Optimal operation of simple refrigeration cycles Part I: Degrees of freedom and optimality of sub-cooling

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Abstract

The paper focuses on the operation of simple refrigeration cycles. With equipment given, there are, from a control or operational point of view, five steady state degrees of freedom; the compressor power, the heat transfer in the condenser and evaporator, the choke valve opening and the active charge in the cycle. With a given load (e.g. given cooling duty) the compressor power is set. Furthermore, it is usually optimal to maximize the heat transfer. The two remaining degrees of freedom (choke valve and active charge) may be used to set the degree of super-heating and sub-cooling. It is found that super-heating should be minimized, but sub-cooling is found to be optimal. For a simple ammonia cycle, savings in compressor power are about 2 %. In this paper, refrigeration (cooling) cycles are considered, but the same principles apply to heat pumps.

Key words: Operation, self-optimizing control, vapour compression cycle

1 Introduction

Cyclic processes for heating and cooling are widely used and their power ranges from less than 1 kW to above 100 MW. In both cases vapour compression cycle is used to “pump” energy from a low to a high temperature level.

The first application, in 1834, was to produce ice for storage of food, which led to the refrigerator found in most homes (Nagengast, 1976). Another well-known system is the air-conditioner (A/C). In colder regions a cycle operating in the opposite

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direction, the "heat pump", has recently become popular. These two applications have also merged together to give a system able to operate in both heating and cooling mode.

In Figure 1 a schematic drawing of a simple cycle is shown together with a typical pressure-enthalpy diagram for a sub-critical cycle. The cycle works as follows:

The low pressure vapour (4) is compressed by supplying work W_s to give a high pressure vapour with high temperature (1). The vapour is cooled to its saturation temperature in the first part of the condenser, condensed in the middle part and possibly sub-cooled in the last part to give the liquid (2). In the choke valve, the pressure is lowered to its original value, resulting in a two-phase mixture (3). This mixture is vaporized and possibly super-heated in the evaporator (4) closing the cycle.

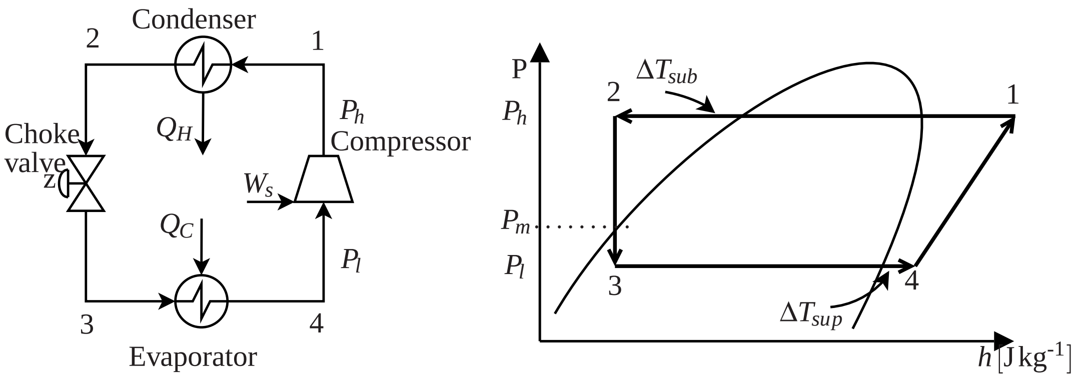


Fig. 1. Simple refrigeration or heat pump cycle with typical pressure-enthalpy diagram indicating both sub-cooling and super-heating

The choke valve may be replaced by an expander for improved efficiency, but this is not considered here. The coefficient of performance for a refrigeration cycle (refrigerator, A/C) is defined as

$$COP = \frac{Q_c}{W_s} = \frac{\dot{m}(h_4 - h_3)}{\dot{m}(h_1 - h_4)} \tag{1}$$

The COP is typically around 3 which indicates that 33% of the heat duty is added as work (e.g. electric power).

In this paper, the objective is to optimize the operation of a given cycle (Figure 1) in terms of maximize the COP, or specifically to minimize the compressor power W_s for a given cooling load Q_c. We consider only steady state operation. The model equations are summarized in Table 1. Note that pressure losses in piping and equipment are neglected. We also assume that the temperature of the hot (T_H) and cold (T_C) source are constant throughout the heat exchanger. This assumption holds for a cross flow heat exchanger. In practice, there may be some operational constraints,

Table 1
Structure of model equations

Heat exchangers (condenser and evaporator)
$Q = U \cdot \int \Delta T \, \mathrm{d}A = \dot{m} \cdot (h_{out} - h_{in})$
$P = P_{sat}(T_{sat})$
$m = \rho / V$
Valve
$\dot{m} = z \cdot C_V \sqrt{\Delta P \cdot \rho} \qquad h_{out} = h_{in}$
Compressor
$W_s = \dot{m}(h_{out} - h_{in}) = \dot{m} \cdot (h_s - h_{in}) / \eta$

for example, maximum and minimum pressure constraints, which are not considered here.

In industrial processes, especially in cryogenic processes such as air separation and liquefaction of natural gas (LNG process), more complex refrigeration cycles are used in order to improve the thermodynamic efficiencies. These modifications lower the temperature differences in the heat exchangers and include cycles with mixed refrigerants, several pressure levels and cascaded cycles. Our long term objective is to study the operation of such processes. However, as a start we need to understand the simple cycle in Figure 1.

