Degrees of freedom and optimal operation of simple heat pump cycles

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Abstract

Cycles for heating and cooling have traditionally been studied in detail when it comes to thermodynamics and design. However, there are few publications on their optimal operation which is the theme of this paper. We consider the degrees of freedom for operation. Unlike in open systems, the "active charge" in the cycle has a steady-state effect, and we discuss how this degree of freedom may be utilized during operation. Specifically, we find that the active charge may be used to achieve sub-cooling in the condenser. As far as we know, sub-cooling has not been considered in the literature, but we find that it typically saves about 2% in compression power.

Key words: Operation, self-optimizing control, vapour compression cycle

1 Introduction

Cyclic processes for heating and cooling are widely used in many applications and their power ranges from less than 1 kW to above 100 MW. Most of these applications use the vapour compression cycle to "pump" energy from a low to a high temperature level.

The first application, in 1834, was cooling to produce ice for storage of food, which led to the refrigerator found in most homes Nagengast (1976). Another well-known system is the air-conditioner (A/C). In colder regions a cycle operating in the opposite direction, the "heat pump", has recently become popular. These two applications have also merged together to give a system able to operate in both heating and cooling mode.

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A schematic drawing of a simple cycle is shown in Figure 1 together with a typical pressure-enthalpy diagram for a sub-critical cycle. The cycle works as follows:

The low pressure vapour (4) is compressed by supplying work W_s to give a high pressure vapour with high temperature (1). This stream is cooled to the saturation temperature in the first part of the condenser, condensed in the middle part and possibly sub-cooled in the last part to give the liquid (2). In the expansion choke, the pressure is lowered to its original value, resulting in a two-phase mixture (3). This mixture is vaporized and possibly super-heated in the evaporator (4) closing the cycle.

The coefficients of performance for a heating cycle (heat pump) and a cooling cycle (refrigerator, A/C) are defined as

$$COP_h = \frac{Q_h}{W_s} = \frac{\dot{n}(h_1 - h_2)}{\dot{n}(h_1 - h_4)} \quad \text{and} \quad COP_c = \frac{Q_c}{W_s} = \frac{\dot{n}(h_4 - h_3)}{\dot{n}(h_1 - h_4)} \tag{1}$$

respectively. Heat pumps typically have a COP of around 3 which indicates that 33% of the gained heat is addet as work (eg. electric power).

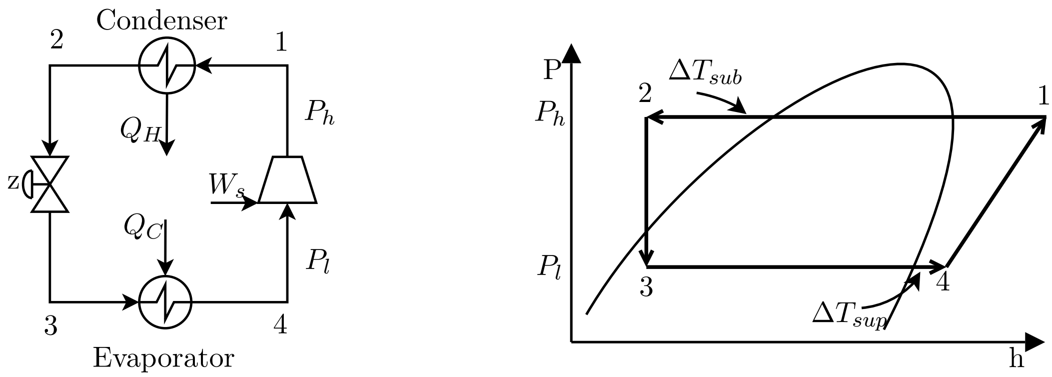


Fig. 1. Schematics of a simple vapour compression cycle with typical pressure-enthalpy diagram indicating both sub-cooling and super-heating

In industrial processes, especially in cryogenic processes such as air separation and liquefaction of natural gas (LNG process), more complex cycles are used in order to improve the thermodynamic efficiencies. These modifications lower the temperature differences in the heat exchangers and include cycles with mixed refrigerants, several pressure levels and cascaded cycles. Our long term objective is to study the operation of such processes. However, as a start we need to understand the simple cycle in Figure 1.

An important result from this study is the degree of freedom analysis given in Section 2. This is more or less directly applicable to more complex designs. We find that the charge plays an important role in operation of cyclic processes. A cyclic process will not have boundary conditions on pressures, which for an open process will determine the pressure and holdup internally, so controlling the charge in the system is important as it indirectly sets the pressure level in the system.

Although there is a vast literature on the thermodynamic analysis of such cycles, there are few authors who discuss the operation and control of closed cycles. Some discussions are found in text books such as Stoecker (1998), Langley (2002) and Dossat (2002), but these mainly deal with more practical aspects. Svensson (1994) and Larsen et al. (2003) discuss operational aspects. A more comprehensive recent study is that of Kim et al. (2004) who consider the operation of trans-critical CO_2 cycles. They discuss the effect of "active charge" and consider alternatives for placing the receiver.

In the literature, it is generally taken for granted that there should be no sub-cooling and super-heating ($\Delta T_{sub} = 0$ and $\Delta T_{sup} = 0$) in an optimal cycle. For example, (Stoecker 1998, page 57) states that

> The refrigerant leaving industrial refrigeration condensers may be slightly sub-cooled, but sub-cooling is not normally desired since it indicates that some of the heat transfer surface that should be be used for condensation is used for sub-cooling. At the outlet of the evaporator it is crucial for protection of the compressor that there be no liquid, so to be safe it is preferable for the vapor to be slightly super-heated.

