CONTROL STRUCTURE SELECTION FOR SERIAL PROCESSES WITH APPLICATION TO pH-NEUTRALIZATION

A. Faanes*, S. Skogestad†

Department of Chemical Engineering
Norwegian University of Science and Technology
N–7491 Trondheim, Norway
† Fax : +47 73594080 and e-mail:skoge@chembio.ntnu.no
* also affiliated with Norsk Hydro ASA, Research Centre, N-3907 Porsgrunn, Norway

Keywords: Control structure, Serial process, Multivariable control, Feedforward, Feedback

Abstract

In this paper we aim at obtaining insight into how a multivariable feedback controller works, with special attention to serial processes.

1 Introduction

Before designing and implementing a multivariable controller, there are some questions that are important to answer: What will the multivariable controller really attempt to do? Will a multivariable controller significantly improve the response as compared to a simpler scheme? What must the multivariable controller take into account to succeed? How accurate a model is needed?

There is a fundamental difference between feedforward and feedback controllers with respect to their sensitivity to uncertainty. Feedforward control is sensitive to static uncertainty, while feedback is not. On the other hand, aggressively tuned feedback controllers are very sensitive to small uncertainties in the crossover frequency region. Similar differences with respect to uncertainty can be found for multivariable controllers. Traditional single loop controllers are predominantly based on feedback, whereas model based multivariable controllers usually have a significant component of feedforward action (for example the decoupling elements of the controllers).

In this paper we discuss these questions for an important class of processes: *The serial processes.* The structure of this class of processes makes it suitable for a discussion of these questions for the multivariable controllers. A serial process consists of a series of one way interacting units. The states in one unit influence the states in the downstream unit, but *not* vice verse. This is very common in the process industry, where the outlet flow of one process enters into the next. One example, which will be studied in section 4, is neutralization performed in several tanks in series. Examples of processes that are not serial are processes with some kind of recycle of material or energy. Even for such processes, however, parts of the process may be modelled as a serial process, if the outlet variations of the last unit is dampened through other process units before it is recycled, so that no significant correlation can be found between the outlet variations and the variations in the disturbances to the first unit.

The characteristics of serial processes can be utilized when considering control structures and multivariable controllers for such processes. The multivariable controller can be divided into three types of controller blocks: Local feedback, feedback from downstream units and "feedforward" from upstream units.

This division of the controller blocks has two purposes. First, it allows each block to be implemented in simpler multivariable controllers, using conventional controllers. In some cases the multivariable controller can be implemented as combinations of conventional single loop controllers. Second, it gives insight into the behavior of the control system.

In section 2 we develop the model structure for serial processes, and discuss some of the specific properties of this structure. In section 3 multivariable control of serial processes is discussed, and the ideas are illustrated through an example with pH neutralization in three stages (section 4).

2 Model structure of serial processes

In this section we look closer at serial processes and develop a general transfer function model.

Definition 1 *A serial process can be divided into a series*

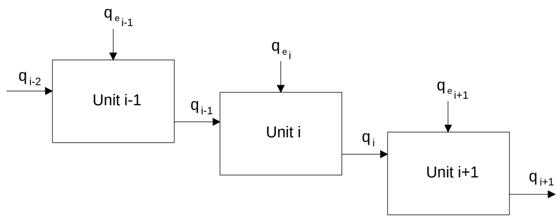

Figure 1: A serial process.

of subprocesses or units in such a way that the states in each unit are entirely decided by the states in the unit itself, the states in the upstream unit, and by external inputs to the unit.

An example of a serial process is a process where mass and/or energy flow from one process unit to another, and there is no recycling of mass or energy. From this formulation we can express the model for unit no. i:

$$\frac{d}{dt}x_i = f_i\left(x_i, x_{i-1}, u_i, d_i\right) \tag{1}$$

where x_i and $x_{i-1}$are the state vectors for unit i and unit $i-1$ respectively, and the external input is divided into a vector of manipulated inputs, u_i, and disturbances, d_i. We linearize this equation around a working point, introducing $A_j^i = \partial f_i / \partial x_j$, $B_i = \partial f_i / \partial u_i$ and $E_i = \partial f_i / \partial d_i$ and let the variables be the deviation from their working point. Applying Laplace transformation, and recursively inserting for variables from previous tank, we obtain:

$$y(s) = G(s)u(s) + G_d(s)d(s) \tag{2}$$

We have defined the total output vector, $y(s)$, as all the states, $u(s)$ as all the inputs, $d(s)$ as all the disturbances.

