A Method for Selection of Controlled Variables and Robust Setpoints

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Abstract

The idea of self-optimizing control is "to find a function c of the process variables, which when held constant, leads automatically to the optimal adjustments of the manipulated variables, and with it the optimal operating conditions" (Morari *et al.* 1980). One problem is that in general it is not clear off hand whether such a self-optimizing controlled variable set exists. Skogestad (2000) presents a method for selection of controlled variables, based on steady-state economics. In this paper we extend this method to include the choice of setpoints by robust optimization. As a case study we consider a reactor-separator-recycle process. For this process the control structures based on Luybens rule ("fix a flow in every recycle loop") give infeasibility if we use the nominal optimal setpoints, but it is feasible with acceptable loss with robust optimal setpoints.

1 Introduction

This paper is concerned with the *implementation* of an optimal control policy. We consider a strategy where the optimization layer sends setpoints for the controlled variables to be implemented by the control layer, see figure 1.
There are two classes of problems:

- *Constrained*: The optimal solution lies at active constraints for all expected disturbances
- *Unconstrained* (the focus of this paper): One or more of the optimization degrees of freedom are unconstrained for all or some expected disturbances.

Two important decisions are to be made:

- Decision 1: *Selection of controlled variables (c)*: This is a *structural* decision which is made off-line before implementing the control strategy.
- Decision 2: *Selection of setpoints (c_s)* for the controlled variables. This is a *parametric* decision which is usually done on-line.

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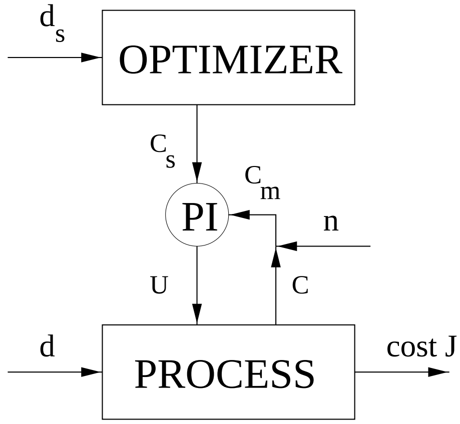

Figure 1: A typical optimization system incorporating local feedback: The process is disturbed (d) and the control system tries to keep the controlled variables (c) at their setpoints (c_s). Steady-state optimization is performed regularly to track the optimum by updating the setpoints.

For the constrained problem, we usually select the active constraints as controlled variables (Maarleveld and Rijnsdorp 1970). Then selection of setpoints (Decision 2) is the only issue. An exception is when the constraints are difficult to measure or difficult to control due to poor dynamics. This is thoroughly discussed by Perkins and coworkers, e.g. (Narraway *et al.* 1991). Another exception is when the optimally active constraint is moving. In this case one may select unconstrained variables with good self-optimizing properties.

For the unconstrained problem, the selection of what to control (Decision 1) is crucial. A main issue is that the controlled variables should yield feasible operation, that is, not violate any constraints for the expected disturbances and implementation errors. Otherwise, there may be both dynamic and steady-state problems such as instability, input saturation and operation outside constraints. To avoid such problems it may be necessary to back-off from the nominal optimum. The required back-off and the corresponding economic loss depend on the selected controlled variables. The required back-off can be reduced by using logic, model predictive control and/or on-line optimization. A good choice of controlled variables may reduce the need for these remedies and give a simpler and cheaper system.

2 Some definitions

2.1 Optimal operation

From a steady-state point of view optimal operation for a given disturbance (d) can be found by solving the following problem:

$$\begin{aligned} &\min_{x,u} J(x,u,d) \\ &f(x,u,d) = 0 \\ &g(x,u,d) \leq 0 \end{aligned} \tag{1}$$

The scalar objective function J describes the quality (cost) of operation, f is the process model, g is the inequality constraints connected to operation, u is the independent variables (inputs) we can affect, d is the independent variables (disturbances) we cannot affect and x consists of internal variables, e.g. states. The inequality constraints usually consist of upper and lower bounds on the output and input variables.