In this study, we find that super-heating is not optimal. However, contrary to popular belief, we find that with given equipment, sub-cooling in the condenser may give savings in energy usage (compressor power) in the order of 2%. It is normally assumed that the high pressure P_h and the hot source temperature T_H are directly coupled, but sub-cooling gives some decoupling. The optimality of sub-cooling is discussed in Section 4. An ammonia case study is presented to obtain numerical results.

We consider only steady state operation in this paper, as this has the most influence on the operating costs.

2 Degrees of freedom in simple vapour compression cycles

2.1 Design versus operation

Table 1 shows typical specifications for the simple cycle in Figure 1 in design (find equipment) and in operation (given equipment). The five design specifications include the load, the two pressures, and the degree of sub-cooling and super-heating. Based on these five design specifications we may obtain the following four equipment parameters which can be adjusted during operation: compression work (W_s) valve opening (z) and effective areas (UA-values) for the two heat exchangers. Initially, we were puzzled because we could not identify the missing fifth equipment parameter to be adjusted during operation. However, we finally realized that we can manipulate the "active charge" in the cycle, which indirectly sets the "pressure level" in the cycle. By "active charge" is meant the mass accumulated in the process equipment, that is, mainly in the two heat exchangers in Figure 1. The fact that the charge is an independent variable is unique for closed systems since there is no boundary condition for pressure.

Table 1
Typical specifications in design and operation

	Given	#
Design	Load (e.g. Q_h), P_l, P_h, ΔT_{sup} and ΔT_{sub}	5
Operation	W_s (load), choke valve opening (z),	
	UA in two heat exchangers and active charge	5

2.2 Active charge and holdup tanks

For the simple cycle in Figure 1 we have the following total material balance:

$$m_{tot} = \underbrace{m_{evap} + m_{con}}_{\text{Active charge}} + m_{valve} + m_{comp} + m_{tank} \tag{2}$$

With no filling, emptying or leaks, the total mass m_{tot} is fixed. We have not included a holdup tank in Figure 1, but in practice it is common to include a variable liquid level (tank; receiver) in the cycle. m_{tank} is then the overall mass in this tank(s). Normally the holdups in the valve and compressor are neglected and we get:

$$m_{tot} = \underbrace{m_{evap} + m_{con}}_{\text{Active charge}} + m_{tank} \tag{3}$$

With a given volume of the equipment, the "pressure level" is indirectly given by the active charge. With constant active charge, we assume that a change in m_{tank} (e.g. by filling or leaking) does not affect the operation of the cycle. This implies that the the tank(s) must contains both liquid and gas in equilibrium. Then we can move mass to or from the tank without affecting the pressure, and thus without affecting the rest of the cycle.

In addition to making operation independent of total charge in the system, the extra tank introduces an additional degree of freedom. This can be seen from Equation 3: With m_{tot} constant, we can by altering the liquid level in the tank (m_{tank}), change the active charge in the rest of the system (condenser and evaporator). This shows that the liquid level in the tank has an indirect steady state effect, and can therefor be used for control purposes, of course provided that we have means of changing it.

Although it is possible to introduce several tanks in a cycle, we only have one material balance for each cycle, so this will not add any steady-state degrees of freedom with respect to the total holdup.

Rule 1 *In each closed cycle, we have one degree of freedom related to the active charge, which may be indirectly adjusted by introducing a variable liquid level (tank; receiver) in the cycle.*

Rule 2 *In each closed cycle, there will be one liquid holdup that does not need to be explicitly controlled, because the total mass is fixed. This is usually selected as the largest liquid volume in the closed system. The remaining liquid levels (holdups) must be controlled.*

Remark 1 Rule 2 does not mean that we cannot control all the liquid volumes in the system (including the largest one), but it just states that it is not strictly necessary. In fact, controlling all the liquid volumes, provides a way for explicitly controlling the active charge in the cycle, which may be a good option in some cases.
Remark 2 Introducing additional liquid tanks may be useful for operation, but at least for pure fluids, these will not introduce any additional steady-state degrees of freedom because we can move mass from one tank to another without affecting operation. Also, to avoid that tanks fill up or empty, these additional levels will need to be controlled (Rule 2), either by self-regulation or feedback control.
Remark 3 In *mixed refrigerant* cycles two tanks may be used to indirectly change the composition of the circulating refrigerant. In this case the two tanks have different composition so moving mass from one tank to another does affect operation. This is utilized in the auto-cascade process (Neeraas et al. (2001)). For more complex cycles the maximum number of degrees of freedom related to tank holdups is the number of components in the refrigerant.

