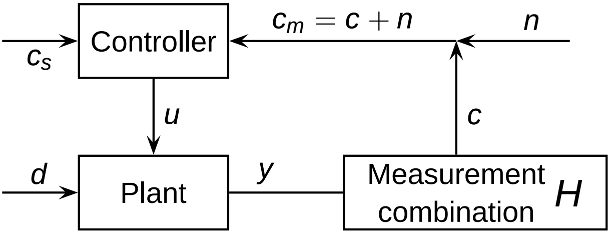
Example: Classical (implicit) MPC.

Paradigm 2

Pre-computed solutions based on off-line optimization. Typically, the measurements are used to (indirectly) update the inputs using feedback control schemes. Focus of this work.

Examples: Explicit MPC and self-optimizing control.

What variables should we control?



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Choice of H such that acceptable operation is achieved with constant setpoints (c_s constant).

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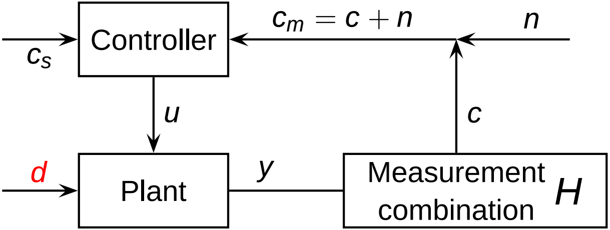

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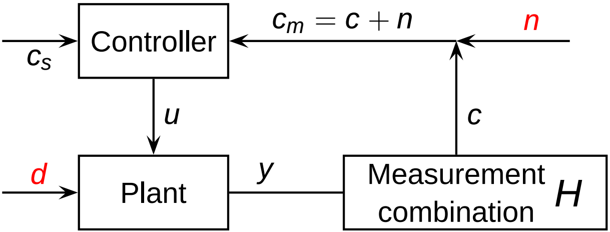

Self-optimizing control

Choice of H such that acceptable operation is achieved with constant setpoints (c_s constant).

- Optimal c_s is invariant with respect to disturbances d
- Insensitive to measurement errors n

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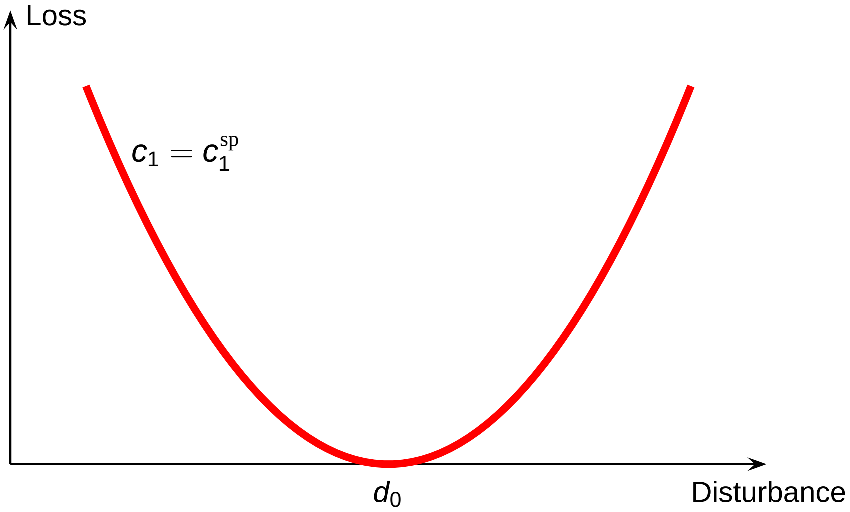