2.2 Feasibility

From a steady-state point of view operation is feasible when the following constraints are fulfilled:

$$\begin{aligned} f(x,u,d) &= 0 \\ g(x,u,d) &\leq 0 \end{aligned} \tag{2}$$

A constant setpoint policy is feasible if, with constant setpoints for the controlled variables ($c(x,u,d) = c_s + e$), none of the constraints are violated for expected variations in disturbances ($d \in D$) and implementation errors ($e \in E$).

Mathematically, this means there is a feasible solution to the following system of equations for all expected disturbances ($d \in D$) and implementation errors ($e \in E$).

$$\begin{aligned} f(x,u,d) &= 0 \\ g(x,u,d) &\leq 0 \\ c(x,u,d) &= c_s + e \end{aligned} \tag{3}$$

where c is the variables that we try to keep constant (equal c_s). The implementation error e is the sum of the measurement error ($c_m - c$) and the control error ($c_s - c_m$), see figure 1. We distinguish between hard and soft constraints. Soft constraints may be violated in transients, but not at steady-state (average). Hard constraints must neither be violated in transients nor at steady-state. We then have

- For controlled variables related to soft constraints we should only include the steady-state implementation error which with integral action equals the steady-state measurement error.
- For controlled variables related to hard constraints we must also include the worst-case dynamic control error.

2.3 Back-off

Back-off from nominal optimal operation is sometimes needed to achieve feasible operation. *The "back-off"* is *the difference between the actual setpoints and the nominal optimal setpoints* $b = c_s - c_{s,opt}(d_0)$. Figure 2 demonstrates the need for back-off. The cost is shown as function of controlled variables. At nominal point the cost is presented by line $J(d_0, c)$. If a disturbance happens, the system changes. The new cost is presented by line $J(d_1, c)$. Outside the lines constraints are violated and the operation is infeasible. If we use nominal optimal setpoints ($c_{s,opt}(d_0)$), we get infeasibility. To avoid this we may change setpoints by doing some back-off (b).

There are two types of back-off:

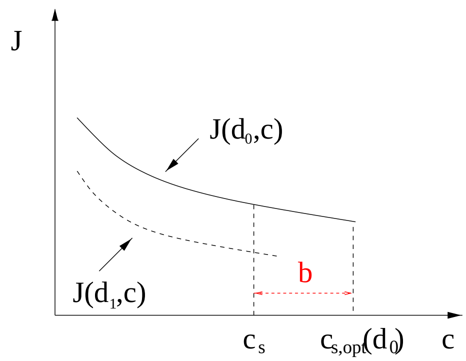

Figure 2: Cost (J) as function of controlled variable (c) for different disturbances (d_0,d_1)

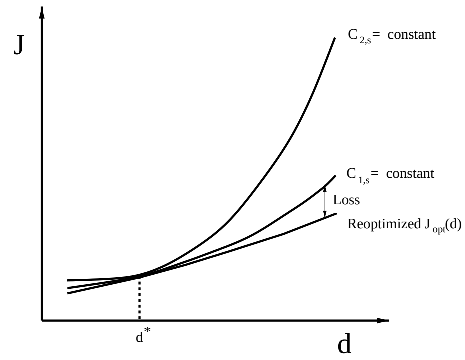

Figure 3: The loss is the difference between the cost by keeping a set of controlled variables constant ($c_{1,s}$ or $c_{2,s}$) and the cost by re-optimizing for a given disturbance (d)

1. *"Simple" back-off* is moving away from active constraints as given by the implementation errors.

"Simple" back-off is used for variables at active output constraints. Back-off for variables at active input constraints is often not necessary, because it is impossible to violate the input constraints, e.g. to have a negative valve opening

2. *"Complex" back-off* is used in problems where "simple" back-off is not sufficient.

"Complex" back-off may be required for (1) unconstrained problems where the optimal value of c is unconstrained for all or some operating points (i.e. for some disturbances) or (2) constrained problems where the optimal value of c moves from one set of active constraints to another during operation.

