NTNU

Robust operation by controlling the right variable combination

Presented at AIChE Annual Meeting, Indianapolis, USA, November 7th 2002

Vidar Alstad†

and

Sigurd Skogestad

Department of Chemical Engineering

Norwegian University of Science and Technology

Trondheim, Norway

†vidaral@chemeng.ntnu.no

Outline

- Introduction and motivation
  - What is self-optimizing control
  - Requirements for controlled variables
- Procedure for selecting combinations of measurements
  - How to find optimal combination?
  - Which measurements to select?
- Example:
  - Divided wall (Petlyuk) distillation column
- Conclusion
- References

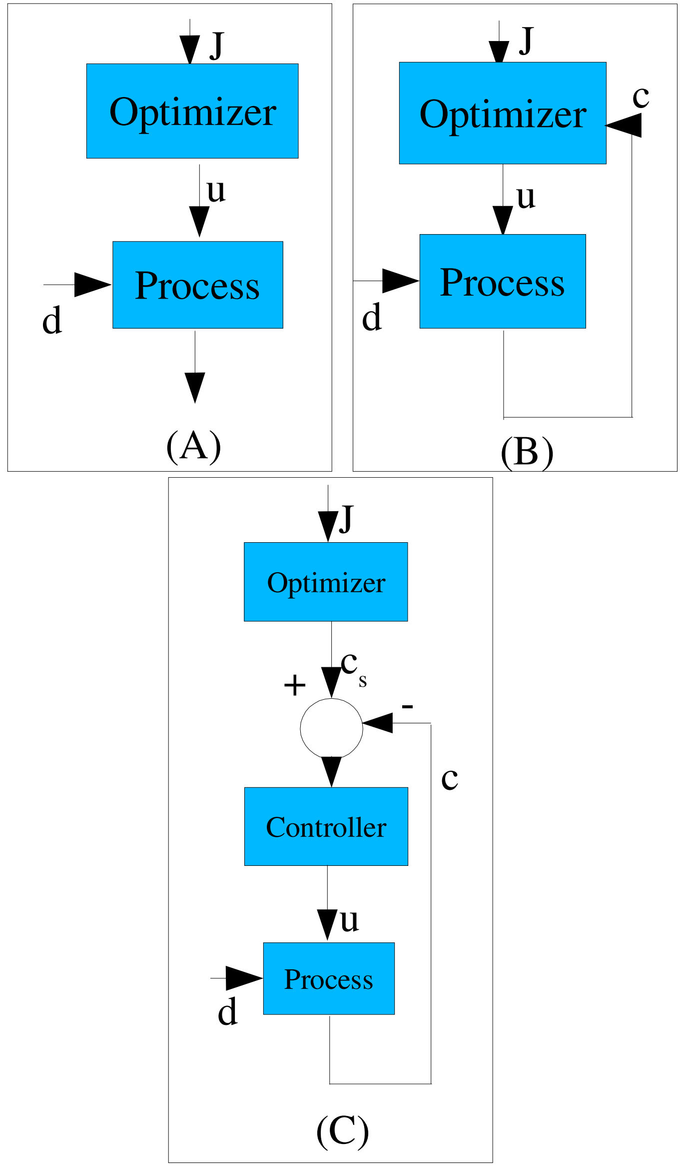
Introduction and motivation

- Optimal operation for a given disturbance d:

$$\min_u J(x, u, d)$$

$$f(x, u, d) = 0$$
$$g(x, u, d) \leq 0$$
$$x \in X, d \in D$$

- **How to implement?**
  - Open loop structure (u_s) (A)
  - Optimizing controller (B).
  - *Self-optimizing control: Simple feedback control (C)*
- Controlling the right variables; key element in overcoming uncertainty

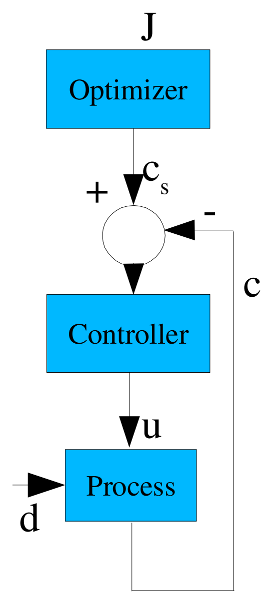


Self-optimizing control-Basics

- Define loss:

$$L = J(c_s + n, d) - J_{opt}(d)$$

- Self-optimizing control (Skogestad, 2000)
  - Self-optimizing control is when acceptable loss can be achieved using constant setpoints (c_s) for the controlled variables c (without re-optimizing when disturbances occur).