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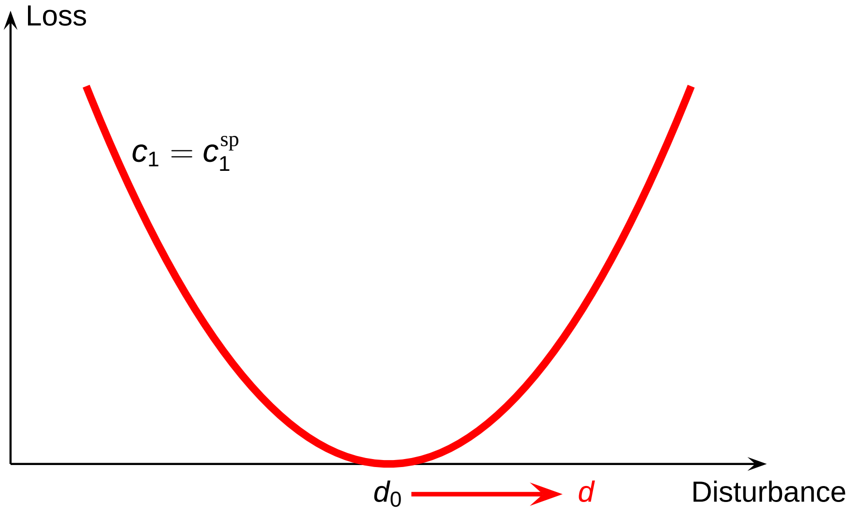

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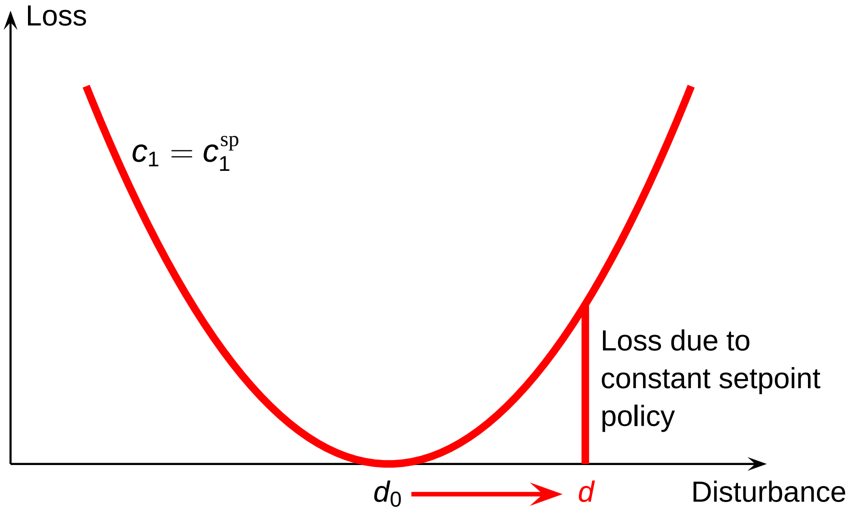

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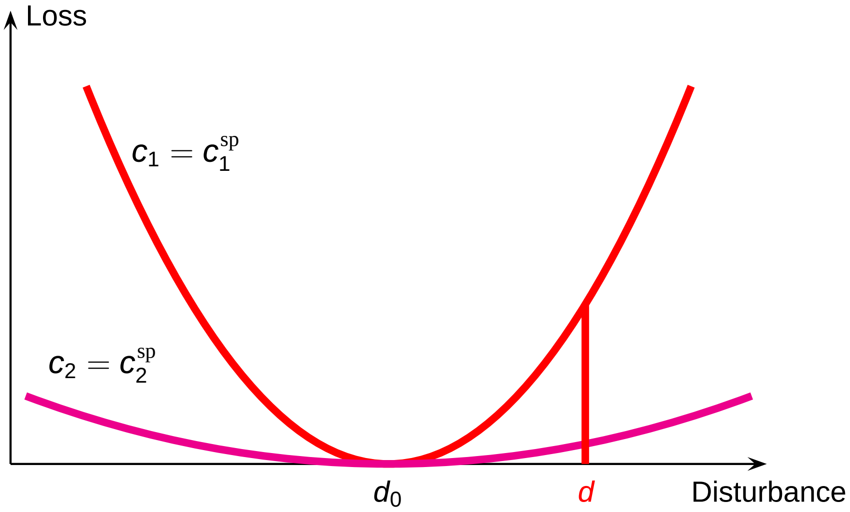