$$G(s) = \begin{bmatrix} \left(sI - A_1^1\right)^{-1} B_1 & 0 & 0 & \cdots & 0 \\ \left(sI - A_2^2\right)^{-1} A_1^2 \left(sI - A_1^1\right)^{-1} B_1 & \left(sI - A_2^2\right)^{-1} B_2 & 0 & \cdots & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ \left(sI - A_n^n\right)^{-1} \prod_{r=1}^{n-1} \left[A_{n-r}^{n-r+1} \left(sI - A_{n-r}^{n-r}\right)^{-1}\right] B_1 & \left(sI - A_n^n\right)^{-1} \prod_{r=1}^{n-2} \left[A_{n-r}^{n-r+1} \left(sI - A_{n-r}^{n-r}\right)^{-1}\right] B_2 & \cdots & \cdots & \left(sI - A_n^n\right)^{-1} B_n \end{bmatrix} \tag{3}$$

and G_d is identical but with B_i replaced by E_i(the disturbances to each unit are assumed independent).

We see that $G(s)$ and $G_d(s)$ are both lower block triangular. From (3), we can deduce the following properties:

- The state vector of a process unit is not influenced by control inputs and disturbances to downstream units.
- The influence from a control input or a disturbance which enters an upstream unit, q, is dampened by the transfer function $\left(sI - A_i^i\right)^{-1} \prod_{r=1}^{i-q} \left[A_{i-r}^{i-r+1} \left(sI - A_{i-r}^{i-r}\right)^{-1}\right]$ before it reaches the output of unit i.
- $G(s)$ and $G_d(s)$ are block diagonal at infinite frequency ($s \to \infty$).

Often for a serial process, it is the states in the last unit that is of importance, since the final product is taken out here. In an optimization of the operation of the process, only the states in the last tank and the control inputs to all units need to be represented in the objective.

3 Control structures for serial processes

In the previous section we introduced the concept of serial processes. Equations (2) and (3) summarizes the linearized model. If a full, multivariable controller is used to control this process, the blocks of this controller can be given special characteristics. If we for simplicity assume that the set-points are zero, and we want to control all the outputs, the control inputs are given by:

$$u(s) = K(s)y(s) \tag{4}$$

where $K(s)$ is the controller.

We divide the controller, $K(s)$, into $n \times n$ blocks of the same size as the blocks in $G(s)$:

$$K(s) = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1n} \\ K_{21} & K_{22} & \dots & K_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ K_{n1} & K_{n2} & \dots & K_{nn} \end{bmatrix} \tag{5}$$

These controller blocks can be divided into three groups:

Blocks on the diagonal (K_{ii}) These blocks use local control, where inputs to the unit are used to control outputs of the same unit.

Blocks above the diagonal (K_{ij}, $j > i$) These blocks represents feedback from the outputs of downstream units. Intuitively, when the dampening through the different units is large, or if there is transport delays in the units, these blocks seem ineffective since the local feedback always will be quicker. There are, however, several cases when it may prove useful:

1. We have no relevant control inputs downstream so local control is impossible.
2. The control inputs downstream are constrained, and insufficient to compensate the disturbances.
3. The downstream inputs are expensive to use.

In the latter two cases the upstream inputs can be used to (slowly) drive the downstream inputs to zero or to some other ideal resting value. This is called input resetting and is normally used for systems where we have more control inputs than outputs ([skoge2], page 416).

Of course, the feedback from downstream units may also come into use if the downstream actuators are slow, so that it actually is more efficient to operate the upstream control inputs.

Blocks below the diagonal (K_{ij}, $i > j$) Through these blocks an output from an upstream unit directly affects the input in a downstream unit. Since upstream units act as disturbances to downstream units, these controller blocks may be viewed as *"feedforward" elements*. However, strictly speaking, a feedforward element is defined as a link between "measured disturbances and manipulated inputs" (see e.g. [skoge2]), and in our case the "disturbance" is actually an output from an upstream unit. In particular, if we use a full controller with blocks above the diagonal, then the upstream unit will be affected by the downstream units, and the term "feedforward " element is somewhat misleading.

3.1 Some special controllers

3.1.1 Full controller

With a full controller, as in (5), the loop transfer function becomes

$$L = K(s)G(s) = \begin{bmatrix} G_{11}K_{11} & G_{11}K_{12} & \cdots & G_{11}K_{13} \\ G_{21}K_{11} + G_{22}K_{21} & G_{21}K_{12} + G_{22}K_{22} & \cdots & G_{21}K_{13}+G_{22}K_{23} \\ G_{31}K_{11} + G_{32}K_{21} + G_{33}K_{31} & G_{31}K_{12} + G_{32}K_{22} + G_{33}K_{32} & \cdots & G_{31}K_{13}+G_{32}K_{23}+G_{33}K_{33} \end{bmatrix} \quad (6)$$

In this case the stability of the closed-loop system is affected by all elements in the controller K (and in G).