An important result from this study is the degree of freedom analysis given in Section 2. We find that the "active" charge plays an important role in operation of cyclic processes. This is also directly applicable to more complex designs. Unlike an open process, a closed cyclic process does not have boundary conditions on pressures imposed by the flows in and out of the system. Instead the pressure level is indirectly given by the external temperatures, heat exchanger sizes, load and the active charge. *The active charge is defined as the total mass accumulated in the process equipment in the cycle, mainly in the condenser and evaporator, but excluding any adjustable mass in liquid receivers (tanks).*

The effect of a change in active charge on operation depends on the specific design. Intuitively, it seems that an increase in active charge must increase the pressure, and indeed this is true in most cases. For example, this is the case for the models used in this paper with plug-flow in the heat exchangers. Then more liquid in the condenser gives more sub-cooling which, effectively reduces cooling and pressure increases. Similarly more liquid in the evaporator gives less super-heating effectively increasing heat transfer and pressure increases. However, there may be designs where the effect of charge on pressure is opposite. For example, consider a well-mixed flooded condenser where the heat transfer coefficient U to liquid is larger than to vapour. An increase in charge (liquid) may then improve cooling and pressure decreases. In any case, the main point is that the “active” charge is a degree of freedom that affects the operation of the system, and this paper focuses on how to use it effectively.

Although there is a vast literature on the thermodynamic analysis of refrigeration cycles, there are very few authors who discuss their operation and control. Some discussions are found in text books such as Stoecker (1998), Langley (2002) and Dossat (2002), but these mainly deal with more practical aspects. Svensson (1994) and Larsen et al. (2003) discuss operational aspects. A more comprehensive recent study is that of Kim et al. (2004) who consider the operation of trans-critical CO_2 cycles. They discuss the effect of “active charge” and consider alternatives for placing the receiver.

The paper also discuss super-heating and sub-cooling. In the literature, it is generally taken for granted that there for a given cycle should be no sub-cooling and super-heating ($\Delta T_{sub} = 0\,^{\circ}$C and $\Delta T_{sup} = 0\,^{\circ}$C) in optimal operation. For example, Stoecker (1998, page 57) states that

> *The refrigerant leaving industrial refrigeration condensers may be slightly sub-cooled, but sub-cooling is not normally desired since it indicates that some of the heat transfer surface that should be be used for condensation is used for sub-cooling. At the outlet of the evaporator it is crucial for protection of the compressor that there be no liquid, so to be safe it is preferable for the vapor to be slightly super-heated.*

In this study, we confirm that super-heating is not optimal. The issue of sub-cooling is less clear. Of course, sub-cooling in itself is always optimal, as less refrigerant needs to be circulated. The issue is whether sub-cooling is optimal for a given cold source temperature and a given condenser area, because sub-cooling will reduce the temperature driving forces which must be compensated by increasing the pressure. We find, contrary to popular belief, that with given equipment, sub-cooling in the condenser may give savings in energy usage (compressor power) in the order of 2%. An ammonia case study is presented to obtain numerical results.

2 Degrees of freedom in simple cycles

2.1 Design versus operation

Table 2 shows typical specifications for the simple refrigeration cycle in Figure 1 in design (find equipment) and in operation (given equipment). The <u>five</u> design specifications include the load, the two pressures, and the degree of sub-cooling and super-heating. Based on these five design specifications, external conditions and an assumed isentropic efficiency for the compression, we may obtain the following <u>four</u> equipment parameters which can be adjusted during operation: compression work (W_s) valve opening (z) and effective heat transfer (including UA-values) for the two heat exchangers. Initially, we were puzzled because we could not identify the missing fifth equipment parameter to be adjusted during operation. However, we finally realized that we can manipulate the ”active charge” in the cycle, which affects the operation. The fact that the charge is an independent variable is unique for closed systems since there is no (external) boundary condition for pressure which would otherwise set the active charge.

Table 2
Typical specifications in design and operation

	Given	#
Design	Load (e.g. Q_h), P_l, P_h, ΔT_{sup} and ΔT_{sub}	5
Operation	W_s (load), choke valve opening (z), effective heat transfer (e.g. UA) in two heat exchangers and active charge	5

2.2 Active charge and holdup tanks

For the simple cycle in Figure 1 we have the following overall material balance:

$$m_{tot} = \underbrace{m_{\text{evap}} + m_{\text{con}} + m_{\text{valve}} + m_{\text{comp}}}_{m_{\text{active}}} + m_{\text{tanks}} \tag{2}$$

Normally the holdups in the valve and compressor are neglected and we get:

$$m_{tot} = \underbrace{m_{\text{evap}} + m_{\text{con}}}_{m_{\text{active}}} + m_{\text{tanks}} \tag{3}$$

With no filling, emptying or leaks, the total mass m_{tot} is fixed. We have not included a holdup tank in Figure 1, but in practice it is common to include a tank or receiver with variable liquid mass. It is assumed that a change in m_{tanks} (e.g. by filling or leaking) with a constant active charge (m_{active}) does not affect the operation of the cycle. This implies that the tank must contain both liquid and gas in equilibrium (saturated). Then we can move mass to or from the tank without affecting the pressure, and thus without affecting the rest of the cycle. Thus the liquid tank makes operation independent of the total charge in the system.

More importantly, the extra tank introduces an additional degree of freedom. This can be seen from Equation 3: With m_{tot} constant, we can by changing the mass (liquid) in the tank (m_{tank}), change the active charge (m_{active}). This shows that m_{tank}

has an indirect steady state effect on the active charge, and can therefore be used for control purposes, of course provided that we have means of changing it.

Although it is possible to introduce several tanks in a cycle, we only have one material balance for each cycle, so from Equation 3 this will not add any steady-state degrees of freedom with respect to the active charge.

Rule 1 *In each closed cycle, we have one degree of freedom related to the active charge, which may be indirectly adjusted by introducing a variable liquid level (tank; receiver) in the cycle.*

Rule 2 *In each closed cycle, there will be one liquid holdup that does not need to be explicitly controlled, because the total mass is fixed. This is usually selected as the largest liquid volume in the closed system. The remaining liquid levels (holdups) must be controlled (to avoid overfilling or emptying of tanks).*

Remark 1 Note that in Rule 2 it says "does not need" rather than "must not". Thus, Rule 2 does not say that we cannot control all the liquid volumes in the system (including the largest one), but it just states that it is not strictly necessary. In fact, controlling all the liquid volumes, provides a way for explicitly controlling the active charge in the cycle (Rule 1).