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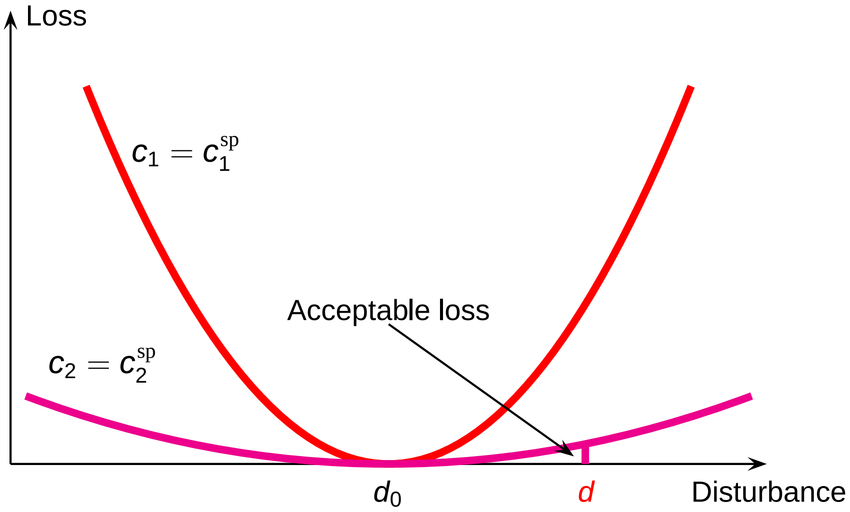

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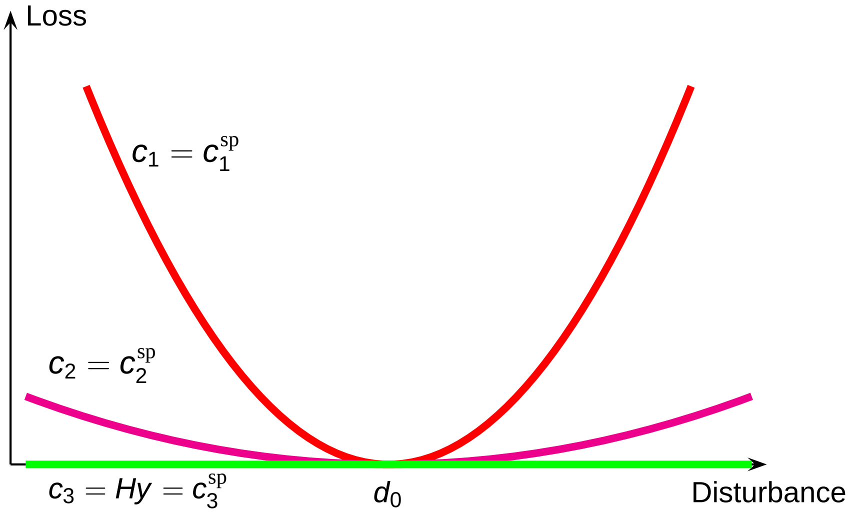


Nullspace method for QP problems

The nullspace method in [Alstad and Skogestad(2007)] is restated for QP's

Theorem (Nullspace method for QP)

- *Consider the quadratic problem*

$$\min_u J = \begin{bmatrix} u & d \end{bmatrix} \begin{bmatrix} J_{uu} & J_{ud} \\ J_{ud}^{\mathrm{T}} & J_{dd} \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix} \tag{1}$$

  If there exists $n_y \geq n_u + n_d$ independent measurements, then the optimal solution to (1) has the property that there exists variable combinations $c = Hy$ that are invariant to the disturbances d.