3.1.2 Lower block triangular controller

In this case the loop transfer function becomes:

$$L = \begin{bmatrix} G_{11} & 0 & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} K_{11} & 0 & 0 \\ K_{21} & K_{22} & 0 \\ K_{31} & K_{32} & K_{33} \end{bmatrix} = \begin{bmatrix} G_{11}K_{11} & 0 & 0 \\ G_{21}K_{11} + G_{22}K_{21} & G_{22}K_{22} & 0 \\ G_{31}K_{11} + G_{32}K_{21} + G_{33}K_{31} & G_{32}K_{22} + G_{33}K_{32} & G_{33}K_{33} \end{bmatrix} \quad (7)$$

Here the stability of the closed-loop system is determined only by the blocks on the diagonal, that is we have closed-loop stability if and only if each of the diagonal closed-loop blocks $(I + G_{ii}K_{ii})^{-1}$ are stable. In this case the controller elements below the diagonal have most of the properties of feedforward elements, for example, that they do not affect closed-loop stability. Also, as is generally the case with feedforward control, the response is strongly dependent on model error, also at steady-state. In summary for lower block triangular controllers, the controller elements will have very different properties:

The diagonal elements are feedback elements, where most of the benefits may be achieved simply by using sufficiently high gains and an accurate process model is not needed. The main problem is that too high gain may give closed-loop instability. With a nominal model for unit i, $y_i = Gu_i + G_d y_{i-1}$, and an actual plant model $y'_i = G'u_i + G'_d y_{i-1}$, a feedback controller gives the following error when run on the actual plant: $y'_i - r_i = S_i \frac{1}{1+E_i T_i} (G'_d y_{i-1} - r_i)$ (eq. (5.71) and (5.72) in [skoge2] with $d = y_{i-1}$). S_i and T_i are sensitivity and complementary sensitivity functions, respectively, and E_i relative error in G. So here the model error may be dampened by the feedback since effective feedback control gives $|S_i| << 1$. For frequencies where $|S_i| \approx 1$ and $|T_i| > 0$, the model error influences the control error, and may even influence stability ([skoge2]).

The elements below the diagonal are feedforward elements, where benefits can be achieved only if we have an accurate model. The elements have no effect on stability. Using the same models as in last point, a perfect feedforward controller gives the following error when run on the actual plant: $e'_i = y'_i - r_i = \left(1 - \frac{G'/G'_d}{G/G_d}\right) G'_d y_{i-1} - \left(1 - \frac{G'}{G}\right) r_i$ (eq. (5.70) in [skoge2] with $d = y_{i-1}$). So the relative errors in G and G/G_d directly influence the control error.

These differences are particularly clear here, but similar differences occur for most multivariable controllers. Such insights are important, e.g. when evaluating how the controller is affected by model error.

3.2 Final control only in last unit

In many serial processes, only the output from the last unit is important for the overall plant economics. The outputs in upstream units are controlled to improve control performance in the final unit. Assuming that we have control inputs to several (or all) of the units, this means that we actually have a plant with more inputs than outputs. In such cases, we often adjust the setpoints in upstream units such that the inputs in downstream units are reset to some ideal resting value.

We may then use the following control elements:

Local control ($i = j$)	$u_i = k_{ii}(s)\,[r_i - y_i]$
Feedforward ($i > j$)	$u_i = k_{ij}^{FF}(s) y_j$
Input resetting ($j = i + 1$)	$r_i = k_{ij}^{IR}(s)\left[r_{u_j} - u_j\right]$

Note that we here have restricted the input resetting to operate between neighbor units, but this is not strictly required. Applying local control in the three units, feedforward from unit 1 and 2 to units 2 and 3, and input