2.4 Optimal back-off

The required back-off and economic loss depend on the selected controlled variables. The optimal back-off can be determined by *robust optimization* (Glemmestad *et al.* 1999), where we find the setpoints (c_s) that minimize an economic criterion (usually steady-state) and satisfy constraints for the expected disturbances (D) and implementation errors (E):

$$\begin{aligned} \min_{x,u,c_s} \int_{e\epsilon E} \int_{d\epsilon D} w(d,e) J(x,u,d) \; \mathrm{d}d \; \mathrm{d}e \\ f(x,u,d) = 0 \\ g(x,u,d) \leq 0 \\ c(x,u,d) = c_s + e \\ d \in D \\ e \in E \end{aligned} \tag{4}$$

The optimal back-off is then:

$$b_{opt} = c_{s,robust} - c_{s,opt}(d_0) \tag{5}$$

where $c_{s,robust}$ is the robust optimal setpoints and $c_{s,opt}(d_0)$ is the nominal optimal setpoints. Nominal optimal setpoints are found by solving equation 1 with respect to nominal disturbances (d_0).

2.5 Self-optimizing control

A set of controlled variables have good self-optimizing properties, when constant setpoints yield acceptable operation for expected variation in disturbances and implementation errors (Skogestad 2000). More precisely, the loss L should be acceptable. The loss for a given disturbance (d) is the difference between the cost by keeping a set of controlled variables constant ($J(c_s+e,d)$) and the cost by re-optimizing ($J_{opt}(d)$):

$$L = J(c_s + e, d) - J_{opt}(d) \tag{6}$$

Figure 3 shows loss as function of disturbances for different sets of controlled variables.

3 Robust optimization

We will now discuss robust optimization, see equation 7, in more detail.

3.1 Dimensions

The problem is infinite dimensional. However, it can be simplified by choosing a limited number of operation points (i).

$$\begin{aligned} \min_{x_i,u_i,c_s} \sum_i w_i J(x_i,u_i,d_i) \\ f(x_i,u_i,d_i) &= 0 \\ g(x_i,u_i,d_i) &\leq 0 \\ c(x_i,u_i,d_i) &= c_s + e_i \\ d_i &\in D \\ e_i &\in E \end{aligned} \tag{7}$$

If we chose the nominal point and the corner points for expected disturbances and implementation errors, see figure 4, this gives
$(2n_d + 2n_u + 1) \times (n_u + n_x) + n_u$ optimization variables,
$(2n_d + 2n_u + 1) \times (n_u + n_x)$ equality constraints and
$(2n_d + 2n_u + 1) \times n_g$ inequality constraints.
It is clear that the problem becomes very large even for a modest number of states, disturbances and controlled variables. In addition, the grids need to be dense enough to include important nonlinearities. The optimization problem may be solved by using a subspace optimization algorithm.

An alternative way to do robust optimization is to optimize for expected worst-case disturbances. This gives a smaller problem to solve, but it is not obvious which are the worst-case disturbances. In addition minimizing the expected objective value is preferable from an economic point of view.