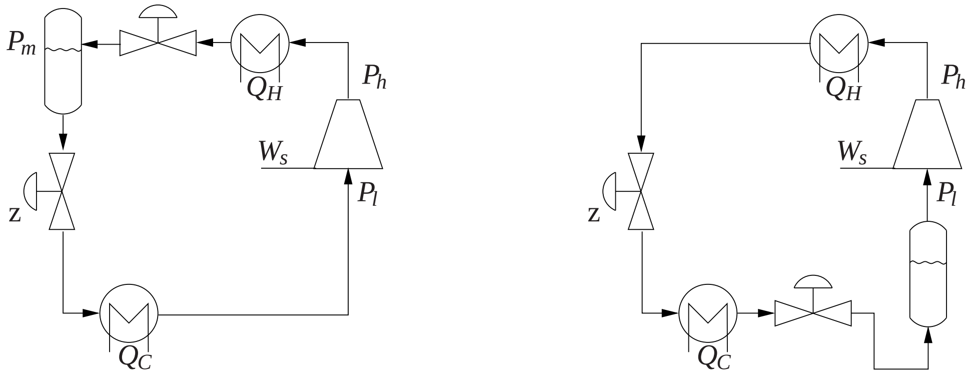
Remark 2 Introducing additional liquid tanks may be useful for operation, but at least for pure fluids, these will not introduce any additional steady-state degrees of freedom because we can move mass from one tank to another without affecting operation. Also, to avoid that tanks fill up or empty, these additional levels must be controlled (Rule 2), either by self-regulation or feedback control.

Remark 3 In *mixed refrigerant* cycles two tanks may be used to indirectly change the composition of the circulating refrigerant. In this case the two tanks have different composition so moving mass from one tank to another does affect operation. This is utilized in the auto-cascade process (Neeraas et al. (2001)). For more complex cycles the maximum number of degrees of freedom related to tank holdups is the number of components in the refrigerant.

2.2.1 Adjusting the active charge

In order to freely adjust the active charge, we need to introduce a liquid tank (receiver) plus an extra valve. Kim et al. (2004) discuss alternative locations for the variable tank holdup (liquid receiver). In Figure 2, we show cycles for the two main cases where the tank is placed (a) on the high pressure side after the condenser and (b) on the low pressure side after the evaporator. Other placements and combinations are possible, but these are only variations of these two and will not add any steady-state degrees of freedom for pure refrigerants.

The most obvious way of introducing a means for adjusting the tank holdup is to add an extra valve before the tank as shown in Figure 2. In Figure 2(a), the liquid tank is located at an intermediate pressure P_m after the condenser. Since the extra valve is on the "same side" as the expansion valve (choke), the pressure drop over



(a) Liquid tank and extra valve on high pressure side

(b) Liquid tank and extra (non-optimal) valve on low pressure side

Fig. 2. Simple cycle with variable active charge

the extra valve will not effect the efficiency of the cycle. Since P_m is assumed to be the saturation pressure at the tank temperature, the exit stream from the condenser must be sub-cooled. Thus, in Figure 2(a), the pressure drop across the valve may be used to adjust the degree of sub-cooling in the condenser. To understand how the extra valve creates sub-cooling, consider the pressure-enthalpy diagram in Figure 1. The receiver (tank) with saturated liquid operates at saturation pressure P_m, and the pressure drop for the extra valve introduces a pressure drop $P_h - P_m$. As seen from Figure 1, the corresponding operating point 2 at the exit of the condenser must then be at a sub-cooled state.

Another possibility is to place the tank after the evaporator, as shown in Figure 2(b). With this design the stream exiting the evaporator is not fully evaporated and by lowering the pressure through the extra valve the vapour exiting the valve becomes saturated (see pressure-enthalpy diagram). However, in this case the valve introduces a pressure drop that must be compensated by increasing the compression power, so a valve here is generally not optimal.

A low pressure tank may not be desirable from a practical point of view, since the vapour velocity will be highest at this point in the cycle and the extra equipment will increase the pressure drop.

2.2.2 Extra valve removed

An extra valve is generally required to freely adjust the active charge. However, in many practical cases the extra valve in Figure 2(a) and 2(b) is removed. What effect does this have?

- High pressure tank without valve. Without the valve we have at steady state the same thermodynamic state at the exit of the condenser as at the exit from the tank.

Thus, the exiting stream from the condenser will be saturated liquid. The most common design is shown in Figure 3, where the tank and condenser are merged together so that the saturated liquid from the condenser drains into the receiver. As we will show, this is not generally optimal. Thus, in this design we have used a degree of freedom ("fully open valve") to set the degree of sub-cooling to zero (not optimal).

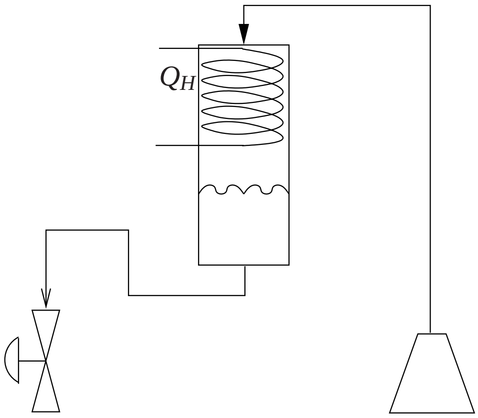


Fig. 3. Condenser with saturation at outlet (non-optimal)

- Low pressure tank without valve (Figure 4(a)). This gives saturated vapour to the compressor. Fortunately, this is generally optimal for the cycle as a whole, because the inlet temperature to the compressor should be as low as possible to minimize vapour volume and save compression power. Thus, in this design we have used a degree of freedom ("fully open valve") to set the degree of superheating to zero (optimal). Two designs are shown in Figure 4(a), one with a separate receiver and one using a flooded evaporator. The designs are equivalent thermodynamically, but the heat transfer coefficient and pressure drop will be different.