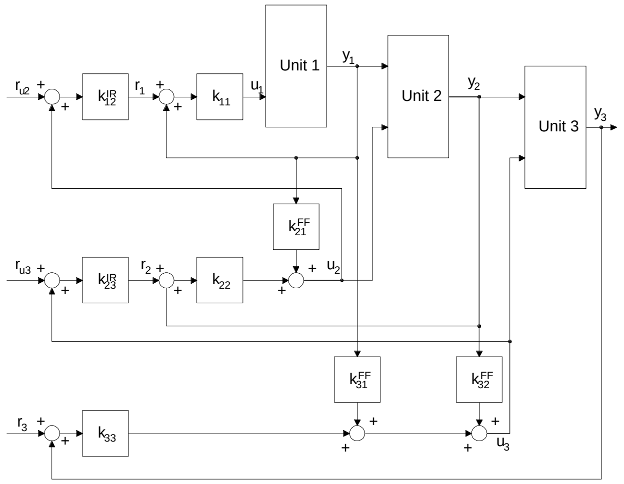

Figure 2: Serial units with a combination of local control, feedforward control and input resetting.

resetting from unit 3 to unit 2 and from unit 2 to unit 1, the equivalent multivariable controller is:

$$K(s) = \begin{bmatrix} -k_{11}\left(1 + k_{12}^{IR}k_{21}^{FF} - k_{12}^{IR}k_{22}k_{23}^{IR}k_{31}^{FF}\right) & k_{11}k_{12}^{IR}k_{22}\left(1 + k_{23}^{IR}k_{32}^{FF}\right) & -k_{11}k_{12}^{IR}k_{22}k_{23}^{IR}k_{33} \\ k_{21}^{FF} - k_{22}k_{23}^{IR}k_{31}^{FF} & -k_{22}\left(1 + k_{23}^{IR}k_{32}^{FF}\right) & k_{22}k_{23}^{IR}k_{33} \\ k_{31}^{FF} & k_{32}^{FF} & -k_{33} \end{bmatrix} \tag{8}$$

$$K_r(s) = \begin{bmatrix} k_{11}k_{12}^{IR} & -k_{11}k_{12}^{IR}k_{22}k_{23}^{IR} & k_{11}k_{12}^{IR}k_{22}k_{23}^{IR}k_{33} \\ 0 & k_{22}k_{23}^{IR} & -k_{22}k_{23}^{IR}k_{33} \\ 0 & 0 & k_{33} \end{bmatrix} \tag{9}$$

with $u(s) = K(s)y(s) + K_r(s)\left[r_{u_2}, r_{u_3}, r_3\right]^T$, where r_3 is the set point for the controlled output in unit 3, whereas r_{u_2} and r_{u_3} are the ideal resting values for the inputs in tank 2 and 3.

The final controller in (8) and (9) may seem very complicated, but it can usually be tuned in a rather simple cascaded manner. The feedforward elements are normally the fastest acting and should normally be designed first. The local feedback controllers can be tuned almost independently. Finally, the slow input resetting is added. It will not affect closed-loop stability if it is sufficiently slow.

4 Example: pH neutralization

Neutralization of strong acids or bases is often performed in several steps. The reason for this is mainly that the pH control in one tank cannot be quick enough to compensate for disturbances [skoge1]. In [mcmill], an analogy from golf is used: the difficulty of controlling the pH in one tank is compared to getting a hole in one. Using several tanks, and smaller valves for addition of reagent for each tank, is compared to the easier task of reaching the hole with a series of shorter and shorter strokes.

In this example control structures for neutralization of a strong acid by use of three tanks in series are discussed. The aim of the control is to keep the outlet pH from last tank constant despite changes in inlet pH or flow. This is obviously a serial process, since the flow goes from one tank to another. For each tank the pH can be measured, and the reagent can also be added to each tank. Referring to Figure 1, the three units (i-1, i and i+1) correspond to the three tanks (1, 2 and 3).

To study this process we model each tank as described in [skoge1]. In each tank we model the excess H^+ concentrations, that is $c = c_{H^+} - c_{OH^-}$. This gives bilinear models, which are further linearized around a stationary working point so that methods from linear control theory can be used. We get two states in each process unit (tank), namely the concentration, c, and the level. The disturbances enter tank 1 only. We here assume that there is a delay of 5 seconds for the effect of a change in inlet acid or base flow or inlet concentration to reach the outflow of the tank, e.g. due to incomplete mixing, and a further delay of 5 seconds until the change can be measured. In the linear state space model these transportation delays are modeled by Padé-approximations of 4th order. There is assumed no further delay in the pipes between the tanks. We assume that the levels are controlled by the outflows using a P controller such that the time constant for the level is about 1/10 the time constants for the concentrations.