- *H may be found from $HF = 0$, where $F = \frac{\partial y^{opt}}{\partial d^{\mathrm{T}}}$*

Classical (implicit) MPC

For a given $x(t)$, one solves the quadratic problem

$$\min_{U} J(U, x(t)) = x_{t+N_y}^{\mathrm{T}} P x_{t+N_y} + \sum_{k=0}^{N_y-1} \left[x_{t+k}^{\mathrm{T}} Q x_{t+k} + u_{t+k}^{\mathrm{T}} R u_{t+k} \right]$$

subject to

- Output and input constraints.
- Discrete model

$$\begin{aligned}
x_t &= x(t) & \\
x_{t+k+1} &= A x_{t+k} + B u_{t+k}, & k \geq 0 \\
y_{t+k} &= C x_{t+k}, & k \geq 0 \\
u_{t+k} &= K x_{t+k}, & N_u \leq k \leq N_y
\end{aligned}$$

Explicit MPC [Bemporad et al.(2002)]

- The “classical” MPC problem can, by substitution, be written as a quadratic problem:

$$\min_{U} J(U, x(t)) = \begin{bmatrix} U^{\mathrm{T}} & x(t)^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} H & F \\ H & Y \end{bmatrix} \begin{bmatrix} U \\ x(t) \end{bmatrix}$$

$$\text{s.t. } GU \leq W + Ex(t)$$

- The initial state $x(t)$ is considered to be a parameter and a parametric program is solved.
- The solution of the parametric program gives regions in the state space.
- Given an algorithm for deciding the current region (i), one implements a continuous piece-wise affine control law

$$u = F^i x + g^i.$$

Link between explicit MPC and self-optimizing control

Let

$$d = x_0 \quad \text{and} \quad y = \begin{bmatrix} u \\ x \end{bmatrix}$$

The optimal combination

$$c = Hy$$

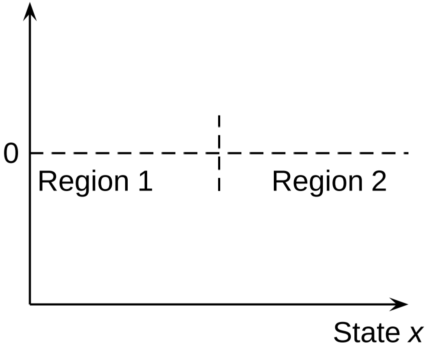
can be written as the feedback law

$$c = u - (Kx + g)$$

and H (or K) can be obtained from nullspace method

When to switch region?

- The invariants can be used to track region changes
- By monitoring neighboring regions we switch regions when $c_i - c_j$ changes sign



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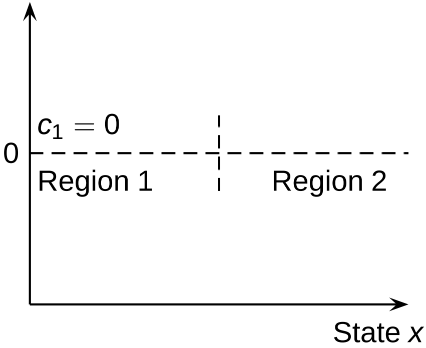

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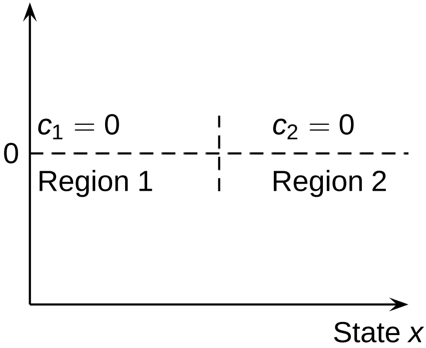