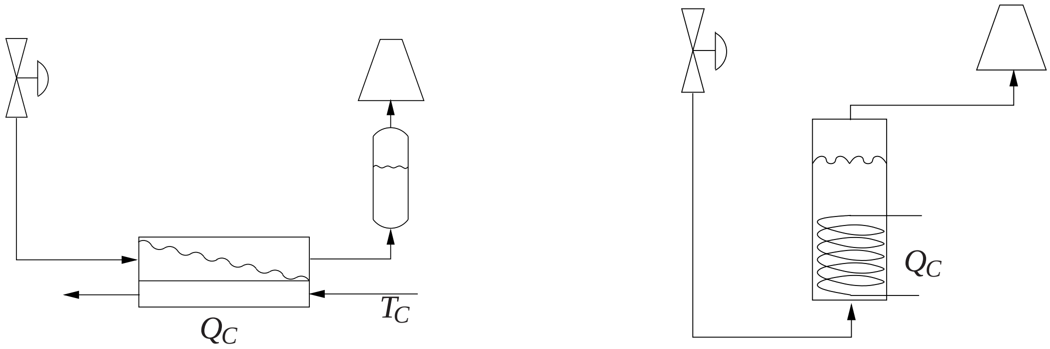
In summary, removing the valve gives saturation at the exit of the heat exchanger. In the case of high-pressure liquid tank we get a sub-optimal design if we remove the valve, whereas for the low-pressure tank we get an optimal design if the extra valve is removed.

2.3 Degrees of freedom for operation

In summary, we have the following five operational or control degrees of freedom for a simple refrigeration cycle (Figure 1):

1 Compressor power W_s. We assume here that it is used to set the "load" for the cycle.

2, 3 Effective heat transfer. There are two degrees of freedom related to adjusting the heat transferred in the condenser and evaporator. This may be done in many



(a) With separate receiver (b) Flooded evaporator

Fig. 4. Evaporator with saturation at outlet (optimal)

ways, for example, by introducing bypasses, changing the flowrates of coolant or using a flooded condenser or evaporator to change the effective UA-value. However, we generally find that it is optimal to maximize the effective heat transfer in the condenser and evaporator. There are exceptions where it may not be optimal to maximize the heat transfer in the condenser and evaporator, for example because, of costs related to pumps, fans or coolants, but these degrees of freedom are not considered in the following.
4 Choke valve opening (z)
5 Active charge (see Section 2.2)

In practice, we are then with a given load and maximum heat transfer left with two steady state degrees of freedom. These are the choke valve opening (z) and the active charge (m_{active}). These may be used to set the degree of super-heating and degree of sub-cooling. The pressure levels (P_h and P_l) are indirectly determined by the given (maximum) value of the heat transfer.

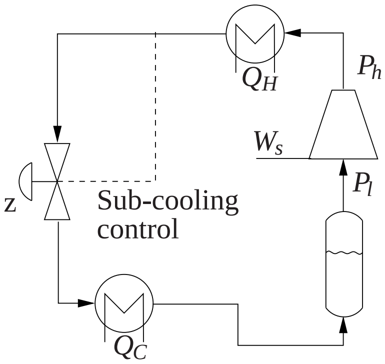
3 Discussion of some designs

As discussed in more detail in Section 4, we find that the thermodynamic efficiency is optimized by having no super-heating and some sub-cooling. With this in mind, we next discuss some alternative designs.

3.1 Optimal designs

Two potentially optimal designs are shown in Figure 5. The reason we say “potentially optimal” is because they will only be optimal if we use the optimal value for the sub-cooling and super-heating.

To avoid super-heating, we have in Figure 5(a) and 5(b) a low-pressure tank (re-



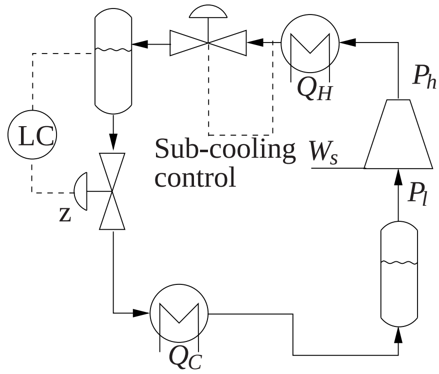


Fig. 5. Two potentially optimal designs that allow for sub-cooling

ceiver) after the evaporator. This tank will give saturated vapour out of the evaporator at steady state (optimal), and also by trapping the liquid it will avoid that we get liquid to the compressor during transient operation. To avoid super-heating we must have vapour-liquid equilibrium in the tank. This may be achieved by letting the vapour bubble through the tank. An alternative design is the flooded evaporator in Figure 4(b).

At the high-pressure side, we show optimal designs with both (a) no receiver and (b) a receiver and an extra valve. In (a) the choke is used to control the degree of sub-cooling (ΔT_{sub}). Also other control policies are possible, for example, keeping the choke valve position at its optimal value or controlling the pressure, but controlling ΔT_{sub} was found by Jensen and Skogestad (2005) to be a good self-optimizing controlled variable.

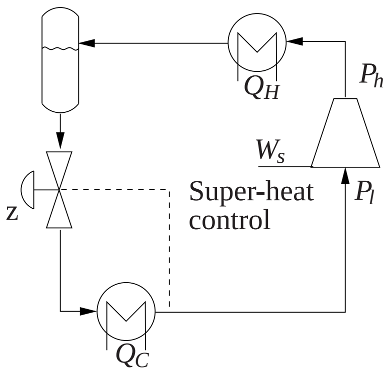
The design in Figure 5(b) is thermodynamically equivalent to Figure 5(a), but the addition of the tank may prevent that we get two-phase flow with vapour “blow out” through the choke. We here have two adjustable holdups, so from Rule 2 one of them must be controlled. In Figure 5(b) is shown the case where the choke valve is used to control the level in the high pressure tank, but alternatively it could control the level in the low pressure tank.