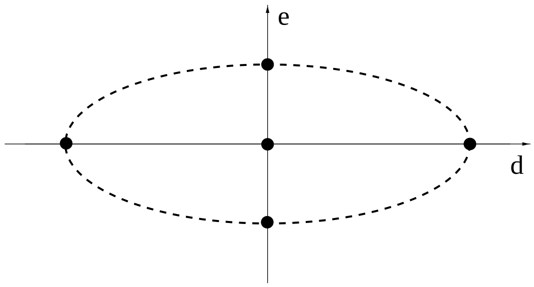

Figure 4: Operating points

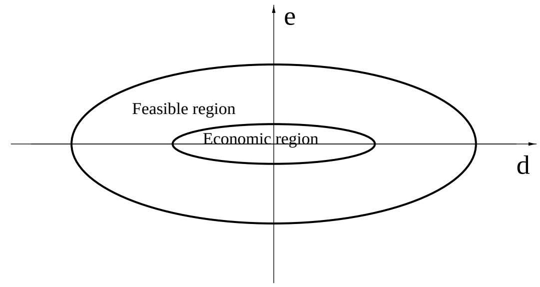

Figure 5: Feasible and economic region

3.2 Weights

The weights in the objective function (cost) can be chosen in different ways. Using a nominal objective ($w_0 = 1$, $w_i = 0$ when $i \neq 0$) gives no difference between sets of controlled variables where nominal optimal setpoints are feasible. Using an average objective with respect to all operation points considered ($w_i = 1$ for all i) may give a too conservative operation. Preferably, the weights should be chosen equal to the probability for operation in the respective points. However, feasibility may be very important. This can be handled by distinguishing between an economic and a feasible region, see figure 5. The constraints must be fulfilled in the feasible region, whereas the cost is average in the economic region.

4 Method for selecting controlled variables and robust setpoints

In the method presented by Skogestad (2000) the nominal optimal setpoints were used to identify promising sets of controlled variables. Here we focus on achieving feasible operation by implementing setpoints found by robust optimization ("optimal back-off"). We use a five step procedure

1. *Initial system analysis*:
   Identify the number of degrees of freedom, define objective function and constraints, identify main disturbances and candidates for controlled variables, optimize at nominal and for expected disturbances, see equation 1.

2. *Identify candidate controlled variables sets*:
   Eliminate variables with no steady-state effect, use active constraint control, eliminate variables with large losses by using short-cut loss evaluation, eliminate variables based on process insight.

3. *Evaluate loss and select setpoints for different sets of controlled variables, by using nominal optimization and "simple" back-off.*

4. *Evaluate loss and select setpoints for different sets of controlled variables, by using robust optimization*, see equation 7.

5. *Final evaluation and selection of control structures*:
   Stabilization, controllability analysis, selection of control configuration and simulation of proposed control structures.

The method is applied to a reactor, separator and recycle process in next section.

An alternative to initial screening (step 2) before evaluating the loss (step 3 and 4) is mathematical programming to find sets of controlled variables which imply small losses. The robust optimization is then the inner problem in a MINLP-problem. If including a controllability test (step 5) for different controlled variable sets, the selection of controlled variables is done automatically.

5 Example: Reactor, separator and recycle process

The process equipment consists of reactor, distillation column and liquid recycle (Papadourakis *et al.* 1987). There is no inert in the feed, and no purge is required. We apply model parameters presented by Wu and Yu (1996). Larsson (2000) identify promising sets of controlled variables for this process using nominal optimal setpoints.

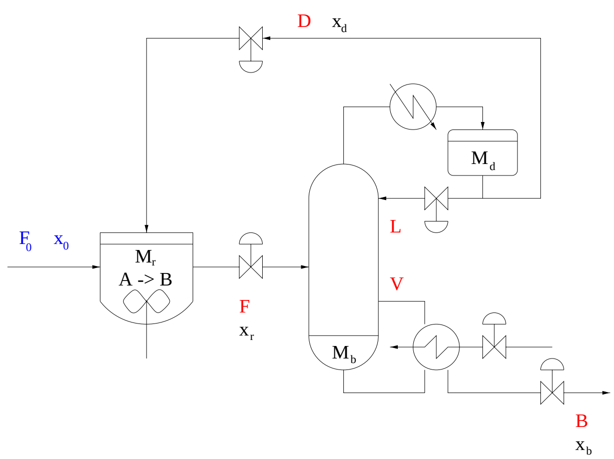

Figure 6: Reactor/separator process with liquid recycle

The computations are done by Matlab. Optimization toolbox is used in nominal and robust optimization and μ-analysis and synthesis toolbox is used during controllability analysis.