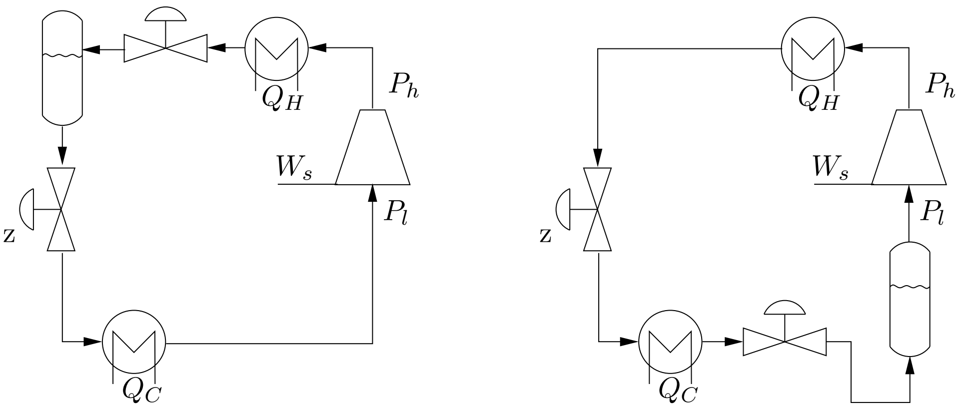
2.2.1 Adjusting the holdup with an extra valve

Kim et al. (2004) discuss alternative locations for the variable tank holdup (liquid receiver). In Figure 2, we show cycles for the cases where the tank is placed (a) on the high pressure side after the condenser and (b) on the low pressure side after the evaporator. Other placements and combinations are possible, but these are only variations of these two and will not add any steady-state degrees of freedom for pure refrigerants.

The most obvious way of introducing a means for adjusting the tank holdup, is to add an extra valve before the tank as shown in Figure 2.

In Figure 2(a), the tank is located at an intermediate pressure P_m after the condenser. In this case the extra valve is on the same side as the expansion valve (choke), so the pressure drop over the extra valve will not effect the efficiency of the cycle. The pressure P_m is assumed to be the saturation pressure at the tank temperature, an exit stream from the condenser will then have to be sub-cooled. Thus, with the tank after the condenser (Figure 2(a)), the pressure drop across the valve may be used to adjust the degree of sub-cooling in the condenser. As discussed below, it is possible to eliminate the valve, but if we then keep the tank we can not get sub-cooling. As found later in this paper, some sub-cooling appears to be optimal in most cases.

Another possibility is to place the tank after the evaporator, as shown in Figure 2(b). However, in this case the valve introduces a pressure drop which must be compensated by increasing the compression power, so a valve here is generally not optimal.



(a) Liquid tank and valve at high pressure side

(b) Liquid tank and valve at low pressure side. Note that the extra valve is **not** optimal

Fig. 2. Simple cycle with variable active charge

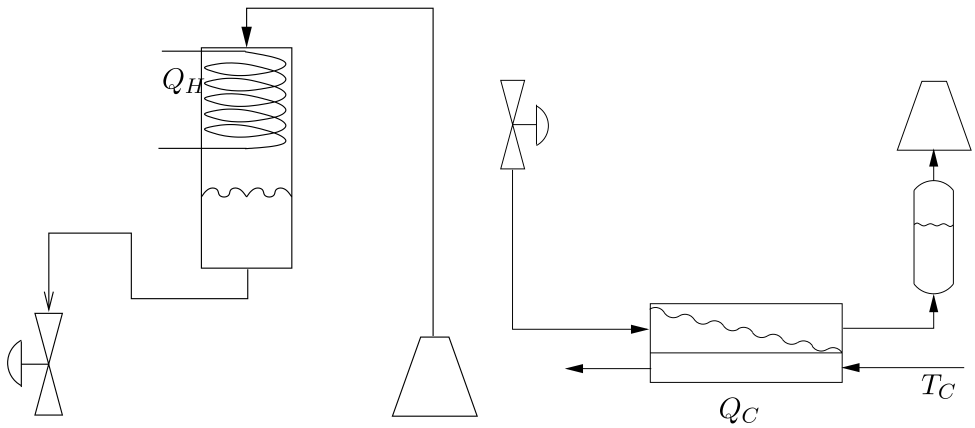
2.2.2 Extra valve removed

In most practical cases the extra valves in Figure 2(a) and 2(b) are removed. What effect does this have?

- High pressure tank without valve (see Figure 3(a) where the tank and condenser are merged together): Without the valve, we will have at steady state the same thermodynamic state at the exit of the condenser as at the exit from the tank. Thus, the exiting stream from the condenser will be saturated liquid. As we will show, this is not generally optimal. Thus, in this design we have used a degree of freedom (“no valve”) to set the degree of sub-cooling to a non-optimal value. Nevertheless, this design is commonly used in most applications.

- Low pressure tank without valve (see Figure 3(b)): With a liquid tank after the evaporator we get saturated vapour to the compressor. Fortunately, this is generally optimal for the cycle as a whole, because the inlet temperature to the compressor should be as low as possible to minimize vapour volume and save compression power. Thus, in this design we have used a degree of freedom (“no valve”) to set the degree of super-heating to the optimal value.

So in the case of high pressure liquid tank we get a sub-optimal design if we remove the valve, whereas for the low pressure tank we get an optimal design if the extra valve is removed. It is also possible to remove the tanks as discussed later.



(a) Condenser without sub-cooling

(b) Evaporator without super-heating

Fig. 3. Condenser and evaporator with valve removed and saturation at outlet

2.3 Degrees of freedom for operation

During operation the equipment is given. Nevertheless, we have some operational or control degrees of freedom.

