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Controllability of Processes with Large Gains

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Introduction

Are large process gains a problem in terms of input-output controllability? Two main types of input errors are discussed: input (load) disturbance and input (valve) inaccuracy caused by limited input resolution. This work is motivated by the results in [1, 2, 3, 4, 5].

References

[1] T. J. McAvoy and R. D. Braatz. Controllability of process with large singular values. *Ind. Eng. Chem. Res.*, 42:6155–6165, 2003.

[2] M. Morari. Design of resilient processing plants III - a general framework for the assessment of dynamic resilience. *Chemical Engineering Science*, 38:1881–1891, 1983.

[3] S. Skogestad. Simple analytic rules for model reduction and PID controller tuning. *Journal of Process Control*, 13:291–309, 2003.

[4] S. Skogestad and I. Postlethwaite. *Multivariable Feedback Control: Analysis and design.* John Wiley & Sons, Chichester, UK, 1996.

[5] J. E. Slotine and W. Li. *Applied Nonlinear Control.* Prentice-Hall International Editions, New Jersey, USA, 1991.

Input Load Disturbance

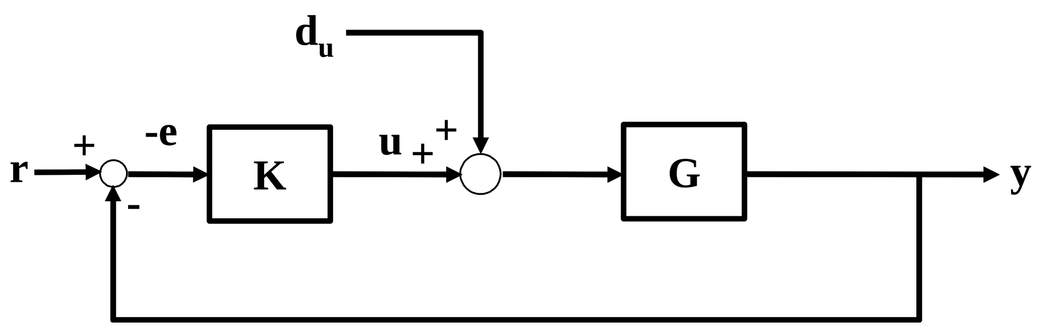

Figure 1. Block diagram of a feedback control system.

For performance we must require $|SGd_u| < y_{max}$, $\forall\omega$. This gives the controllability requirement:

$$|G(j\omega_S)| < \frac{y_{max}}{|d_u|}, \quad (1)$$

where $|S(j\omega_S)| = 1$ and typically $\omega_S \approx 0.5/\theta$ (θ is the effective delay in the loop).

Example, pH neutralization: $|G(0)| > 10^4$. Use many tanks to get $G(s)$ high order so it drops off.

Limited Input Resolution

Limited input resolution is represented in Figure 2 by a quantizer in which $u_q = q \cdot \text{round}(u/q)$, where q is the quantization step.

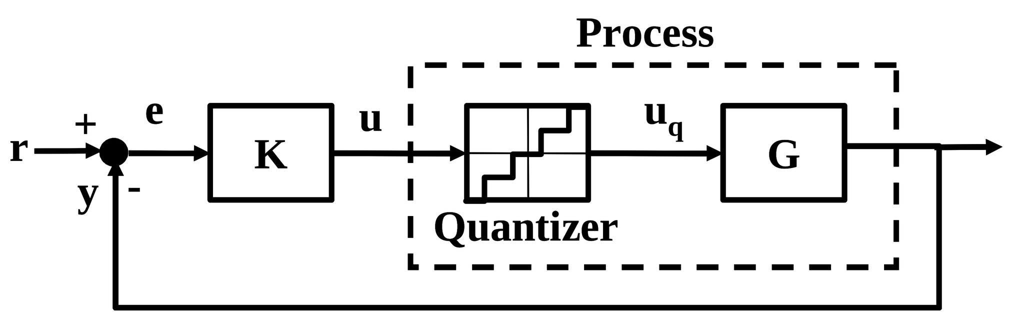

Figure 2. Feedback control configuration for limited input resolution.

