Figure 1: Tennessee Eastman process flowsheet

it has been important for the development of plantwide control as a field. Many authors has used it to demonstrate their procedure for the design of a control system, e.g. (McAvoy and Ye, 1994), (Lyman and Georgakis, 1995), (Ricker, 1996), (Luyben et al., 1997), (Ng and Stephanopoulos, 1998), (Tyreus, 1999). To summarize, most authors do not control all the variables which are constrained at the optimum, thus they can not operate optimally in the nominal case. Most control reactor pressure, reactor level, reactor temperature and composition of B (inert) in reactor feed or in purge. It is common to control stripper temperature, separator temperature, and composition of C and/or A in reactor feed.

Stepwise procedure for self-optimizing control

The main objective of operation, in addition to stabilization, is to optimize the economics of the operation subject, e.g in terms of minimizing the economic cost function J. To achieve truly optimal operation we would need a perfect model, we would need to measure all disturbances, and we would need to solve the resulting dynamic optimization problem on-line. This is unrealistic, and the question is if it is possible to find a simpler implementation which still operates satisfactorily (with an acceptable loss). More precisely, the *loss* L is defined as the difference between the actual value of the cost function and the truly optimal value, i.e. $L = J - J_{\text{opt}}$. *Self-optimizing control* is when we can achieve an acceptable economic loss with constant setpoint values for the controlled variables (without the need to reoptimize when disturbances occur). This sounds very simple, but it is not necessarily clear for a given problem what these controlled variables should be. The main objective of this paper is to search for a set of controlled variables which results in self-optimizing control for the Tennessee Eastman process. We will apply the stepwise procedure for self-optimizing control of (Skogestad, 2000). The main steps are

1. Degree of freedom analysis
2. Definition of optimal operation (cost and constraints)
3. Identification of important disturbances
4. Optimization
5. Identification of candidate controlled variables
6. Evaluation of the loss with constant setpoints for the alternative combinations of controlled variables (caused by disturbances or implementation errors)
7. Final evaluation and selection (including controllability analysis)

	Manipulated variables	12
	D feed flow	
	E feed flow	
	A feed flow	
	A + C feed flow	
	Compressor recycle flow	
	Purge flow	
	Separator liquid flow	
	Stripper liquid product flow	
	Stripper steam flow	
	Reactor cooling water flow	
	Condenser cooling water flow	
	Agitator speed	
-	Levels without steady state effect	2
	Separator level	
	Stripper level	
-	Equality constraints	2
	Product quality	
	Production rate	
=	Degrees of freedom at steady state	8
-	Active constraints at the optimum	5
	Reactor pressure	
	Reactor level	
	Compressor recycle valve	
	Stripper steam valve	
	Agitator speed	
=	Unconstrained degrees of freedom	3

Table 1: Degrees of freedom and active constraints.

(Skogestad, 2000) applied this procedure to a reactor case and a distillation case, but in both cases there were only one unconstrained degree of freedom, so the evaluation in step 6 was managable. However, for the Tennessee Eastman process there are three unconstrained degrees of freedom, so it is necessary to do some more effort in step 5 to reduce the number of alternatives. We present below some general criteria that are useful for eliminating controlled variables.

Degrees of freedom analysis and optimal operation

The process has 12 manipulated variables and 41 measurements. All the manipulated variables have constraints and there are "output" constraints, including equality constraints on product quality and product rate. (Downs and Vogel, 1993) specify the economic cost J [\$/h] for the process, which is to be minimized. In words,

$$J = (unreacted\ feed) + (steam\ costs) + (compression\ costs)$$

The first term dominates the cost. An analysis, see Table 1, show that there are eight degrees of freedom at steady state which may be used for steady-state opti-

mization. (Ricker, 1995) solved the optimization problem using the cost function of (Downs and Vogel, 1993) and gives a good explanation on what happens at the optimum. At the optimum there are five active constraints and these should be controlled to achieve optimal operation (at least nominally). This leaves three unconstrained degrees of freedom, which we want to select such that a constant setpoints policy results in an acceptable economic loss (self-optimizing control).
We consider the following three disturbanves:

- Disturbance 1: Change in A/C ratio in feed 4
- Disturbance 2: Change in %B (inert) in feed 4
- Throughput disturbances: Change in production rate by ±15 %.

We use the same contraints (and safety margins) as given by (Ricker, 1995). The optimal (minimum) operation cost is 114.323 $/h in the nominal case, 111.620 $/h for disturbance 1, and 169.852 $/h for disturbance 2. We define an "acceptable loss" to be 6 $/h when summed over the disturbances.

Selection of controlled variables

What should we control? More precicely, we have 8 degrees of freedom at steady state, and we want to select 8 controlled variables which are to be controlled at constant setpoints. We can choose from 41 measurements and 12 manipulated variables, so there are 53 candidate variables. Even in the simplest case, where we do not consider variable combinations (such as differences, ratios, and so on), there are

$$\frac{53 \cdot 52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 886 \cdot 10^6$$

possible combinations. It is clearly impossible to evaluate the loss with respect to disturbances and implementation errors for all these combinations. The following criteria are proposed to reduce the number of alternatives. Most of them are rather obvious, but nevertheless we find them useful.