3.2 Non-optimal designs

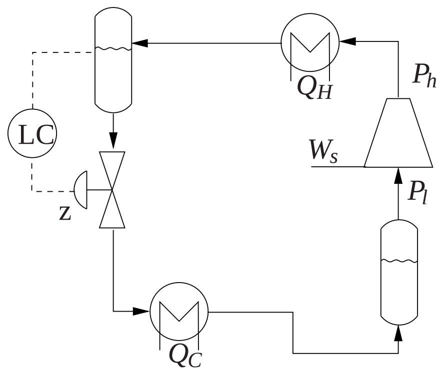
Three non-optimal designs are shown in Figure 6. Figure 6(a) shows the design used in most applications except that the tank and condenser are often integrated as shown in Figure 3. This common design has two errors compared to the optimal solution: 1) There is no sub-cooling in the condenser and 2) there is super-heating in the evaporator. The super-heat control is in practice accomplished with a thermostatic expansion valve (TEV). In theory, one could get optimality by setting the setpoint for super-heating to zero, but in practice this is not possible because this could give liquid out of the evaporator. The setpoint for super-heating is typically about 10 °C.

In Figure 6(b) we have two liquid tanks, one after the evaporator and one after the condenser. This design is better since there is no super-heating in the evaporator, but one error remains: There is no sub-cooling in the condenser. Note that we need to control one of the liquid levels in accordance with Rule 2.

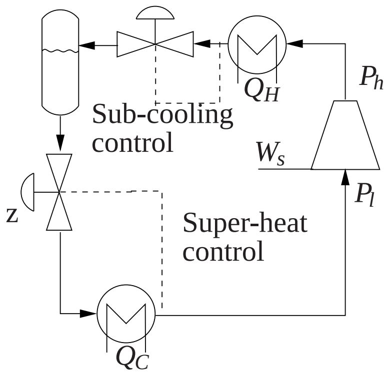
Another non-optimal design is shown in Figure 6(c). Here we have introduced the possibility for sub-cooling, but we have super-heating which is generally not optimal.



(a) Non-optimal 1. This design has two errors: 1) No sub-cooling and 2) Super-heating



(b) Non-optimal 2. This design has one error: No sub-cooling



(c) Non-optimal 3, This design has one error: Super-heating

Fig. 6. Three non-optimal designs

4 Optimality of sub-cooling

We have several times made the claim that sub-cooling may be optimal. To justify this somewhat controversial claim, we start by considering a specific example.

4.1 Ammonia case study

The objective is to cool a storage building by removing heat (Q_C) as illustrated in Figure 7. The cycle operates between a cold medium of air inside the building ($T_C = T_{\mathrm{room}}$) and hot medium of ambient air ($T_H = T_{\mathrm{amb}}$). The steady state heat loss from the building is 20 kW and the cooling load Q_C is indirectly adjusted by the temperature controller which adjusts the compressor work (W_s) to maintain $T_C = T_C^s$.

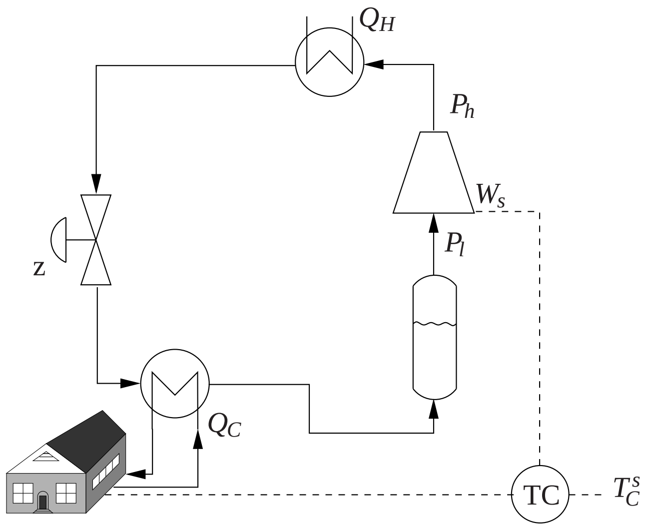


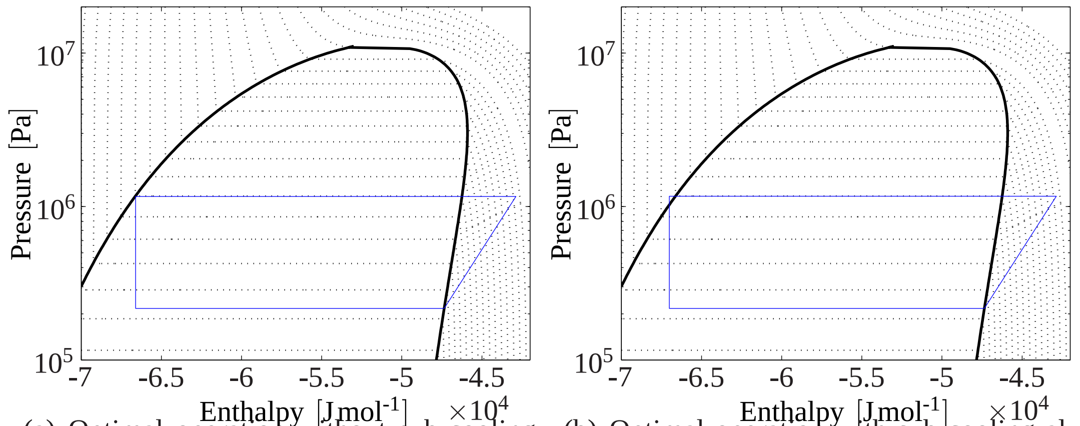
Fig. 7. Cold warehouse with ammonia refrigeration unit

Some data for the cycle:

- Ambient temperature $T_H = 25\,^\circ\mathrm{C}$
- Indoor temperature setpoint $T_C^s = -12\,^\circ\mathrm{C}$
- Isentropic efficiency for compressor is 95 %
- Heat transfer coefficients (U) are 1000 and 500 $\mathrm{W\,m^{-2}\,K^{-1}}$ for the evaporator and condenser, respectively
- Heat exchangers with areas given in Table 3
- Thermodynamic calculations based on SRK equation of state

The equipment is given and we have 5 steady-state operational degrees of freedom (Section 2). With a given load and maximum heat transfer, we have two remaining steady state degrees of freedom, which may be viewed as the degree of sub-cooling

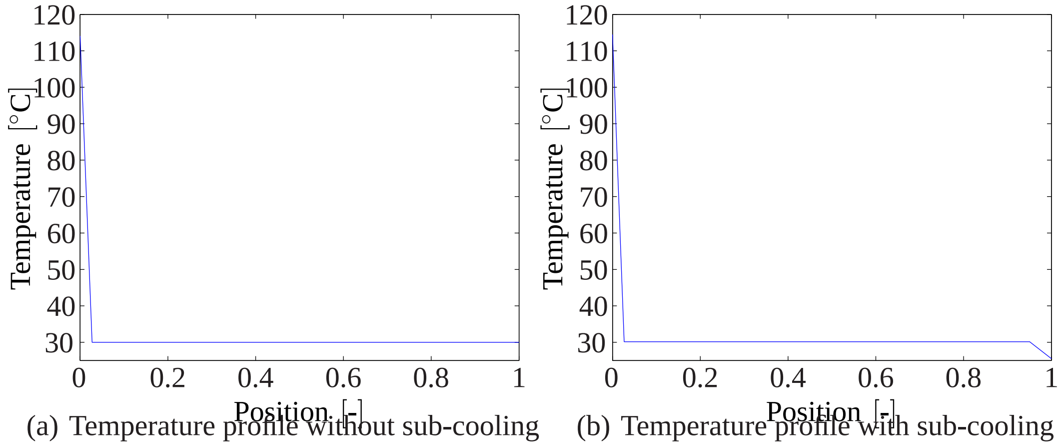
(ΔT_{sub}) and the degree of super-heating (ΔT_{sup}). The performance of the cycle, measured by the compressor power W_s, was optimized with respect to the two degrees of freedom. We find as expected that super-heating is not optimal, but contrary to popular belief, the results in Table 3 show that sub-cooling by 4.66 °C reduces the compression work W_s by 1.74 % compared to the case with saturation out of the condenser. The high pressure P_h increases by 0.45 %, but this is more than compensated by a 2.12 % reduction in flowrate. The sub-cooling increases the condenser charge M_{con} by 5.01 %. Figure 8 shows the corresponding pressure enthalpy diagram for the two cases and Figure 9 shows the temperature profile in the condenser. Similar results are obtained if we use other thermodynamic data, if we change the compressor efficiency or if we let UA be smaller in the sub-cooling zone.



(a) Optimal operation without sub-cooling (Figure 6(b))

(b) Optimal operation with sub-cooling allowed (Figure 5)

Fig. 8. Pressure-enthalpy diagrams with and without sub-cooling



(a) Temperature profile without sub-cooling

(b) Temperature profile with sub-cooling

Fig. 9. Temperature profile in condenser

The improvement of 2 % would be larger if the pressure drop in the piping and equipment was accounted for in the model, because the volumetric flowrate in the low pressure side of the cycle is reduced by having sub-cooling. The pressure losses on the high pressure side will also be slightly reduced (because of smaller flowrate

Table 3
Optimal operation with and without sub-cooling

	No sub-cooling	Optimal sub-cooling
W_s [W]	4648	4567
Q_C [kW]	20	20
$\dot{m}$ [kg s^{-1}]	0.0177	0.0173
M_{con} * [kg]	0.301	0.316
ΔT_{sub} [°C]	0.00	4.66
ΔT_{sup} [°C]	0.00	0.00
$\Delta T_{\text{min, con}}$ [°C]	5.00	0.491
P_h [bar]	11.63	11.68
P_l [bar]	2.17	2.17
A_{con} [m^2]	8.70	8.70
A_{vap} [m^2]	4.00	4.00

* Evaporator charge has no effect because of saturation (no super-heating) in the evaporator

and higher pressure), but this is less important for the efficiency of the cycle.

4.2 Explanation

The irreversible isenthalpic expansion through the choke valve gives a thermodynamic loss. The reason for the improvement in efficiency by sub-cooling is that loss is reduced because less vapour is formed, see Figure 8. This more than compensates the increased irreversible loss due to larger temperature difference in the condenser. To understand this in more detail consider Figure 10 which shows a conceptual pressure enthalpy diagram of a typical vapour compression cycle. We have indicated a cycle without sub-cooling (solid line) and the same cycle with sub-cooling (dotted line). Note that since we in the latter case have a higher condenser pressure (and therefore also a higher temperature in the condensing section) we will with given equipment (UA-values) have more heat transfer, which gives a lower outlet temperature. The condenser outlet will follow the line “Con. out” with increasing pressure. The line will asymptotically approach the hot source temperature T_H and we want to find the optimal operating point on this line.