1 The compression power W_s. We assume here that it is used to set the “load” for the cycle.

2, 3 Effective heat transfer area (UA). There are two degrees of freedom related to adjusting the heat transfer, which may thought of as adjusting (reducing) the effective UA value in each heat exchanger. This may be done in many ways, for example, by introducing bypasses or using flooded condenser or evaporator. However, we generally find that it is optimal to maximize the effective UA. Thus, these degrees of freedom are not considered in the following.

4 Adjustable choke valve (z); see Figure 1
5 Adjustable active charge.

In summary, with a given load we are in practice left with two steady state degrees of freedom. These are the choke valve opening and the active charge. These may be used to set the degree of super-heating and degree of sub-cooling. The pressure levels (P_h and P_l) are indirectly determined by the given (maximum) value of the heat transfer $Q = UA\Delta T$ as determined by the two UA values.

3 Discussion of some designs

As discussed in more detail in Section 4, we find that the thermodynamic efficiency is optimized by having no super-heating and some sub-cooling.

3.1 Optimal designs

Two potentially optimal designs are shown in Figure 4. The reason we say "potentially optimal" is because they will only be optimal if we use the optimal value for the sub-cooling.

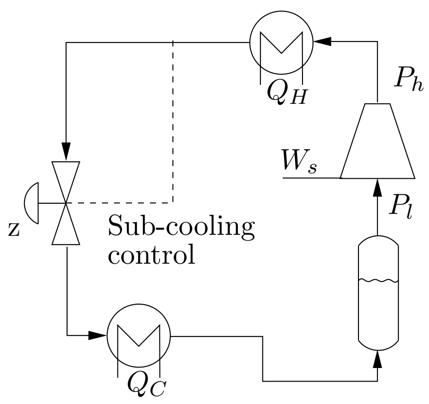
In Figure 4(a) we have a low pressure tank (receiver) between the evaporator and compressor which ensures that the vapour entering the compressor is saturated. A demister may be added to avoid that liquid droplets enter the compressor. The choke valve may be used to control the degree of sub-cooling (ΔT_{sub}) as shown in Figure 4(a). Also other control policies are possible, for example, keeping the choke valve position constant or controlling the pressure, but controlling ΔT_{sub} was found by Jensen & Skogestad (2005) to be a good self-optimizing controlled variable.

In Figure 4(b) we have added a high pressure tank and valve after the condenser. Thermodynamically this design is equivalent to Figure 4(a), but the addition of the tank may prevent that we get two-phase flow with vapour "blow out" through the choke. In this case, it seems reasonable to use the "new" valve to control the sub-cooling as shown in Figure 4(b). We now have two adjustable holdups, so from Rule 2 one of them must be controlled. In

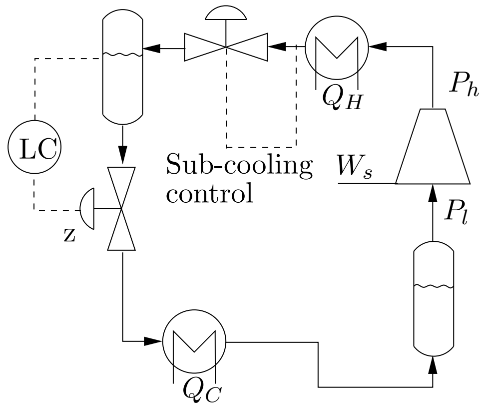
Figure 4(b), we show the case where the choke valve is used to control the level in the high pressure tank, but alternatively we could control the level in the low pressure tank.

A third potentially optimal design (not shown) would be to remove the valve in Figure 4(b), and instead add a sub-cooling heat exchanger before the choke. This may also be accomplished by having only one heat exchanger where the liquid level covers some of the heat transfer area and there is little mixing in the liquid phase.

To avoid super-heating, we have in Figure 4(a) and 4(b) a tank after the evaporator. This tank will give saturated vapour out of the evaporator at steady state, and also by trapping the liquid it will avoid that we get liquid to the compressor during transient operation. To avoid super-heating we must have vapour-liquid equilibrium in the tank. This may be achieved by letting the vapour bubble through the tank. An alternative design is to integrate the heat exchanger and the tank as shown in Figure 5. This design is equivalent thermodynamically, but it may not be optimal because the effective heat transfer coefficient (U) may be lower.



(a) Optimal 1



(b) Optimal 2 with extra tank after the condenser

Fig. 4. Two potentially optimal designs

3.2 Non-optimal designs

Figure 6(a) shows the design used in most applications. In practice the tank and condenser are often integrated as shown in Figure 3(a). This design has two errors compared to the optimal solution: 1) There is no sub-cooling in the condenser and 2) there is super-heating in the evaporator. The super-heat control is in practice accomplished with a thermostatic expansion valve (TEV).