With a quantizer, limit cycles oscillations are inevitable if the controller has integral action - independent of the controller tuning.

Proof: At steady state the average $u_{ss} = r/|G(0)|$ does not generally match u_q. With oscillations between two quantization levels, $u_{ss} = u_{q1} \cdot f + u_{q2} \cdot (1-f)$ [f = fraction of time at a given quantizer level].

Sinusoidal oscillations

Consider the following system:

$$G(s) = \frac{100}{(10s+1)(s+1)^2}$$
$$K(s) = 0.04\frac{10s+1}{10s}$$
$$q = 0.03$$
$$r_0 = 1 \quad (2)$$

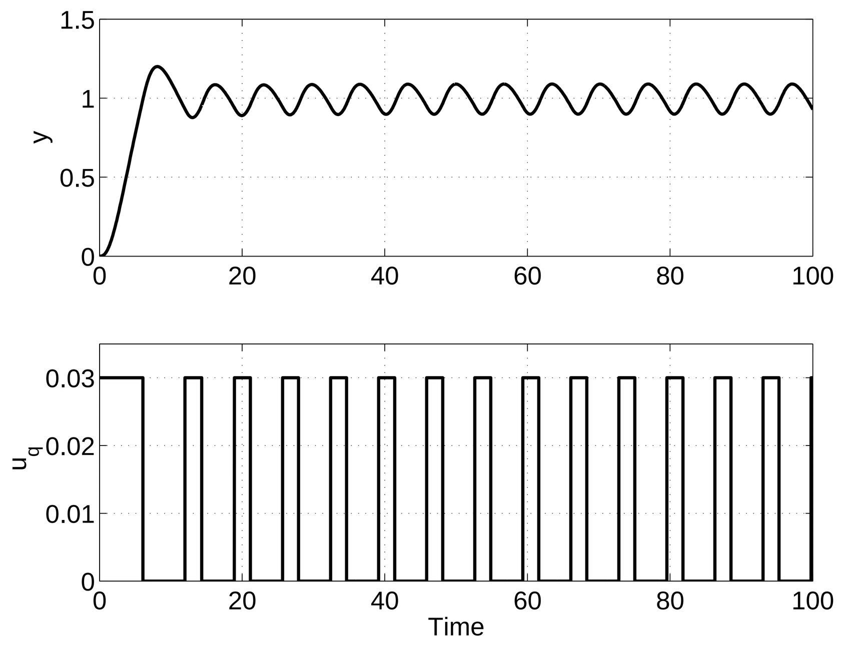

Figure 3. Simulation results for the system (2).

From the figure, the oscillations have magnitude $a = 0.189$ and period $T = 6.72s$. From a describing function analysis, assuming sinusoidal behavior, $a = (4q/\pi) \cdot |G(j\omega_{L,180})|$. This analysis agrees with the simulation in Figure 3: $a = 0.187$ and $T = 2\pi/\omega_{L,180} = 6.28$. The corresponding controllability requirement with $a = y_{max}$ is then:

$$|G(j\omega_{L,180})| < \frac{\pi}{4} \cdot \frac{y_{max}}{q}, \quad (3)$$

where $L = GK$. Typically, $\omega_{L,180} \approx 1.5/\theta$.

Nonsinusoidal oscillations

Consider the following system:

$$G(s) = \frac{100e^{-s}}{10s+1}$$
$$K(s) = 0.04\frac{10s+1}{10s}$$
$$q = 0.03$$
$$r_0 = 0.2 \quad (4)$$