Use active constraint control

We choose to control the active constraints. This reduces the number of controlled variables to be selected from 8 to 3.

Eliminate variables related to equality constraints

The equality constraints must be satisfied, and if there are directly related variables then these must be eliminated from further consideration. The stripper liquid flow (product rate) is directly correlated with production rate which is specified (eliminates 1 manipulated variables and 1 directly related measurement).

Eliminate variables with no steady-state effect

Two variables have no steady-state effect, namely stripper level and separator level (eliminates 2 measurements). (Of course, we need to measure and control these two variables for stabilization, but we are here concerned with the next control layer where the steady-state economics are the main concern).

Eliminate/group closely related variables

The controlled variables should be independent.

- Six of the remaining manipulated variables are measured (A feed, D feed, E feed, A+C feed, stripper liquid flow, purge flow) that is, there is a one to one correlation with a measurement (eliminates 5 mesurements).
- There is a only small differences between controlling the composition in the purge flow and in the reactor feed. We therefore eliminate reactor feed composition (eliminates 6 measurements)

Process insight: Eliminate further candidates

Based on understanding of the process some further variables were excluded form the set of possible candidates for control (since they should *not* be kept constant): pressures in separator and stripper, condenser and reactor cooling water flowrates, reactor and separator cooling water outlet temperatures, and separator liquid flow. Finally the fractions of G in product and H in product should be equal (specified), so by keeping one of these fractions constant, we will idirectly specify their sum, which is optimally about 0.98. However, their sum cannot exceed 1.0, so taking into account the implementation error we should not keep G in product or H in product constant.

Eliminate single variables that yield infeasibility or large loss

The idea is to keep a single variable constant at its nominally optimal value, and evaluate the loss for (1) various disturbances (with the remaining degrees of freedom reoptimized), and (2) for the expected implementation error. If operation is infeasible or the loss is large, then this variable is eliminated from further consideration.
Infeasibility. Keeping one of the following four manipulated variables constant results in infeasible operation for disturbance 2 (inert feed fraction): D feed flow, E feed flow , A+C feed flow (stream 4) and purge flow. This is independent on how the two remaining degrees of freedom are used, see Table 2.
Loss. We have now left 1 manipulated variable (A feed flow) and 17 measurements. Table 3 shows the loss (deviation above optimal value) for fixing one of these 18 variables at a time, and reoptimizing with respect to the re-

Variable	Nominal (constant)	Nearest feasible with disturb. 2
D feed [kg/h]	3657	3671
E feed [kg/h]	4440	4489
A+C feed [kscmh]	9.236	9.280
Purge rate [kscmh]	0.211	0.351

Table 2: Single variables with infeasibility

maning two degrees of freedom. The losses with constant A feed flow and constant reactor feedrate are totally unacceptable for disturbance 1 (eliminates 1 manipulated variable and 1 measurement), in fact, we could probably have eliminated these earlier based on process insight. The remaining 15 measurements yield reasonable losses. However, we have decided to eliminate variables with a loss larger than 6 $/h when summed for the three disturbances. This eliminates the following 5 measurements: separator temperature, stripper temperature, B (inert) in purge, G in purge, and H in purge.

Fixed variable	Dist.1	Dist.2	Throughput +15/-15%
A feed *	709.8	6.8	
Reactor feed*	53.5	0.5	
Recycle	0.0	0.8	0.5 / 0.3
Reactor T.	0.0	0.9	1.2 / 0.7
Sep T*	0.0	0.5	4.2 / 2.3
Stripper T*	0.1	0.3	4.3 / 2.3
Compr. Work	0.0	0.6	0.2 / 0.1
A in purge	0.0	0.7	0.4 / 0.2
B in purge*	0.0	7.4	3.1 / 1.6
C in purge	0.0	0.5	0.1 / 0.1
D in purge	0.0	0.0	0.2 / 0.1
E in purge	0.0	0.4	0.0 / 0.1
F in purge	0.0	0.5	0.0 / 0.0
G in purge*	0.0	0.4	4.1 / 2.2
H in purge*	0.0	0.4	4.2 / 2.2
D in product	0.0	0.1	0.2 / 0.1
E in product	0.0	0.0	1.2 / 0.7
F in product	0.0	1.5	1.4 / 0.8

Table 3: Loss [$/h] with one variable fixed at its nominal optimal value and the remaining two degrees of freedom reoptimized. Variables marked with * have a loss larger than 6 $/h.

Eliminate pairs of constant variables with infeasibility or large loss

We are now left with 11 candidate measurements. that is, $(11 \cdot 10 \cdot 9)/(3 \cdot 2) = 165$ possible combinations of three variables. The next natural step is to proceed with keeping pairs of variables constant, and evaluate the loss with the remaining degree of freedom reoptimized. However, there are 55 combinations of pairs, so this does not result in a large reduction in the number of possibilities. We therefore choose to skip this step in the procedure.