(C)

- Generally two classes of problems
  - Constrained: All DOF optimally constrained (easy)
  - **Unconstrained: Unconstrained DOF (here)**

Self-optimizing control-Basics (cont.)

- Controlled variables *c* to be selected among all available measurements *y*
- Previously:

$$c_1 = y_1, \quad c_2 = y_2$$

- Question: How to find the best combination?

$$c = f_y(y)$$

$$c = h_1 y_1 + h_2 y_2 + h_3 y_3 \cdots = Hy$$

- **How to select H?**

Self-optimizing control-Basics

- Best self-optimizing structure:

$$\min_{H,u}\left[\max_{d,n} L(x,u,d,n,c_s,c)\right]$$

$$\begin{gathered} f(x,u,d)=0 \\ g(x,u,d)\leqslant 0 \\ c=Hy \\ c(x,u,d)=c_s+n \\ d\in D,\ n\in N \end{gathered}$$

- Non-convex and combinatorial optimization problem
- Difficult to solve for any realistic chemical process!
- **Need a much simpler method!**

Candidate controlled variables

- Requirements for good candidate controlled variables (Skogestad & Postlethwaite, 1996)
  1. Its optimal value $\mathbf{c_{opt}(d)}$ is insensitive to disturbances. $\longrightarrow$ $\Delta c_{opt} = 0$
  2. It should be easy to measure and control accurately.
  3. The variable $\mathbf{c}$ should be sensitive to change in inputs.
  4. The selected variables should be independent.

NTNU

New proposed method for selecting variable combinations

- Calculate for small disturbances from the nominal point
  $u_{opt}(d) \longrightarrow y_{opt}(u_{opt}(d))$.

$$\Delta y_{opt} = F(d - d^0)$$

$$F = \frac{d(y_{opt})}{dd}$$

- Linear combination

$$c = Hy$$

- Perfect self-optimizing control

$$\Delta c_{opt} = HF \, \Delta d = 0$$

  - Achieved if

$$HF = 0$$

- Simply choose H such that

$$H \in null(F^T)$$

- As a consequence, we need at least as many measurements:

$$\#y = \#u + \#d$$

Candidate controlled variables (cont.)

- Requirements for good candidate controlled variables (Skogestad & Postlethwaite, 1996)
  1. Its optimal value $\mathbf{c}_{opt}(\mathbf{d})$ is insensitive to disturbances. → $\Delta c_{opt} = 0$
  2. It should be easy to measure and control accurately.
  3. The variable **c** should be sensitive to change in inputs. → Implementation error
  4. The selected controlled variables should be independent.

Candidate controlled variables (cont.)

- Taylor series expansion of loss function (Skogestad et. al, 1998)
- Two contributions to the loss

$$\Delta c = G \Delta u + G_d \Delta d$$

$$J_{uu} = \frac{\partial^2 J}{\partial u \partial u}$$

$$J_{du} = \frac{\partial^2 J}{\partial d \partial u}$$

$$L = \frac{1}{2} e_u^T J_{uu} e_u$$

$$e_u = u - u_{opt}(d) = (J_{uu}^{-1} J_{du} - G^{-1} G_d)(d - d^0) + G^{-1} n$$

Disturbance effect (OK!)

Implementation error contribution

Proposed approach for reducing implementation error

- Some freedom in choosing measurements to reduce implementation error.
- If #y > #u + #d
  - Select the most independent measurements for use in c=Hy
  - Scaled linearized model
  - Augumented plant
  - Maximize the minimum singular value of augumented plant

$$\Delta y = G^y \Delta u + G_d^y \Delta d$$

$$\Delta y = \widetilde{G}\tilde{u} = [G^y \;\; G_d^y][u \;\; d]^T$$

$$max_{\,y \in Y}\, \underline{\sigma}(\widetilde{G}), \quad \widetilde{G} = [G^y \;\; G_d^y]$$