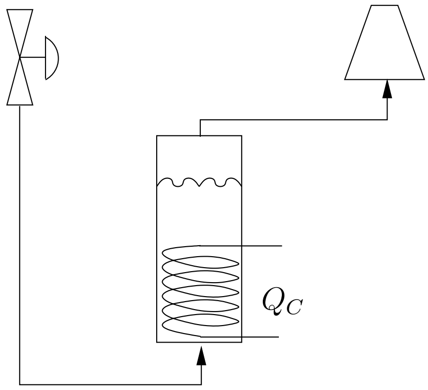


Fig. 5. Flooded evaporator

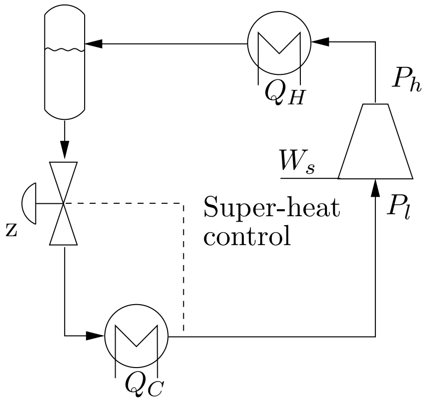
In Figure 6(b) we have two liquid tanks, one after the evaporator and one after the condenser. This design is better since there is no super-heating in the evaporator, but one error remains: There is no sub-cooling in the condenser. Note that we need to control one of the liquid levels in accordance with Rule 2.

Another non-optimal design is shown in Figure 6(c). Here we have introduced the possibility for sub-cooling, but we have super-heating which is generally not optimal.

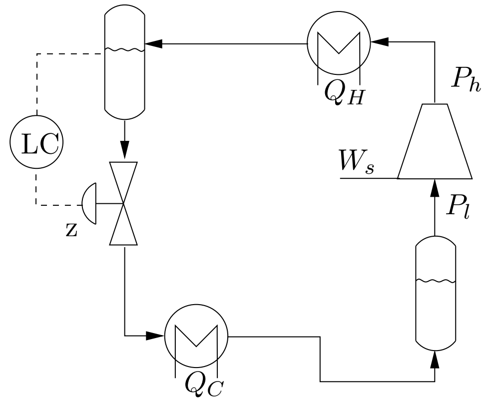
3.3 Internal heat exchange

Internal heat exchange has so far been excluded. There are two possibilities as shown in Figure 7. In Figure 7(a) we add a heat exchanger to super-heat the vapour entering the compressor and sub-cool the liquid before expansion. The sub-cooling is positive because of reduced expansion losses, whereas the super-heating is undesirable because compressor power increases. Depending on the properties of the fluid , this design may be desirable in some cases, even for pure refrigerants (Radermacher (1989)). In the ammonia case study presented below it is not optimal with internal heat exchange, but for a trans-critical CO_2 cycle it is optimal (Nekså et al. (1998)).

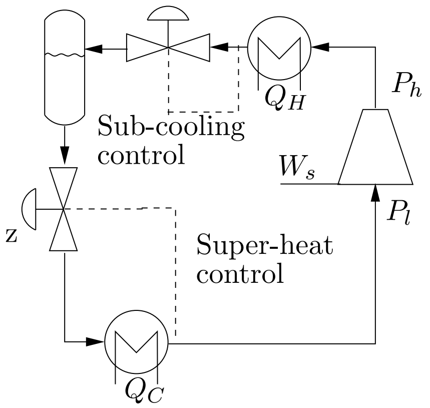
In Figure 7(b) the liquid out of the condenser is sub-cooled by heat exchange with the evaporator. For pure fluids this has no effect (apart from the fact that increased heat transfer area is needed). However, for mixed refrigerants it may be beneficial, and this configuration is frequently used in LNG processes utilizing mixed refrigerants.



(a) Non-optimal 1. This design has two errors: 1) No sub-cooling and 2) Super-heating



(b) Non-optimal 2. This design has one error: No sub-cooling

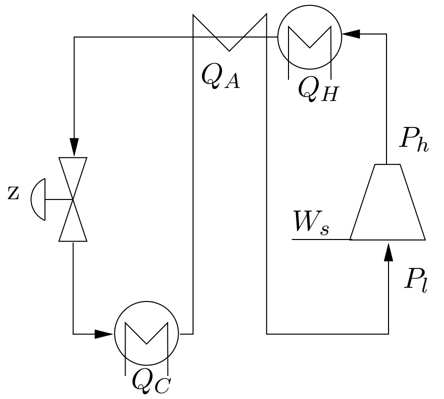


(c) Non-optimal 3, This design has one error: Super-heating

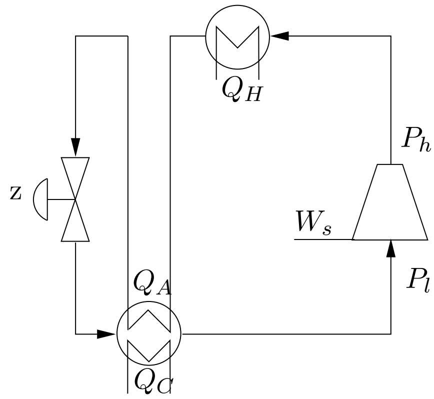
Fig. 6. Three non-optimal designs

4 Optimality of sub-cooling

We have several times made the claim that sub-cooling may be optimal. To justify this controversial claim, we start by considering a specific example.