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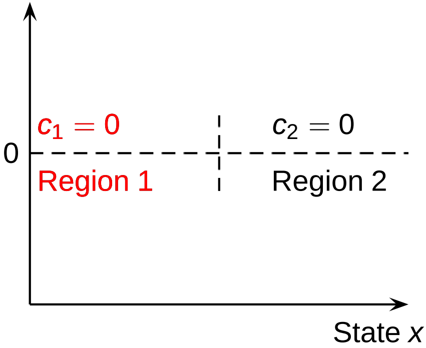

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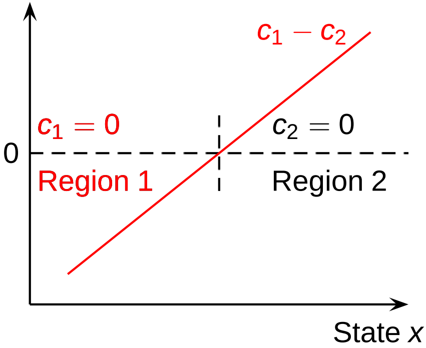

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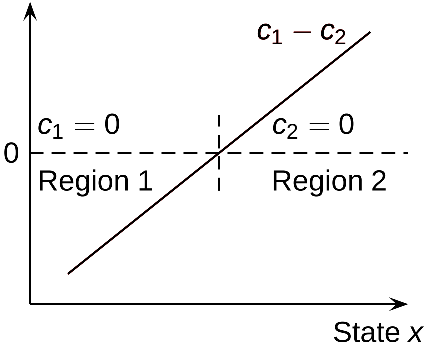

Examples

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$$x_{k+1} = Ax_k + Bu_k$$

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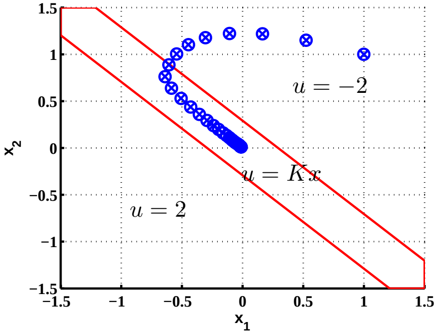
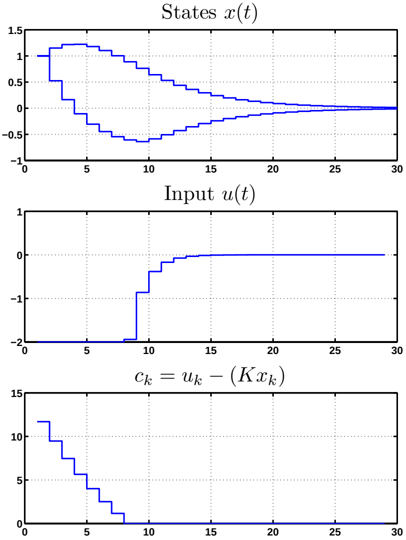
Example 1: Simple SISO system

SISO with input constraint.



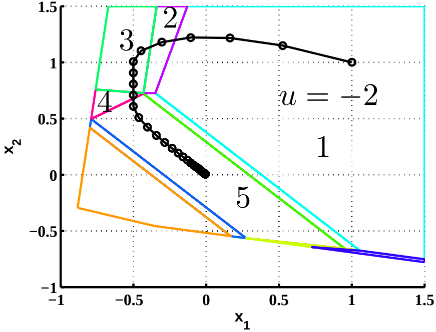
State space partition and simulation from $x_0 = (1, 1)$.