5.1 Initial system analysis

The process has five manipulated variables (valves) which give five degrees of freedom.

$$u^T = [L \ V \ B \ D \ F]$$

However, two of them (the reboiler holdup (M_b) and condenser holdup (M_d)) have no steady-state effect. Then there are three degrees of freedom at steady-state. These may, for example, be selected as the reactor holdup (M_r), product composition (x_b) and recycle composition (x_d). The economic objective is to maximize the profit (the value of the products minus the cost of the utilities and raw materials). Since F_0 is given, B is given and L depends directly on V, this can be simplified to minimize the boilup flow rate:

$$J = V$$

The reactor volume is constrained and there is a purify specification on the product:

$$\begin{aligned} 0 \leq M_r &\leq 2800 \\ x_b &\leq 0.015 \end{aligned}$$

The main disturbances are feed flow rate (F_0) and feed composition (x_0):

$$\mathrm{d}^T = [F_0 \; x_0] = [460 \pm 92 \text{ kmol/h } 0.90 \pm 0.05 \text{ molA/mol}]$$

The 20 candidate controlled variables (9 manipulated variables and measurements and 11 flow ratios) are given below:

$$c^T = [u \; x_r \; x_b \; x_d \; L/F \; V/F \; B/F \; D/F \; V/L \; B/L \; D/L \; B/V \; D/V \; B/D \; F/F_0]$$

The implementation errors are initially assumed as $\pm 10\%$ for flow rates, $\pm 0.5\%$ (absolute) for compositions and $\pm 1\%$ for holdups. Steady-state optimization for the nominal operation and the corner-points in the disturbance regime, see equation 1, show that the product composition (x_b) and the reactor holdup (M_r) are always at their constraints.

5.2 Identify sets of candidate controlled variables

20 candidate controlled variables and three steady-state degrees of freedom give 1140 alternative sets of controlled variables, and we need to reduce the number of sets. We have already eliminated the condenser (M_d) and the reboiler holdup (M_b) which have have no steady-state effect. We choose to control the active constraints. As mentioned, there are two active constraints (c_1=product composition and c_2=reactor holdup). We are then left with 18 candidate controlled variables and 1 steady-state degree of freedom, which give 18 possible sets.

Initial screening is performed by maximizing the steady-state gain ($|G(0)|$) where $G(0)$ is obtained with the active constraints kept constant. The candidate controlled variables are scaled with respect to variation in optimal values and implementation errors. From table 1, we see that x_d and L/F are the most promising controlled variables.

At steady-state the product flow rate must be equal to the feed flow rate ($B = F_0$). Thus, keeping the product flow rate B constant when the feed flow rate changes,

Table 1: Candidate controlled variables ranked by steady-state gain ($|G(0)|$)

Rank	c_3	$\lvert G(0)\rvert \cdot 10^3$
1	x_d	13.1
2	L/F	8.9
3	D/L	7.7
4	D/V	5.8
5	V/L	4.5
6	B/L	4.1
7	V/F	4.0
8	B/D	3.3
9	L	3.0
10	B/F	2.6
11	D	2.6
12	F/F_0	2.5
13	D/F	2.5
14	F	1.9
15	B/V	0
15	V	0
15	x_r	0
15	B	0

does not give feasible steady-state operation. Hence the product flow rate (B) is eliminated as a candidate controlled variable. The product flow rate (B) is given by the component balance of the product:

$$B = kM_r x_r / x_b$$

Here M_r and x_b are controlled at their active constraints, when, as just noted, $B = F_0$. Thus the reactor composition (x_r) is fixed and can be eliminated as a candidate controlled variable. 16 candidate controlled variables and 1 steady-state degree of freedom still remain, which give 16 possible sets.

5.3 Loss evaluation with nominal optimal setpoints

For the remaining 16 alternative sets we evaluate the economic losses imposed by using constant setpoints instead of re-optimization. Average losses (and setpoints) when using *nominal* optimal setpoints are shown in table 2. Simple backoff from active constraints is included. We rank the alternatives based on average loss. Control of x_d (figure 7) closely followed by L/F(figure 8), D/V and D/L, gives the smallest average loss, see also (Larsson 2000). Control of F or D, which follow Luybens rule ("fix a flow in every recycle loop") (Luyben *et al.* 1997), give infeasibility. In addition we have evaluated some alternatives proposed in literature. None of them control the reactor holdup, and none of them yield feasible operation for all disturbances.