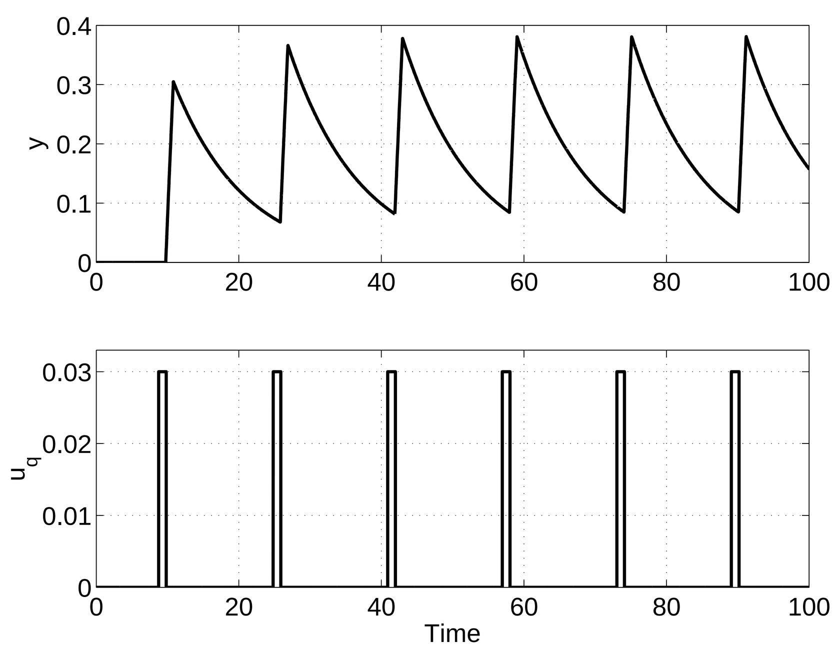

Figure 4. Simulation results for the system (4).

For a first-order plus time delay processes $G(s) = ke^{-\theta s}/(\tau s + 1)$ with a PI-controller $K = K_c(\tau_I s + 1)/\tau_I s$ and $\tau_I = \tau$, the exact amplitude (a) and period (T) of the limit cycles are:

$$a = kq\left(\frac{(1-e^{-(\theta/\tau)/(1-f)}) \cdot (1-e^{-(\theta/\tau)/f})}{1-e^{-(\theta/\tau)[1/(1-f)+1/f]}}\right); \quad T = \theta\left(\frac{1}{1-f} + \frac{1}{f}\right) \quad (5)$$

T varies between 4θ ($f = 0.5$) and ∞ ($f = 0$ or $f = 1$).

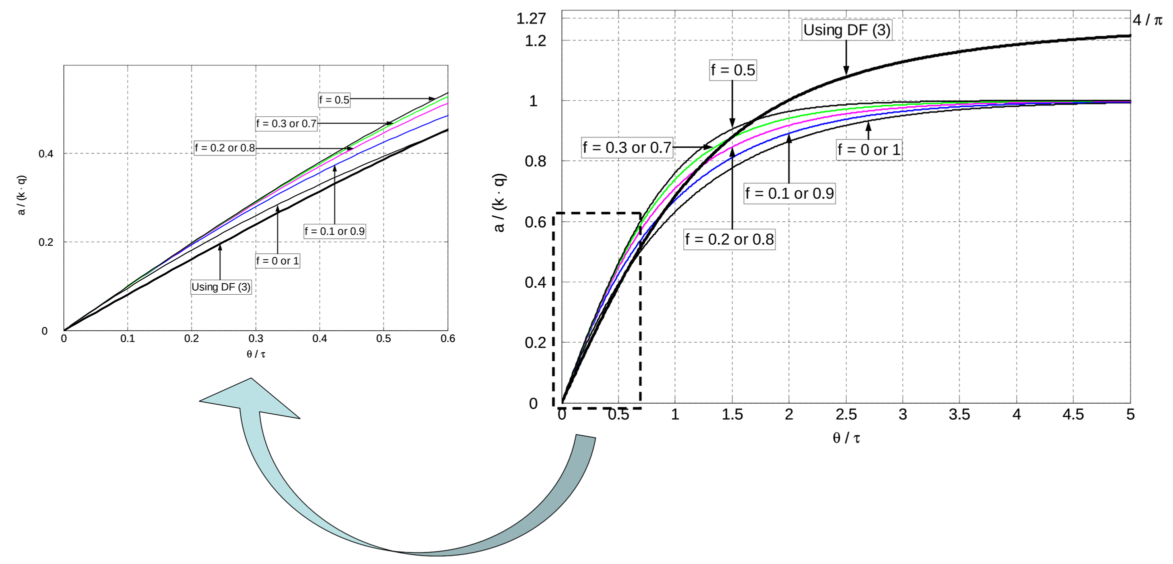

Figure 5. The amplitude in (5) agrees surprisingly well with the describing function analysis (3).