If we consider moving from one operating point to another we require an increase

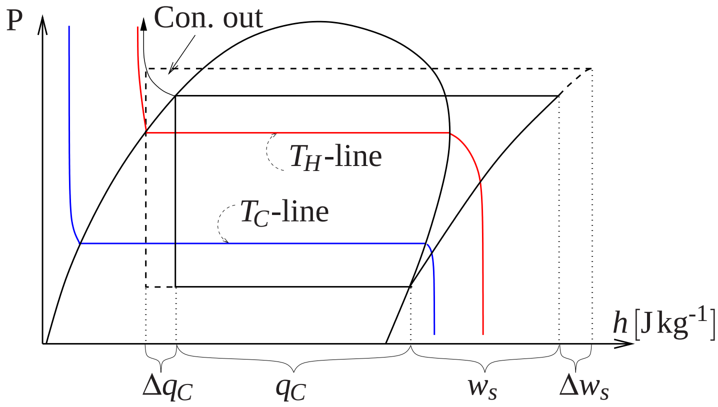


Fig. 10. Pressure-enthalpy diagram for a cycle with and without sub-cooling

in the COP for the change to be optimal:

$$\Delta COP = \frac{q_C + \Delta q_C}{w_s + \Delta w_s} - \frac{q_C}{w_s} > 0 \tag{4}$$
$$COP \cdot \Delta w_s < \Delta q_C \tag{5}$$

where $q_C \cdot \dot{m} = Q_C$ and $w_s \cdot \dot{m} = W_s$. We assume that $Q_C\,[\mathrm{J\,s^{-1}}]$ is given, and that $\dot{m}\,[\mathrm{kg\,s^{-1}}]$ and $q_C\,[\mathrm{J\,kg^{-1}}]$ may vary. We use ΔT_{sub} as the independent variable and introduce differentials. The requirement for improving efficiency is then from Equation 5:

$$\left(\frac{\partial q_C}{\partial \Delta T_{sub}}\right)_{UA} > COP \cdot \left(\frac{\partial w_s}{\partial \Delta T_{sub}}\right)_{UA} \tag{6}$$

According to Equation 6, for an initial COP of 3, the increase in specific duty in the evaporator (Δq_C) should be 3 times larger than the increase in specific compressor power (Δw_s) to give improved performance. In Figure 10 we have that $\Delta q_C \approx \Delta w_s$, so the optimal degree of sub-cooling is clearly less than that indicated by this figure. Note however, that the "Con. out" line is much flatter for smaller Δq_C, so a small degree of sub-cooling may be optimal. The optimum is located at the degree of sub-cooling where the inequality in Equation 6 becomes an equality. In the case study we found that the optimum outlet temperature from the condenser (25.49 °C) is closer to T_H (25 °C) than the saturation temperature (30.15 °C).

Similar considerations on optimizing the pressure P_h have been made earlier for trans-critical CO_2-cycles (Kim et al., 2004). However, for sub-critical cycles like the ammonia cycle studied above, it has been assumed that the pressure is fixed by a saturation condition.

4.3 Discussion of sub-cooling: Why not found before?

The above results on optimality of sub-cooling is contrary to previous claims and popular belief. Why has this result not been found before?

4.3.1 Reason 1: Not allowed by design

The design of the condenser is often as shown in Figure 3, where the saturated liquid drains into a liquid receiver. In this design it is not possible to have sub-cooling.

4.3.2 Reason 2: Infinite area case

The optimal degree of sub-cooling becomes smaller as we increase the heat transfer (UA-values). In particular, with an infinite heat transfer area, sub-cooling is not optimal. In this case the temperature at the condenser outlet is equal to the hot source temperature T_H. Neglecting the effect of pressure on liquid enthalpy, the enthalpy is also given. We then find that $\Delta q_C = 0$ and sub-cooling is not optimal as illustrated in Figure 11.

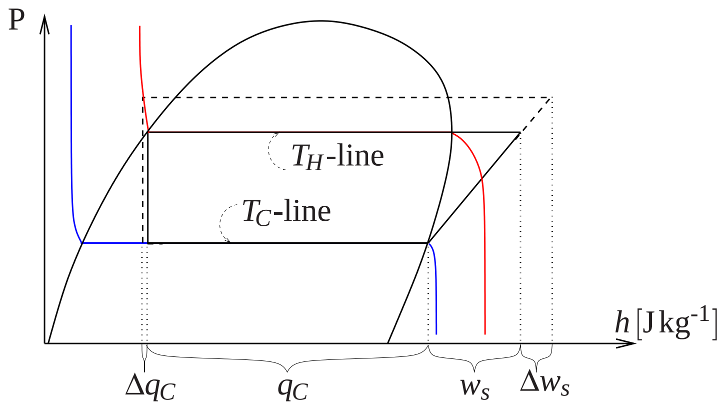


Fig. 11. Pressure-enthalpy diagram for infinite area case where condenser outlet is at hot source temperature T_H

In practice, the enthalpy depends slightly on pressure (as indicated by the curved constant temperature lines in Figure 11) so Δq_C might be larger than zero, but this effect is too small to change the conclusion that sub-cooling is non-optimal with infinite area.

4.3.3 Reason 3: Specifying HRAT

The minimum approach temperature (ΔT_{min} or HRAT) is commonly used as a specification for design of processes with heat exchangers. The idea is to specify ΔT_{min} in order to get a reasonable balance between minimizing operating (energy) costs (favored by a small ΔT_{min}) and minimizing capital costs (favored by a large ΔT_{min}). Although specifying ΔT_{min} may be reasonable for obtaining initial estimates for stream data and areas, it should not be used for obtaining optimal design data - and especially not stream data (temperatures). This follows because specifying ΔT_{min} results in an optimum with no sub-cooling. This can be seen by letting the T_C-line in Figure 11 represent $T_H + \Delta T_{min}$. The condenser outlet temperature is then $T_H + \Delta T_{min}$ and similarly to the infinite area case we get $\Delta q_C = 0$ (neglecting the effect of pressure on liquid enthalpy), and sub-cooling is not optimal.

The results can also be understood because specifying ΔT_{min} favors designs with ΔT being as close as possible to ΔT_{min} throughout the heat exchanger, and this clearly disfavour sub-cooling.