5.4 Loss evaluation with robust optimal setpoints

Use of nominal optimal setpoints may exclude controlled variables that are workable.In the worst case we may not find any feasible sets of controlled variables at

Table 2: Average operation costs and loss when using nominal optimal setpoints

Rank	c_1	c_2	c_3	$c_{1,s}$	$c_{2,s}$	$c_{3,s}$	$J_{average}$	%-loss
Optimal	–	–	–	–	–	–	1231.23	–
1	x_b	M_r	x_d	0.9900	2772	0.8186	1322.35	7.4
2	x_b	M_r	L/F	0.9900	2772	0.8207	1324.20	7.6
3	x_b	M_r	D/L	0.9900	2772	0.6379	1324.17	7.6
4	x_b	M_r	D/V	0.9900	2772	0.3893	1325.87	7.8
5	x_b	M_r	V/F	0.9900	2772	1.3446	1331.10	8.2
6	x_b	M_r	L	0.9900	2772	792.6702	1336.88	8.8
7	x_b	M_r	V/L	0.9900	2772	1.6585	1329.79	9.9
8	x_b	M_r	B/D	0.9900	2772	0.9098	1330.97	11.0
–	x_b	M_r	B/F	–	–	–	infeas	infeas
–	x_b	M_r	B/L	–	–	–	infeas	infeas
–	x_b	M_r	B/V	–	–	–	infeas	infeas
–	x_b	M_r	D	–	–	–	infeas	infeas
–	x_b	M_r	D/F	–	–	–	infeas	infeas
–	x_b	M_r	F	–	–	–	infeas	infeas
–	x_b	M_r	F/F_0	–	–	–	infeas	infeas
–	x_b	M_r	V	–	–	–	infeas	infeas
–	x_b	F	x_d	–	–	–	infeas	infeas
–	x_b	x_r	x_d	–	–	–	infeas	infeas
–	x_b	F/F_0	x_d	–	–	–	infeas	infeas
–	V/B	F/F_0	x_d	–	–	–	infeas	infeas
–	x_b	F/F_0	V/B	–	–	–	infeas	infeas
–	x_b	F/F_0	L/D	–	–	–	infeas	infeas
–	x_b	M_r/F	L/D	–	–	–	infeas	infeas