(a) Internal heat exchange after evaporator (sometimes beneficial also for pure fluids)



(b) Internal heat exchange inside evaporator (no effect for pure fluids)

Fig. 7. Two possible configurations of internal heat exchange

4.1 Ammonia case study

The objective is to cool a storage building by removing heat (Q_C) as illustrated in Figure 8. The cycle operates between a cold medium of air inside the building ($T_C = T_{room}$) and hot medium of ambient air ($T_H = T_{amb}$) removing 20 kW of heat (Q_C) from the building. Some data for the cycle:

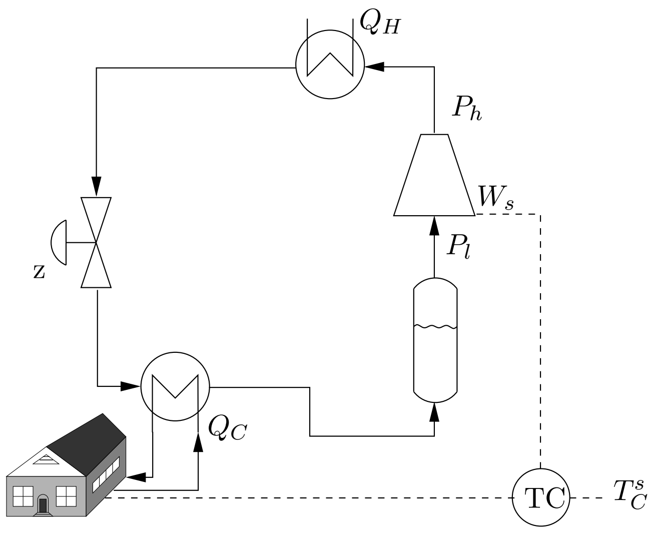


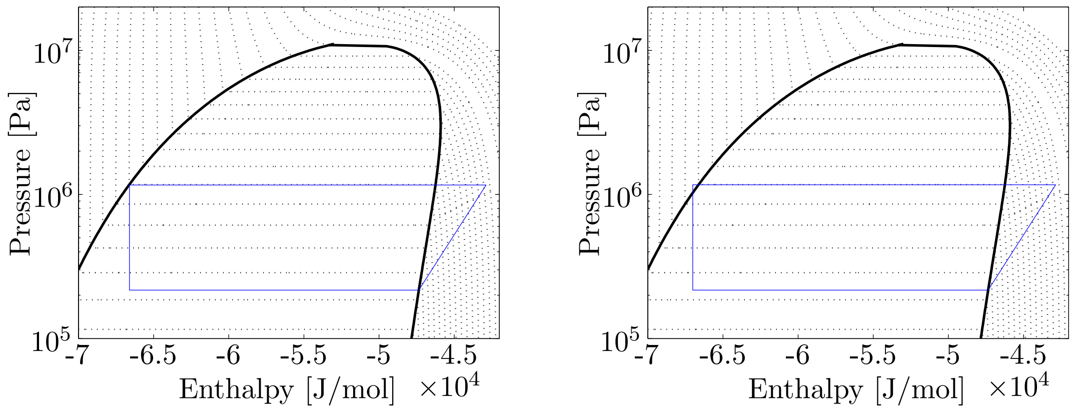
Fig. 8. Cold warehouse with ammonia refrigeration unit

- Ambient temperature T_H = 25 °C
- Indoor temperature set point T_C^s = -12 °C
- Isentropic efficiency for compressor is 95 %
- Heat transfer coefficients (U) are 1000 and 500 $W/(m^2K)$ for the evaporator

and condenser, respectively
- Heat exchangers with areas given in Table 2
- Thermodynamic calculations are based on SRK equation of state

The steady state heat loss from the building is 20 kW and the load Q_C is indirectly adjusted by the temperature controller which adjusts the compressor work (W_s) to maintain $T_C = T_C^s$.

The equipment is given, so we have two remaining steady state degrees of freedom, which may be viewed as the degree of sub-cooling (ΔT_{sub}) and the degree of super-heating (ΔT_{sup}). The results from the optimization with and without sub-cooling are summarized in Table 2. We find that super-heating is not optimal, but contrary to popular belief, we find for this ammonia cycle, that sub-cooling by 4.66 $°C$ reduces the compression work W_s by 1.74%. The high pressure P_h is increases by 0.45%, but this is more than compensated by a 2.12% reduction in flowrate. The sub-cooling increases the condenser charge M_{con} by 5.01% in optimal operation. Figure 9 shows the corresponding pressure enthalpy diagram for the two cases. Figure 10 shows the temperature profile in the condenser for the two cases. Similar results are obtained if we use other thermodynamic data, if we change the compressor efficiency or if we let UA be smaller in the sub-cooling zone.



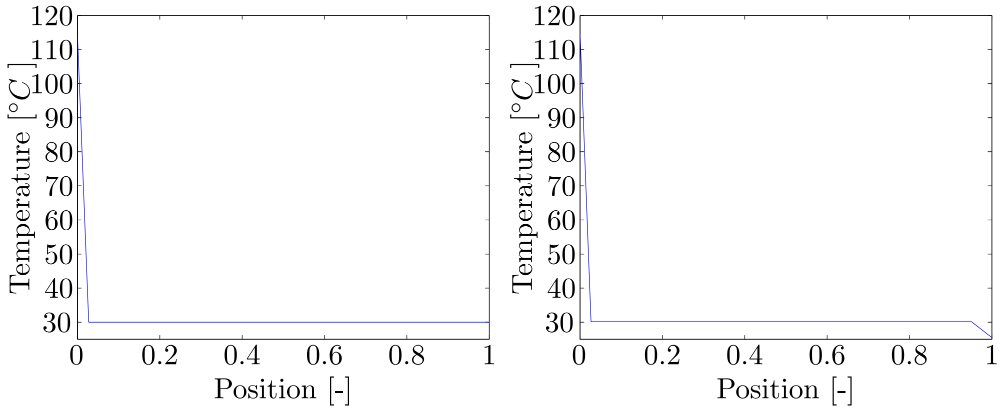
(a) Optimal operation without sub-cooling (Figure 6(a))