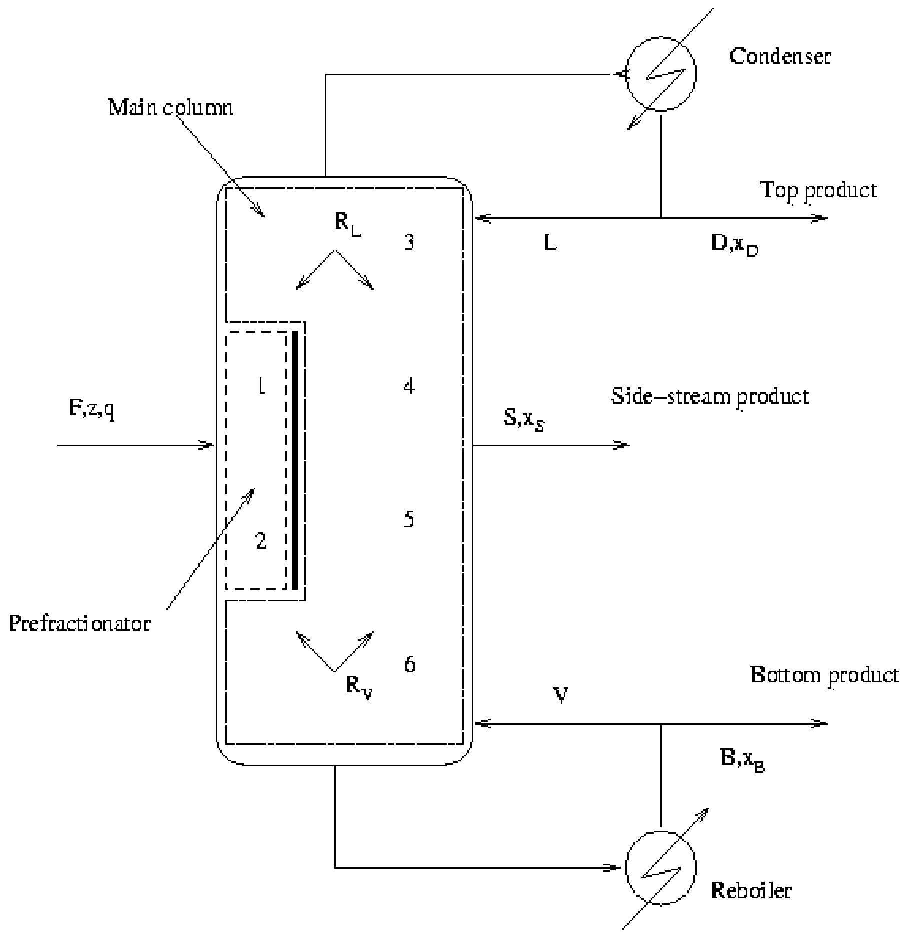
NTNU

Proposed method – Summarized

- Summarized;
  - Select m=n+k measurements that $max_{y \in Y} \underline{\sigma}(\widetilde{G})$
  - Compute F, and find H such that it is spanned by the left null space of F. $H \in null(F^T)$
  - Implies that $\Delta c_{opt} = HF \Delta d = 0$
  - Let the controlled variables be $c = Hy$

- Selection of measurements give “independent measurements”
- Perfect self-optimizing properties locally if we neglect implementation error

Divided wall (Petlyuk) distillation column



- Energy and capital cost savings up to 30% (Smith & Triantafyllou, 1992)
- J=V
- Relative volality:
  $$[\alpha_A \ \alpha_B \ \alpha_C] = [4 \ 2 \ 1]$$
- 5 steady-state degrees of freedom
  $$u = [V \ L \ S \ R_l \ R_v]^T$$
- Major disturbances
  $$d = [z_A \ q]^T = [z_A^0 \pm 0.1 \ \ q^0 \pm 0.1]^T$$
- Active constraints
  $$g = [x_{A,D} \ x_{B,S} \ x_{C,B}]^T$$
- DOF=5-3=2

Divided wall (Petlyuk) distillation column (cont.)