Example Figure 4: the measured and calculated values (5) are $a = 0.30$ and $T = 16.07s$. For this case, (3) is a bit optimistic as it gives $a = 0.24$ and $T = 4\theta = 4s$. Conclusion: (3) is a nice bound!

How to Avoid Oscillations

The oscillations in Figure 2-4 can be reduced by the following means:

a. Change the valve (smaller q);
b. Redesign the process (smaller effective delay θ);
c. Remove integral action (but P-control may give poor performance);
d. Introduce fast forced cycling, e.g., $d_u = \sin\omega t$ (may wear out the valve, see Figure 6).

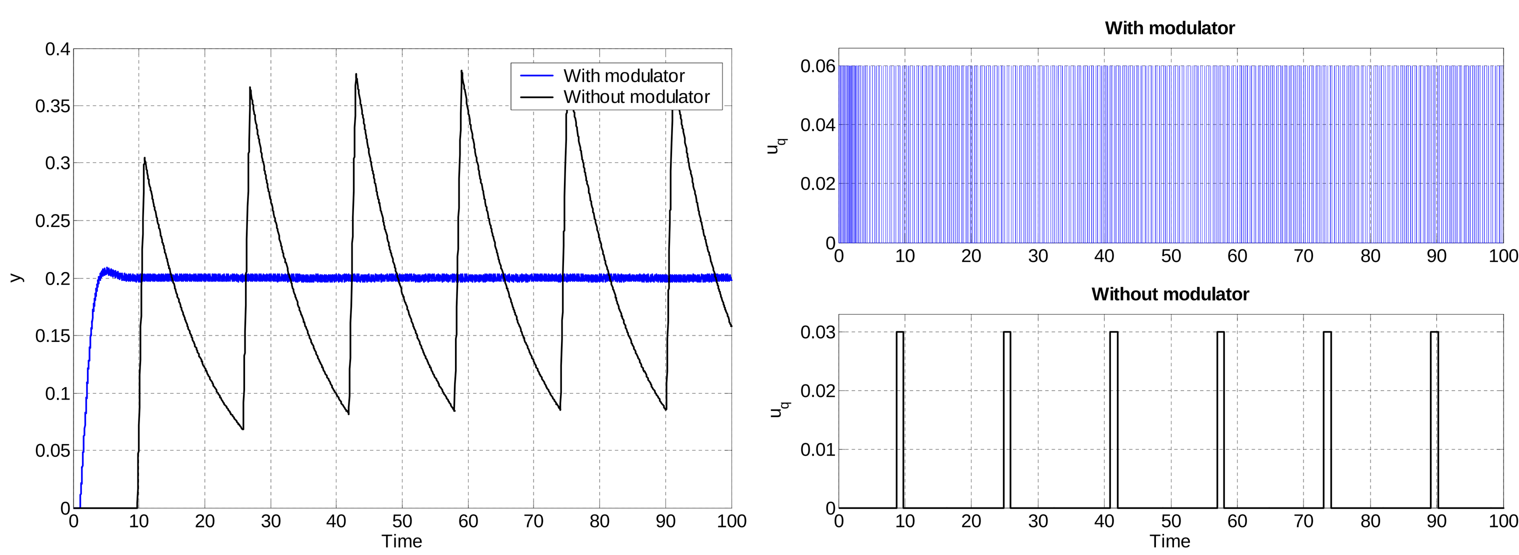

Figure 6. Simulation results for the system given by (4) using a modulator.

Conclusion

Large steady-state gain $|G(0)|$ is not a problem.

Large gain $|G(j\omega)|$ at bandwidth frequencies should be avoided:

1. With input load disturbances, a high gain implies the need for a high bandwidth which cannot always be achieved.
2. For limited input resolution, high gains give a large amplitude of the resulting limit cycles.
3. Controllability: (1) is more restrictive than (3) if $|d_u| > q/2.4$ (normally the case).