(b) Optimal operation with sub-cooling allowed (Figure 4)

Fig. 9. Ph-diagrams with and without sub-cooling

4.2 Explanation

Physically, the reason for the improvement in efficiency by sub-cooling is that the irreversible loss through the choke is smaller because less vapour is formed. This more than compensates for the increased irreversible loss in the con-



(a) Temperature profile without sub-cooling

(b) Temperature profile with sub-cooling

Fig. 10. Temperature profile in condenser

Table 2
Optimal operation with and without sub-cooling

	No sub-cooling	Optimal
W_s [W]	4648	4567
Q_C [kW]	20	20
Flow [mol/s]	1.039	1.017
M_{con} [a] [kmol]	17.72	18.61
ΔT_{sub} [$^{\circ}C$]	0.00	4.66
ΔT_{sup} [$^{\circ}C$]	0.00	0.00
ΔT_{min} [$^{\circ}C$]	5.00	0.491
P_h [bar]	11.63	11.68
P_l [bar]	2.17	2.17
A_{con} [m^2]	8.70	8.70
A_{vap} [m^2]	4.00	4.00

[a] Evaporator charge has no effect

denser. To understand this in more detail consider Figure 11 which shows a conceptual pressure enthalpy diagram of a typical vapour compression cycle. We have indicated a cycle without sub-cooling (solid line) and the same cycle with sub-cooling (dotted line). Note that since we in the latter case have a higher condenser pressure (and therefor also a higher temperature in the condensing section) we will with given equipment (UA-values) have a higher heat transfer, which gives a lower outlet temperature. The condenser outlet will follow the line "Con. out" with increasing pressure. The line will asymptotically approach the hot source temperature T_H and we want to find the optimal operating point on this line.

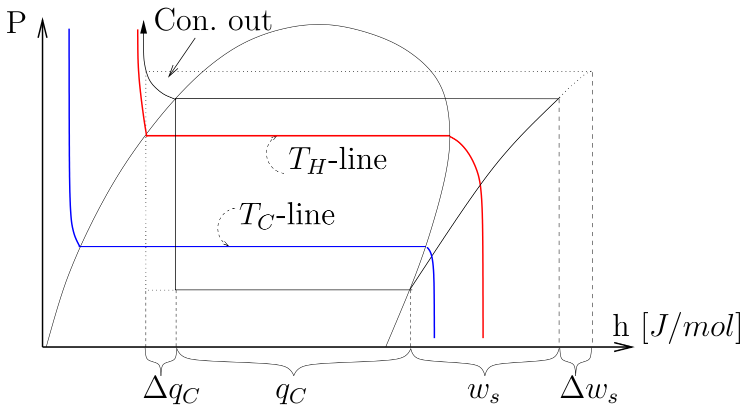


Fig. 11. Pressure-enthalpy diagram for a cycle with and without sub-cooling

If we consider moving from one operating point to another we require an increase in the COP for the change to be optimal:

$$\Delta COP = \frac{q_C + \Delta q_C}{w_s + \Delta w_s} - \frac{q_C}{w_s} > 0 \tag{4}$$

$$COP \cdot \Delta w_s < \Delta q_C \tag{5}$$

where $q_C \cdot \dot{n} = Q_C$ and $w_s \cdot \dot{n} = W_s$. We assume that Q_C [J/s] is given, and that $\dot{n}$ and q_C may vary. We use ΔT_{sub} as the independent variable and introduce differentials. The requirement for improving efficiency is then from Equation 5:

$$\left(\frac{\partial q_C}{\partial \Delta T_{sub}}\right)_{UA} > COP \cdot \left(\frac{\partial w_s}{\partial \Delta T_{sub}}\right)_{UA} \tag{6}$$

According to Equation 6, for an initial COP of 3, the evaporator should have more than 3 times increase in specific duty compared with the compressor to give improved performance. In Figure 11 we have that $\Delta q_C \approx \Delta w_s$, so the optimal degree of sub-cooling is clearly less than that indicated by this Figure . Note however, that the "Con. out" line is much flatter for smaller Δq_C, so a small degree of sub-cooling may be optimal.The optimum is located at the degree of sub-cooling where the inequality in Equation 6 turns into an equality. In the case study we found that the optimum ($25.49°C$) is closer to T_H ($25°C$) than the saturation temperature (30.15).

Similar considerations on optimizing the pressure P_h have been made earlier for trans-critical CO_2-cycles (Kim et al. (2004)). However, for sub-critical cycles it has been assumed that the pressure is fixed by a saturation condition.

4.3 Discussion of sub-cooling: Why not found before?

The above results on optimality of sub-cooling is contrary to previous claims and popular belief. Why has this result not been found before?

4.3.1 Reason 1: Not allowed by design

The design of the condenser is often as shown in Figure 3(a), where the liquid drips down into a liquid reservoir below the condenser as the droplets forms. In this design it is not possible to have sub-cooling.

4.3.2 Reason 2: Infinite area case

If one assumes an infinite heat transfer area, then it is not optimal with sub-cooling. In this case the temperature at the condenser outlet is equal to the hot source temperature T_H. Neglecting the effect of pressure on liquid enthalpy, the enthalpy is also given. We then find that $\Delta q_C = 0$ and sub-cooling is not optimal as illustrated in Figure 12.