- R_v fixed: usually OK

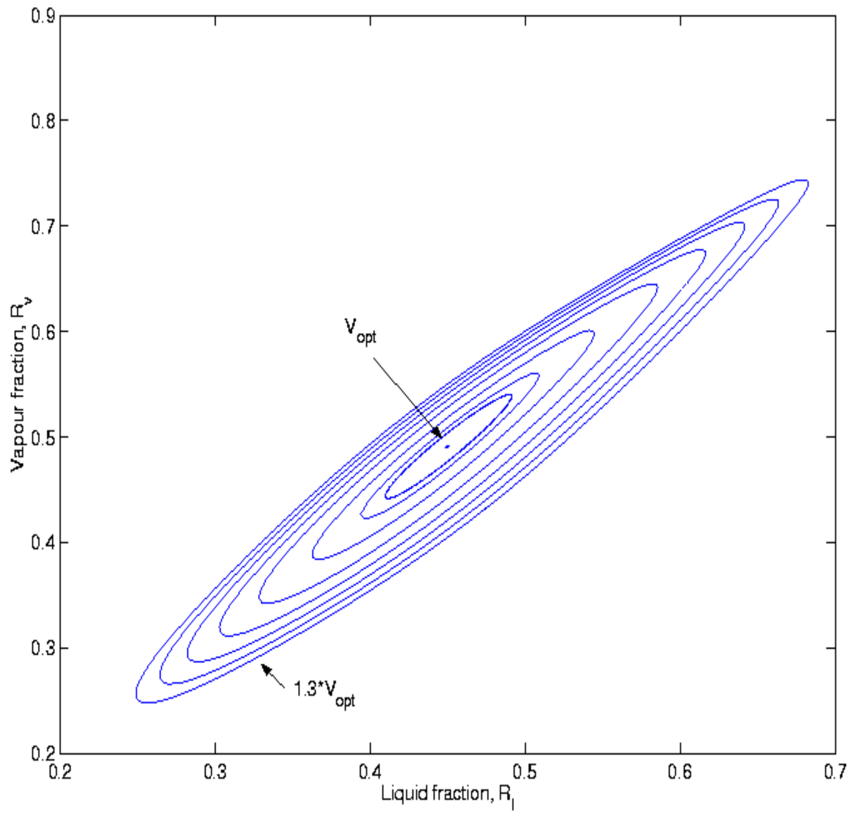
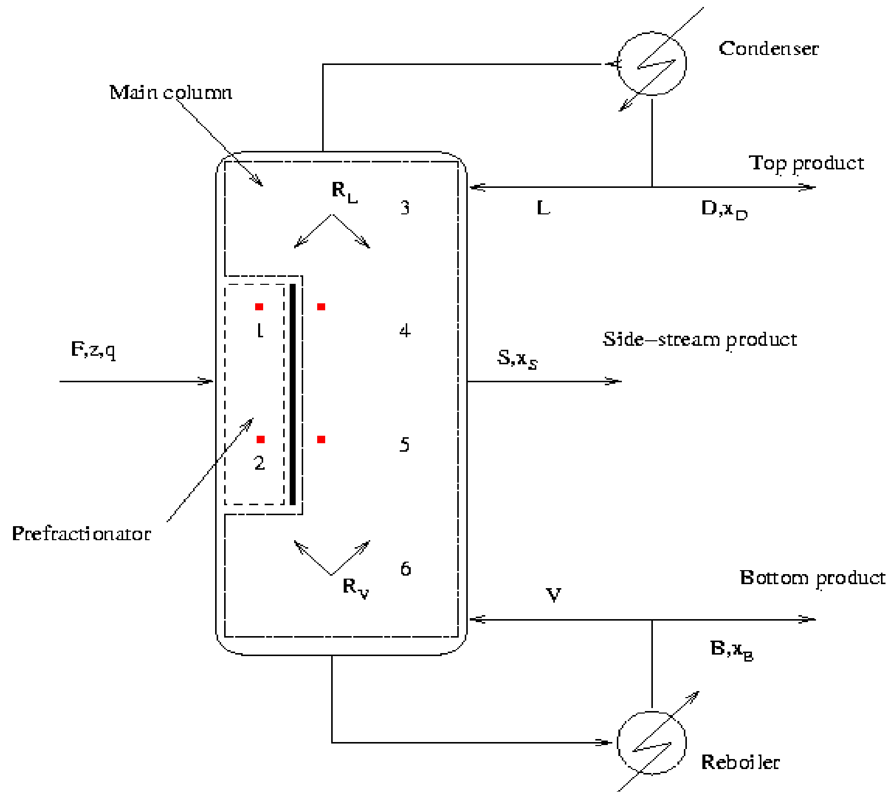


Figure 2: Contour plot J(R_L,R_v)