Final evaluation of loss for remaining combinations

A quick screening indicates that one of the three controlled variables should be reactor temperature, which is the only remaining temperature among the candidate variables. A further evaluation shows that we should eliminate F (byproduct) in purge as a candidate variable, because the optimum is either very "sharp" in this variable, or optimal operation is achieved close to its maximum achievable value (see a typical plot in Figure 2). In either case, operation will be very sensitive to the implementation error for this variable.

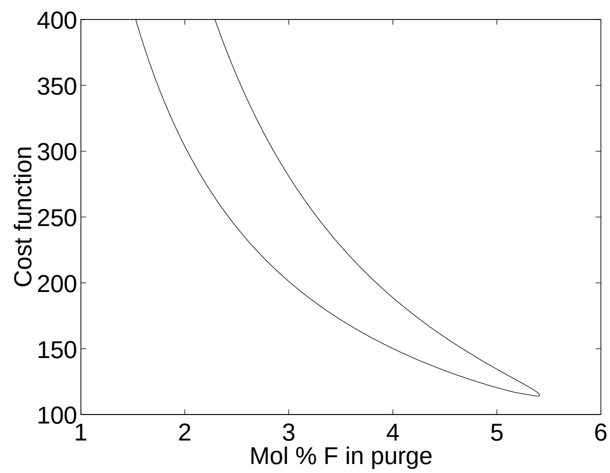

Figure 2: Unfavorable shape of cost function with F (byproduct) in purge as controlled variable. Shown for case with constant reactor temperature and C in purge.

The losses for the remaining $9 \cdot 8/2 = 36$ possible combinations of 2 variables were computed (not shown). Not surprisingly, keeping both recycle flow and compressor work constant results in infeasibility or large loss for disturbance 2 and for feed flow changes. This is as expected, because from process insight these two variables are closely correlated (and we could probably have eliminated one of them earlier). We note that constant F in product in all cases results in a large loss or infeasibility for disturbance 2. This, combined with the earlier finding that we should not control F in purge, leads to the conclusion that it is *not* favorable to control the composition of byproduct (F) for this process. The following four cases have a summed loss of less than 6 [$/h]:

I. Reactor temp., Recycle flow, C in purge (loss 3.8).

II. Reactor temp., Comp. work, C in purge (loss 3.9).

III. Reactor temp., C in purge, E in purge (loss 5.1).

IV. Reactor temp., C in purge, D in purge (loss 5.6).

Evaluation of implementation loss

In addition to disturbances, there will always be a implementation error related to each controlled variable, that is, a difference between its setpoint and its actual value, e.g. due to measurement error or poor control. By plotting plot for "best" case I the cost as a function of the three controlled variables (not shown) we find that the optimum is flat over a large range for all three variables, and we conclude that implementation error will not cause a problem. In comparison, for cases III and IV the cost is sensitive to implementation errors, and we get infeasibility if purge composition of D (case III) or E (case IV) becomes too small.

Should inert be controlled?

A common suggestion is that it is necessary to control the inventory of inert components, that is, in our case, to control the mole fraction of component B (Luyben et al., 1997) (McAvoy and Ye, 1994) (Lyman and Georgakis, 1995) (Ng and Stephanopoulos, 1998) (Tyreus, 1999). However, recall that we eliminated B in purge at an early stage because it gave a rather large loss for disturbance 2 (see Table 3). Moreover, and more seriously, we generally find that the shape of the economic objective function as a function of B in purge is very unfavorable, with either a sharp minimum or with the optimum value close to infeasibility. A typical example of the latter is shown in Figure 3. In conclusion, we do not recommend to control inert composition.

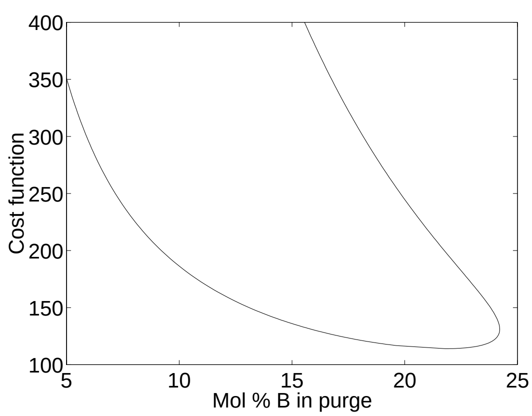

Figure 3: Typical unfavorable shape of cost function with B (inert) in purge as controlled variable (shown for case with constant reactor temperature and C in purge).

Summary

In conclusion, control of reactor temperature, C in purge, and recycle flow or compressor work (cases I or II) result in a small loss for disturbances and a flat optimum (and is thus insensitive to implementation error), and are therefore good candidates for self-optimizing control.
The analysis in the paper is based on steady-state economics, but we have also performed dynamic simulations that show that this proposal may be implemented in practice using a simple decentralized feedback control structure based on PI controllers.

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