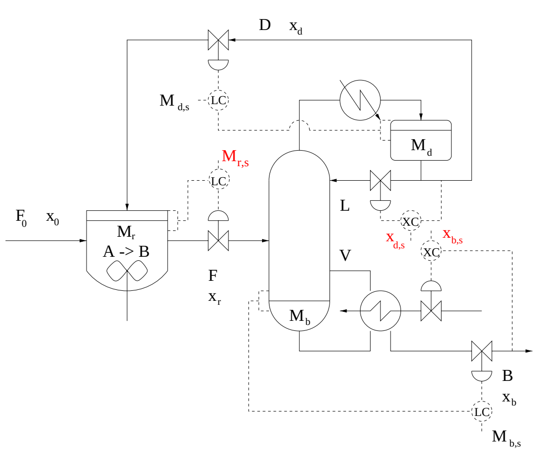

Figure 7: Alternative x_d

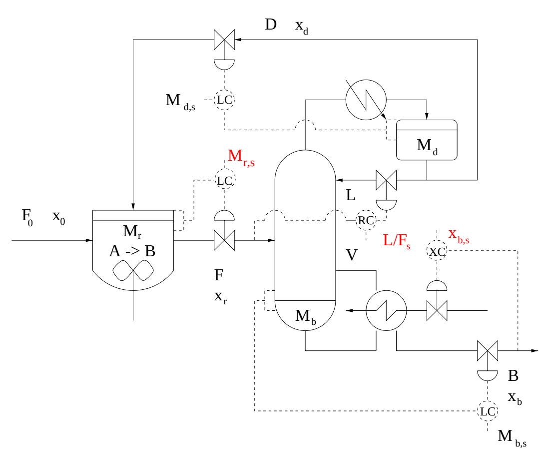

Figure 8: Alternative L/F

Table 3: Average operation costs and loss when using robust optimal setpoints

Rank	c_1	c_2	c_3	$c_{1,s}$	$c_{2,s}$	$c_{3,s}$	$J_{average}$	%-loss
Optimal	–	–	–	–	–	–	1231.23	–
1	x_b	M_r	x_d	0.9900	2772	0.8243	1322.35	7.4
2	x_b	M_r	L/F	0.9900	2772	0.8061	1324.20	7.6
3	x_b	M_r	D/L	0.9900	2772	0.6863	1324.17	7.6
4	x_b	M_r	D/V	0.9900	2772	0.4159	1325.87	7.7
5	x_b	M_r	V/L	0.9900	2772	1.8455	1329.79	8.0
6	x_b	M_r	B/D	0.9900	2772	0.7741	1330.97	8.1
7	x_b	M_r	V/F	0.9900	2772	1.3084	1331.10	8.1
8	x_b	M_r	B/L	0.9900	2772	0.6441	1332.65	8.2
9	x_b	M_r	F/F_0	0.9900	2772	2.3856	1333.47	8.3
10	x_b	M_r	B/F	0.9900	2772	0.4219	1334.96	8.4
11	x_b	M_r	D/F	0.9900	2772	0.5849	1336.65	8.6
12	x_b	M_r	L	0.9900	2772	716.0416	1336.88	8.6
13	x_b	M_r	F	0.9900	2772	1249.7635	1347.75	9.5
14	x_b	M_r	D	0.9900	2772	880.9874	1350.97	9.7
15	x_b	M_r	B/V	0.9900	2772	0.2703	1735.32	40.9
–	x_b	M_r	V	–	–	–	infeas	infeas
16	x_b	F/F_0	V/B	0.9900	2.4963	3.4371	1581.08	28.4
17	x_b	F/F_0	x_d	0.9900	2.4963	0.8522	1585.98	28.8
18	x_b	M_r/F	L/D	0.9900	2.0231	1.2803	1586.64	28.9
19	x_b	x_r	x_d	0.9900	0.4852	0.8522	1588.36	29.0
20	x_b	F/F_0	L/D	0.9900	3.2497	0.5973	1652.97	34.3
21	x_b	F	x_d	0.9900	1954.5	0.7135	1794.04	45.7
22	V/B	F/F_0	x_d	3.9475	2.6167	0.8366	1815.83	47.5

all. We therefore consider use of robust optimal setpoints, see equation 7. We select both the feasible and economic region to include the expected disturbances and implementation errors. We select the nominal point and corner points for expected disturbances and implementation errors as operation points with equal weights (w). We rank the different sets of controlled variables based on their cost in optimum (average loss), see table 3. Interestingly, there are only minor changes compared to table 2 among the best alternatives. However, control of F and D which follow Luybens rule, are now feasible and give acceptable loss. Also alternatives which do not control the reactor holdup, are feasible, but give large losses (28-48%).

Anyway, the conclusion has not changed. The loss is smaller and control is simpler, if we keep x_d or L/F at nominal optimal setpoints rather than controlling D or F at robust optimal setpoints.

5.5 Final evaluation and selection of control structures

We will now check the control properties of the four alternatives with the smallest loss (x_d, L/F, D/L and D/V) and the two alternatives that follow Luybens rule (F and D). The reactor holdup, reboiler holdup and condenser holdup are first stabilized. The controllability analysis reveals no problems for the six alternatives. As the alternatives show small interactions, decentralized control is selected. The pairing of the controlled variable and manipulated variables is based on steady-state relative gain array (RGA), as shown in table 4. Loop 1 and 2 are stabilizing loops. Loop 3 and 4 are used to control active constraints.