Example 2: Same system with state constraint

Additional constraint

$x_{k+1,k+2} \geq -0.5.$



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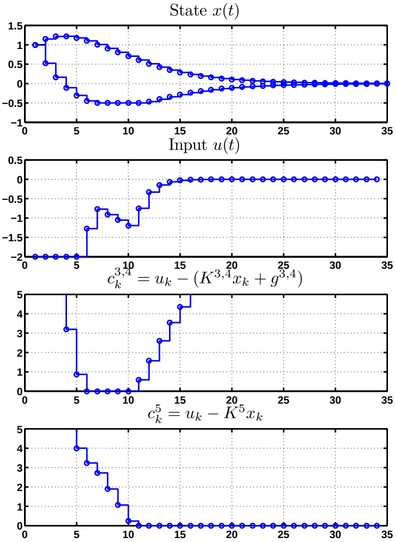

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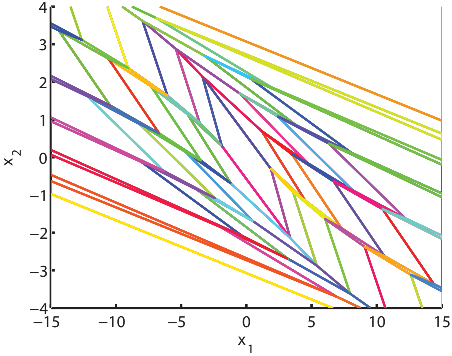
- We merge all regions were the first input is the same.



Bemporad et al.: 57 regions (from 73)

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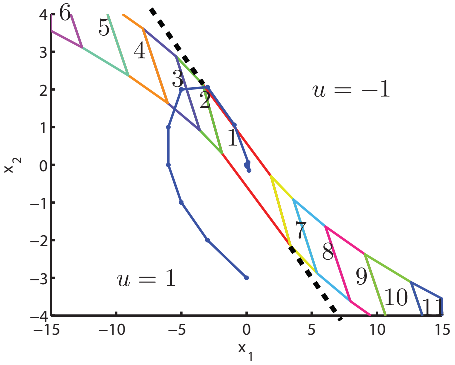

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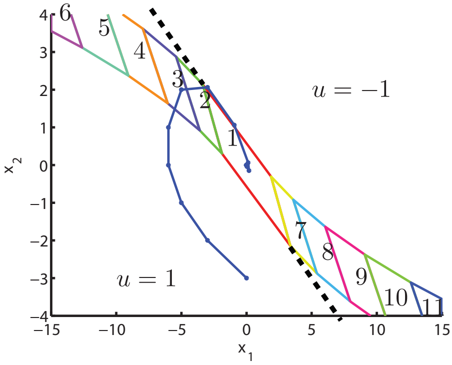

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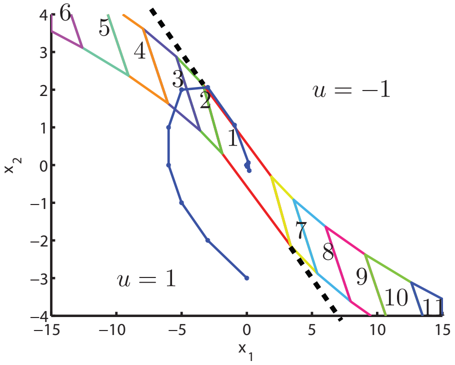
- . . . taking directions into account the “relevant” boundaries form a convex polyhedron.



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Summary of the examples

Examples 1,2

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Example 3: Double integrator

- Problem: Not all merged regions were convex.
- Fix: Used the direction of the process and observed that the relevant boundaries make the regions convex.
- Storage
  - Original partition: 73 regions
  - Bemporad et. al., 2002: need to store 57 regions.
  - Our approach: 11 regions/boundaries needs to be stored.

Extensions

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2. Extend the results to output feedback ($u = Ky$).
3. Explicit expressions for fixed low-order controllers, e.g., MIMO-PID

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See IFAC World Congress, Seoul, July 2008

Conclusion

- MPC: Quadratic optimization problem at each time sample
- From link to self-optimizing control: Must exist invariants $c = Hy$
- Correspond to feedback law: $u = Kx + g$
- Gives new insights
  - Simple feedback solution to MPC must exist (explicit MPC)
  - Region changes identified by tracking variables c from neighboring regions
  - Low-order output feedback, etc.

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