- Second controlled variable:
  - Single temperature: poor
  - Temperature symmetry (Halvorsen , 1998)

$$DT_s = (T_{1,i} - T_{4,i}) - (T_{2,i} - T_{5,i})$$



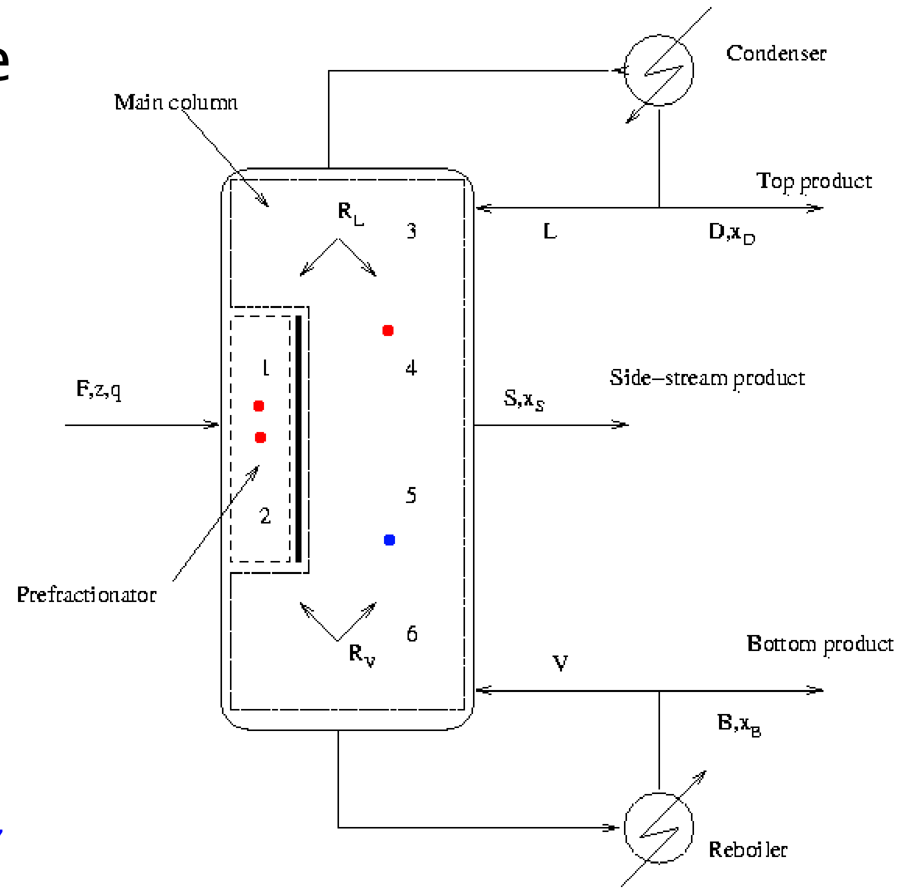
Divided wall (Petlyuk) distillation column (cont.)

R_v-fixed (1 DOF left)

- Following the method outlined above we need at least
  - $\#y = \#u + \#d = 1+2=3$
  - $c_{LC,3}=-0.523\ T_{1,6}+0.27\ T_{2,1}\ +0.83\ T_{4,2}$

R_v-variable (2 DOF left)

- Number of measurements
  - $\#y = \#u + \#d = 2+2=4$
  - $c_{LC,1}=-0.32\ T_{1,6}+0.12\ T_{2,1}\ -0.57\ T_{4,2} + 0.74\ T_{5,7}$
  - $c_{LC,2}=-0.70\ T_{1,6}+0.34\ T_{2,1}\ +0.61\ T_{4,2} - 0.12\ T_{5,7}$