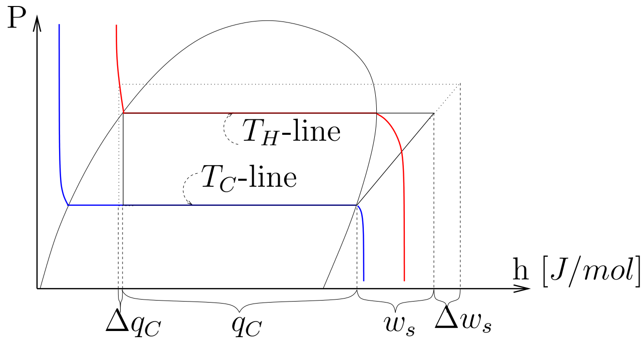


Fig. 12. Pressure-enthalpy diagram for infinite area case where condenser outlet is at hot source temperature T_H

In practice, the enthalpy depends slightly on pressure (as indicated by the curved constant temperature lines in Figure 12) so Δq_C might be larger than zero, but this effect is too small to change the conclusion that sub-cooling is non-optimal with infinite area.

4.3.3 Reason 3: HRAT specification

The minimum approach temperature (ΔT_{min} or HRAT) is commonly used as a specification for design of processes with heat exchangers. The idea is to specify ΔT_{min} in order to get a reasonable balance between minimizing operating (energy) costs (favored by a small ΔT_{min}) and minimizing capital costs (favored by a large ΔT_{min}). Although specifying ΔT_{min} may be reasonable for obtaining initial estimates for stream data and areas, it should not be used for obtaining optimal design data - and especially not stream data (temperatures). This follows because specifying ΔT_{min} will, similarly as the infinite area case, result in an optimum with no sub-cooling. This can be seen by letting the T_C-line and T_H-line in Figure 12 represent lines for $T_C - \Delta T_{min}$ and $T_H + \Delta T_{min}$ respectively. The condenser outlet will then be given by $T_C + \Delta T_{min}$ and again we get that $\Delta q_C = 0$ neglecting the effect of pressure on liquid enthalpy.

Another way of understanding the difference is that we end up with two different optimization problems for design (Equation 7) and operation (Equation 8).

$$\begin{aligned} \min \quad & W_s \\ \text{such that} \quad & T_C - T_C^s = 0 \\ & \Delta T - \Delta T_{\min} \geq 0 \end{aligned} \tag{7}$$

$$\begin{aligned} \min \quad & W_s \\ \text{such that} \quad & T_C - T_C^s = 0 \\ & A_{\max} - A \geq 0 \end{aligned} \tag{8}$$

For the ammonia case study, solving 7 with ΔT_{min}=5°C gives the data for "No sub-cooling" in Table 2. Setting the resulting areas as A_{max}, and solving the optimization problem 8 results in A=A_{max} and the data for "Optimal" in Table 2. We see that specifying ΔT_{min} gives no sub-cooling, whereas fixing the heat exchanger areas to the same value gives 4.66°C sub-cooling.

5 Selection of controlled variable

We have found that it is generally optimal to have no super-heat ($\Delta T_{sup} = 0$) and some sub-cooling ($\Delta T_{sub} > 0$). In practice, no super-heating is easily obtained by use of a design with a low pressure tank as shown in Figure 3(b) and Figure 4. It is less clear how to get the right sub-cooling. In Figure 4 we show a strategy where a valve is used to control the degree of sub-cooling ΔT_{sub}. However, the optimal value of ΔT_{sub} will vary during operation, and also ΔT_{sub} may be difficult to measure and control, so it is not clear that this strategy is good. More generally, we could envisage an on-line optimization scheme where one continuously optimizes the operation (maximizes COP) by adjusting the valves. However, such schemes are quite complex and sensitive to uncertainty, so in practice one uses simpler schemes, like the one in Figure 4, where the valves are used to control some other variable. Such variables could be:

- Valve position setpoint z_s (that is, the valve is left in a constant position)
- High pressure (P_h)
- Low pressure (P_l)
- Temperature out of condenser (T_2)
- Degree of sub-cooling ($\Delta T_{sub} = T_2 - T_{sat}(P_h)$)
- Temperature out of evaporator (T_4)
- Degree of super-heating ($\Delta T_{sup} = T_4 - T_{sat}(P_l)$)
- Liquid level in storage tank (to adjust charge to rest of system)
- Pressure drop across the extra valve if the design in Figure 4(b) is used

The objective is to achieve "self-optimizing" control where a constant setpoint for the selected variable indirectly leads to near-optimal operation Skogestad (2000). The selection of controlled variables is outside the scope of this paper and is presented elsewhere (Jensen & Skogestad (2005)).

6 Conclusion

The "active charge" in a closed cycle has a steady state effect. This is unlike open systems, where we have boundary conditions on pressure. This steady state degree of freedom related to the "active charge" may be used to optimize operation of vapour compression cycles. The key to obtain the extra degree of freedom is to allow for sub-cooling in the condenser. So far it has been assumed that one should avoid sub-cooling in the condenser to maximize the efficiency. However, we find that some sub-cooling may be desirable. For the ammonia case study we get savings in the order of 2%, with the same heat transfer areas, by allowing sub-cooling in the condenser.

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