A third way of understanding the difference is that we end up with two different optimization problems for design (Equation 7) and operation (Equation 8).

$$\begin{aligned} \min \quad & (W_s) \\ \text{subject to} \quad & T_C - T_C^s = 0 \\ & \Delta T_i - \Delta T_{\mathrm{min},i} \geq 0 \end{aligned} \tag{7}$$

$$\begin{aligned} \min \quad & (W_s) \\ \text{subject to} \quad & T_C - T_C^s = 0 \\ & A_{\mathrm{max},i} - A_i \geq 0 \end{aligned} \tag{8}$$

For the ammonia case study, solving 7 with $\Delta T_{min} = 5\,^\circ\mathrm{C}$ gives the data for “No sub-cooling” in Table 3. Setting the resulting areas as A_{max}, and solving the optimization problem 8 results in A=A_{max} and the data for “Optimal sub-cooling” in Table 3. We see that specifying ΔT_{min} gives no sub-cooling, whereas fixing the heat exchanger areas to the same value gives 4.66 °C sub-cooling.

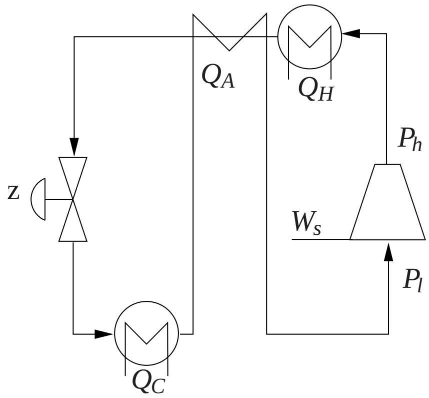
5 Discussion

5.1 Sub-cooling by internal heat exchange

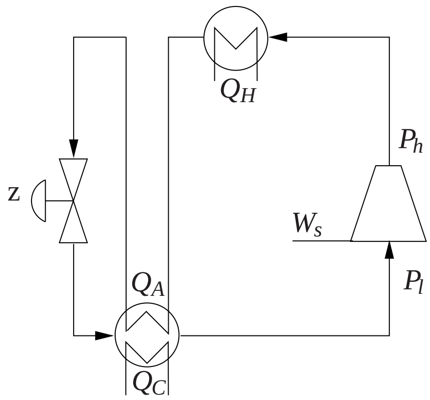
Some sub-cooling in the condenser was found to be optimal, and we here discuss whether other means of obtaining sub-cooling, in particular internal heat exchange, may be beneficial.

Two possibilities are shown in Figure 12. In Figure 12(a) we add a heat exchanger to super-heat the vapour entering the compressor and sub-cool the liquid before expansion. The sub-cooling is beneficial because of reduced expansion losses, whereas the super-heating is undesirable because compressor power increases. Depending on the properties of the fluid, this design may be desirable in some cases, even for pure refrigerants (Radermacher, 1989). In the ammonia case study presented below it is not optimal with internal heat exchange, but for a trans-critical CO_2 cycle it is optimal (Neksaa et al., 1998).

In Figure 12(b) the liquid out of the condenser is sub-cooled by heat exchange with the evaporator. For pure fluids this has no effect (apart from the fact that increased heat transfer area is needed). However, for mixed refrigerants it may be beneficial, and this configuration is frequently used in LNG processes utilizing mixed refrigerants.



(a) Internal heat exchange after evaporator (sometimes beneficial for pure fluids)



(b) Internal heat exchange inside evaporator (no effect for pure fluids)

Fig. 12. Two possible configurations with internal heat exchange

5.2 Selection of controlled variable

We have found that it is generally optimal to have no super-heat ($\Delta T_{sup} = 0\,^\circ\mathrm{C}$) and some sub-cooling ($\Delta T_{sub} > 0\,^\circ\mathrm{C}$). In practice, no super-heating is easily obtained by use of a design with a low pressure tank as shown in Figure 4(a) and Figure 5. It is less clear how to get the right sub-cooling. In Figure 5 we show a strategy where a valve is used to control the degree of sub-cooling ΔT_{sub}. However, the optimal value of ΔT_{sub} will vary during operation, and also ΔT_{sub} may be difficult to measure and control, so it is not clear that this strategy is good. More generally, we could envisage an on-line optimization scheme where one continuously optimizes the operation (maximizes COP) by adjusting the valves. However, such schemes are quite complex and sensitive to uncertainty, so in practice one uses simpler schemes, like the one in Figure 5, where the valve controls some other variable.

Such variables could be:

- Valve position setpoint z_s (that is, the valve is left in a constant position)
- High pressure (P_h)
- Low pressure (P_l)
- Temperature out of condenser (T_2)
- Degree of sub-cooling ($\Delta T_{sub} = T_2 - T_{sat}(P_h)$)
- Temperature out of evaporator (T_4)
- Degree of super-heating ($\Delta T_{sup} = T_4 - T_{sat}(P_l)$)
- Liquid level in storage tank (to adjust charge to rest of system)
- Pressure drop across the extra valve if the design in Figure 5(b) is used

The objective is to achieve "self-optimizing" control where a constant setpoint for the selected variable indirectly leads to near-optimal operation (Skogestad, 2000). The selection of "self-optimizing" controlled variables for simple refrigeration cycles is the main topic in Part II (Jensen and Skogestad, 2007 (to appear).

6 Conclusion

The "active charge" in a closed cycle has a steady state effect. This is unlike open systems, where we have boundary conditions on pressure. To adjust the degree of freedom related to the "active charge" one needs a liquid tank (receiver) in the cycle. The key to make efficient use of this degree of freedom is to allow for sub-cooling in the condenser. So far it has been assumed that one should avoid sub-cooling in the condenser to maximize the efficiency. However, we find that some sub-cooling may be desirable. For the ammonia case study we get savings in the order of 2%, by using the design in Figure 5 that allows for sub-cooling. The savings would be even larger if we compared with the common design in Figure 6(a) which in addition to having no sub-cooling, also gives super-heating.

Nevertheless, the savings in them self are not very large. More importantly, the results show that the active charge is a degree of freedom, and the sub-cooling gives some decoupling between the high pressure P_h and the hot source temperature T_H similar to that found for other cycles, including trans-critical CO_2 cycles.

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