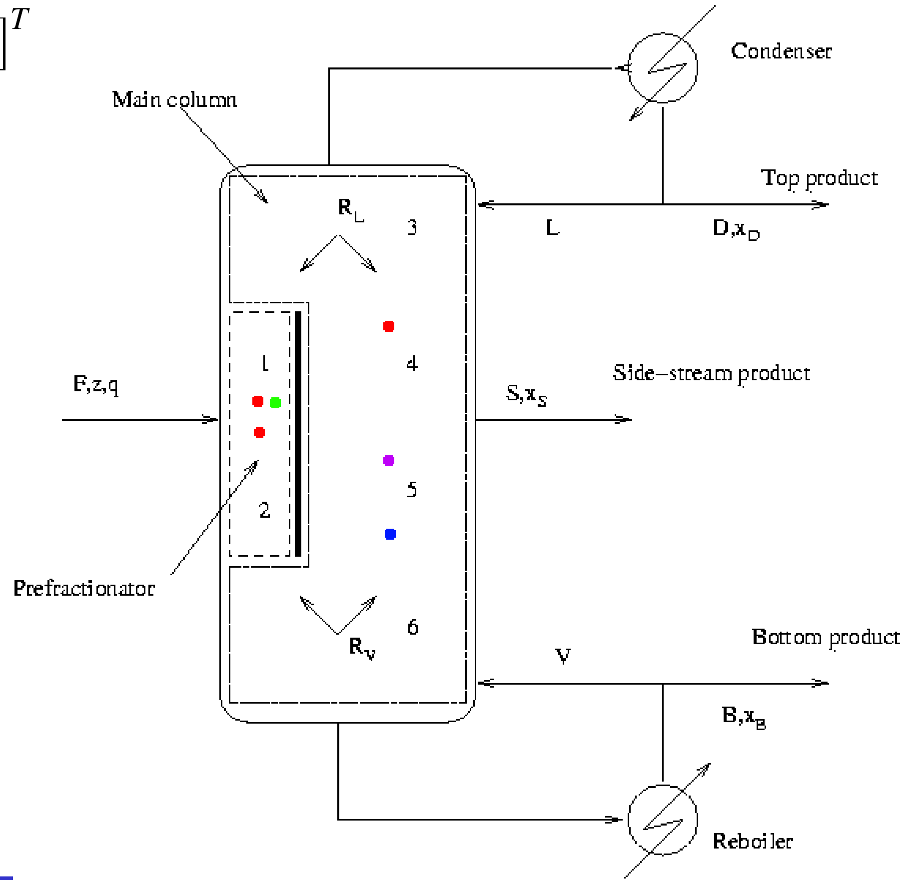
Divided wall (Petlyuk) distillation column (cont.)

- Loss for the proposed structures (combined disturbance and noise norm bounded)

$$d = [z_A \; q]^T = [z_A^0 \pm 0.1 \;\; q^0 \pm 0.1]^T$$

$$n = [n_{R_v} \; n_{R_l} \; n_T]^T = [\pm 0.05 \;\; \pm 0.05 \;\; \pm 0.4]^T$$

c_1	c_2	Rank	Average Loss(%)	Worst case Loss(%)
$c_{LC,1}$	$c_{LC,2}$	1	0.12%	0.32%
R_v	$c_{LC,3}$	2	0.34%	0.97%
R_v	DT_s	3	2.80%	4.35%
R_v	$T_{1,7}$	4	8.60%	16.60%
R_v	R_l	5	11.29%	55.82%
R_v	$T_{5,2}$	6	infeasible	infeasible



Conclusion

- Focus on how to **implement** optimal operation by finding good self-optimizing control structure.
- Proposed a new **simple** method for selecting controlled variables as a linear combination of the measurements with perfect self-optimizing control for small disturbances
- Proposed a method on how to select the necessary measurements
  - #y = #u + #d
  - “independent measurements as possible”
- Local information
- Illustrated the method on a Divided wall (Petlyuk) distillation column
  - Neglible loss by controlling the right variable combination
  - OK to fix R_v

Acknowledgment

- Financial support from The Research Council of Norway is gratefully acknowledged.
- Support of ABB and Norsk Hydro is gratefully acknowledged.

References

- Alstad, V., Skogestad, S., “Robust operation by controlling the right variable combination”, In AIChE Annual Meeting, Indianapolis, 2002.
- Halvorsen, I., “Minimum energy requirements in complex Distillation Arrangements, PhD Thesis, Dep. of Chemical Engineering, Norwegian University of Science and Technology, 2001.
- Smith, R. & Triantafyllou, C., “The design and operation of fully thermally coupled distillation columns”, *Trans. IchemE.*, 1992.
- Skogestad. S., “Planwide control: the search for the optimal control structure”, *J. Proc. Control*, **10**, 487-507, 2000.
- Skogestad. S. & Postlethwaite, I., “Multivariable feedback control”, *John Wiley & Sons*, 1996.
- Skogestad. S. , Halvorsen, I. & Morud, J.C, “Self-optimizing control: The basic idea and taylor series analysis”, In AIChE Annual meeting, Miamy, Fl., 1998.