Table 4: Proposed decentralized control structures based on RGA

Alternative	*Loop* 1	*Loop* 2	*Loop* 3	*Loop* 4	*Loop* 5
x_d	$M_b \leftrightarrow B$	$M_d \leftrightarrow D$	$M_r \leftrightarrow F$	$x_b \leftrightarrow V$	$\mathrm{x}_d \leftrightarrow L$
L/F	$M_b \leftrightarrow B$	$M_d \leftrightarrow D$	$M_r \leftrightarrow F$	$x_b \leftrightarrow V$	L/F$\leftrightarrow L$
D/L	$M_b \leftrightarrow B$	$M_d \leftrightarrow D$	$M_r \leftrightarrow F$	$x_b \leftrightarrow V$	D/L$\leftrightarrow L$
D/V	$M_b \leftrightarrow B$	$M_d \leftrightarrow D$	$M_r \leftrightarrow F$	$x_b \leftrightarrow V$	D/V$\leftrightarrow L$
F	$M_b \leftrightarrow B$	$M_d \leftrightarrow L$	$M_r \leftrightarrow D$	$x_b \leftrightarrow V$	F
D	$M_b \leftrightarrow B$	$M_d \leftrightarrow L$	$M_r \leftrightarrow F$	$x_b \leftrightarrow V$	D

The IMC-tuning is used to select control parameters. Simulations are performed with robust optimal setpoints for step changes in the disturbances, feed flow rate (F_0) and feed composition (x_0). Responses for the reactor holdup and the product composition for increase in the feed flow rate ($\Delta F_0 = +20\%$) are shown in figure 9 and 10. Control of F gives the fastest control and x_d the slowest control.

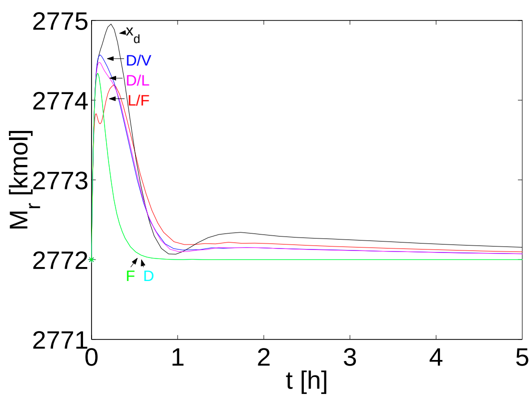

Figure 9: Reactor holdup when $\Delta F_0 = +20\%$

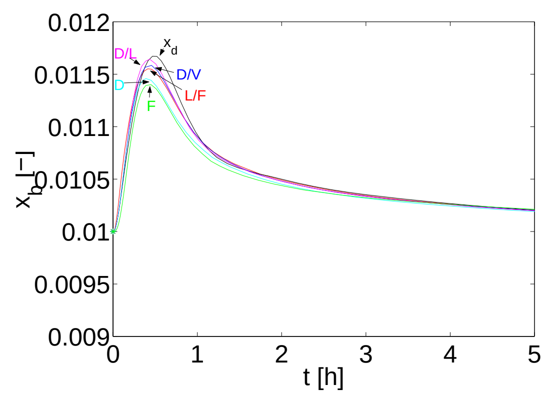

Figure 10: Product composition when $\Delta F_0 = +20\%$

The control deviations are significantly less than the expected implementation errors. If we assume small measurement errors, the study can be performed with some smaller expected implementation errors, which will reduce the economic loss connected to implementation errors. The differences in the control deviation are rather small and will not change the ranking of the alternatives.

6 Conclusion

An extended method for the selection of controlled variables with good self-optimizing control properties which include the choice of setpoints, is presented. We focus on achieving feasible operation by implementing setpoints found by robust optimization ("optimal back-off"). The method is applied to a reactor, separator and recycle process. x_d and L/F show best self-optimizing control properties. Alternatives which follow Luybens rule (F and D), require “complex” back-off and give larger loss than x_d and L/F. Alternatives with variable reactor holdup require “complex” back-off and give large losses.

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