Reply to "Further Theoretical Results on 'Relative Gain Array for Norm-Bonded Uncertain Systems'"

Vinay Kariwala*

Division of Chemical & Biomolecular Engineering, Nanyang Technological University, Singapore 637459

Sigurd Skogestad

Department of Chemical Engineering, Norwegian University of Science and Technology, N-7491, Trondheim, Norway

J. Fraser Forbes

Department of Chemical & Materials Engineering, University of Alberta, Edmonton, Canada T6G 2G6

Sir: In their note, Moaveni and Sedigh[1] noted the following limitations of our earlier results:[2]

(1) The computation of worst-case relative gain for large-scale or complex systems is computationally costly.

(2) The sensitivity problem is not solved, because sufficiently large relative gains result in closed-loop sensitivity issues.

(3) Finally, it cannot detect the appropriate input−output pair when there is no sign change in relative gains and most of them are positive.

While we agree with the first comment, we believe that the latter two comments have arisen because of misinterpretation of our results. Our detailed responses to each of these comments are presented below, in order.

Comment 1: The computation of the magnitude of the worst-case relative gain for an uncertain system requires solving a skewed structured singular value (skewed-μ) problem. Because the exact computation of μ is NP-hard, usually the upper bound on μ is used as an approximation. The upper bound on μ can be computed by solving a linear matrix inequality problem. Thus, the time required to determine the magnitude of the worst-case relative gain increases polynomially with the problem dimensions. For large-scale systems, however, the solution time can still be large.

To alleviate this problem, Moaveni and Sedigh[1] have proposed an alternate upper bound using singular value inequalities. Although easier to compute in some cases, their proposed upper bound suffers from at least three limitations:

(1) The bound can only be used when $\underline{\sigma}(\mathbf{G}) > \bar{\sigma}(\mathbf{\Delta})$. In most cases, the relative gain for the uncertain system is well-defined, even if this condition is violated.

(2) The bound does not take the structure of uncertainty into account and, thus, can be highly conservative. For example, consider the Wood−Berry column,[3] with the uncertainty in the gain matrix being $\mathbf{\Delta} = \alpha\mathbf{G}$ (simultaneous perturbations in every element of $\mathbf{G}$). For this uncertainty description, relative gains do not change with α, which is correctly predicted by the skewed-μ approach.[2] In comparison, the bound proposed by Moaveni and Sedigh[1] suggests that the variation in the relative gain between output y_1 and input u_1 is 0.075, 0.809, and 29.708 for $\alpha = 0.001$, 0.01, and 0.1, respectively, demonstrating the conservatism.

(3) We note that the bound that has been proposed by Moaveni and Sedigh[1] monotonically increases with $\bar{\sigma}(\mathbf{\Delta})$. This implies that the bound on the relative gain can be computed using the uncertainty element with largest $\bar{\sigma}(\mathbf{\Delta})$. In most process control applications, however, only the bounds on the individual elements of $\mathbf{G}$ are available. When every element of $\mathbf{\Delta}$ varies independently between the specified lower and upper bounds, finding $\bar{\sigma}(\mathbf{\Delta})$ itself can be difficult. Thus, the proposed bound can be used for a handful of cases, where either $\bar{\sigma}(\mathbf{\Delta})$ is known a priori or can be computed easily, e.g., for simultaneous perturbations in each element of $\mathbf{G}$.

In summary, although the skewed-μ can be computationally demanding for large-scale systems, the approach of Moaveni and Sedigh[1] fails to provide any reasonable solution to this problem.

Comment 2: Using the skewed-μ approach, the magnitude of the worst-case relative gain can be computed as tightly as possible. As we noted earlier,[2] large worst-case relative gains imply ill-conditioning (sensitivity to input direction) and poor controllability for the uncertain system. It seems to us that Moaveni and Sedigh[1] have misunderstood or at least misinterpreted our results.

Comment 3: For the nominal system, positive relative gains evaluated at steady state are necessary to ensure integrity against loop failures. This necessary condition clearly also carries over to uncertain systems, where the relative gains are required to be positive for every element of the uncertainty set. As was conclusively shown earlier,[2] the relative gain between output y_i and input u_j remain positive over all the elements of the uncertainty set, if the relative gain for the nominal gain matrix is positive and $\mathbf{G}_{ij}$, $\mathbf{G}^{ij}$, and $\mathbf{G}$ remain nonsingular over the uncertainty set. This result implies that, after the nonsingularity of $\mathbf{G}$ and its principal submatrices is established, checking the sign change of relative gain over the uncertainty set is redundant. Thus, computation of bounds on relative gains and checking the overlap between variation bounds is entirely unnecessary to assess the integrity of the uncertain system.

Literature Cited

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* To whom correspondence should be addressed. Tel.: +65-6316-8746. Fax: +47-6794-7553. E-mail address: vinay@ntu.edu.sg.

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