The volumes of the tanks were chosen to $13.6m^3$, the smallest possible volumes according to the discussion in [skoge1]. The acid inflow (disturbance) has $pH = -1$. The pH of the final product in tank 3 should be $pH = 7\pm1$, and we selected the setpoints in tank 1 as 1.65 and in tank 2 as 3.8. The concentrations are scaled so that a variation of $\pm1\,pH$ around these set-points corresponds to a scaled value of ±1. The control inputs and the disturbances are also scaled appropriately. The linear model was used for multivariable controller design, while the simulations are performed on the nonlinear model.

A conventional way of controlling this process is to use local control of the pH in each tank using PID-controllers. Figure 3 shows the response of pH in each tank when the acid concentration in the inflow is decreased from 10mol/l to 5mol/l. As expected from [skoge1], this control system is barely able to give acceptable control. However, the nominal response can be significantly improved with multivariable control.

Figure 4 shows the response with a 3×3 multivariable $\mathcal{H}_\infty$ controller designed with performance weights on the outputs and on the control inputs in all tanks, and with composition into tank 1 as a disturbance. The main reason for the large improvement is the feedforward effect discussed in section 3.

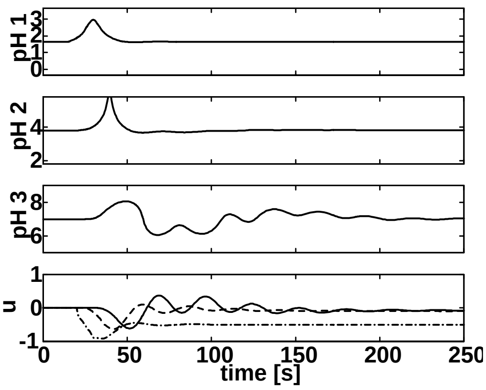

Figure 3: With only local control, PID controllers must be agressively tuned to keep the pH in the last tank within 7 ± 1. (Disturbance in inlet concentration occurs at $t = 10$.)

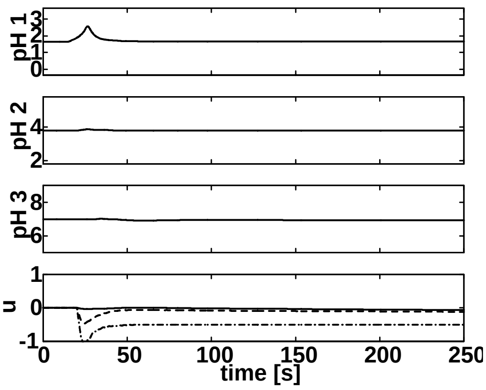

Figure 4: A large improvement in nominal performance is possible with multivariable control. (Disturbance in inlet concentration occurs at $t = 10$)

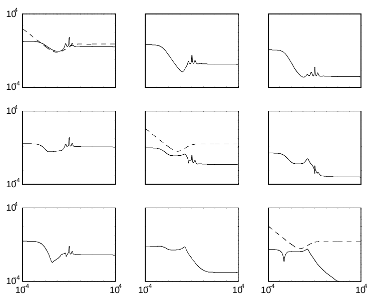

Figure 5: Gain of the control elements of the original 3×3 H_∞ controller. (Local PID controllers are dashed.)

The gain of the elements in the multivariable controller as a function of frequency are shown in Figure 5. The diagonal control elements are the local controllers in each tank, whereas the elements below the diagonal represent the "feedforward" elements. From such plots we get an idea of how the multivariable controller works. For example, we see that the control input to tank 1 (row 1) is primarily determined by local feedback, while in tank 2 it seems that "feedforward" from tank 1 is most decisive for the control input. In tank 3 the control actions are smaller. This is also seen from the simulation in Figure 4 (the solid line in the plot of u).

We observe that none of the control elements have any integrators, even though the simulation in Figure 4 show no steady-state offset. However, if some model error is introduced (20% reduced gain in tank 2 and 3), we do get a steady-state offset. Figure 6 shows the start of the response, it finally ends up slightly above $pH = 8$. Local PID controllers give no such steady-state offset.

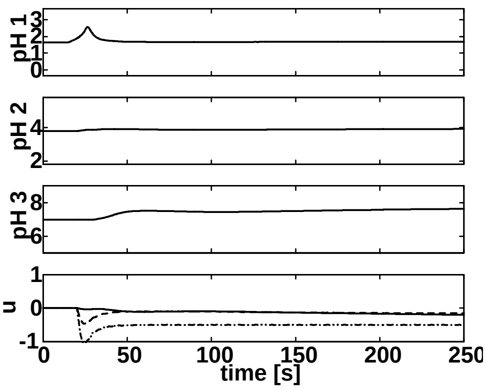

Figure 6: Model error gives steady-state offset with original 3×3 controller.

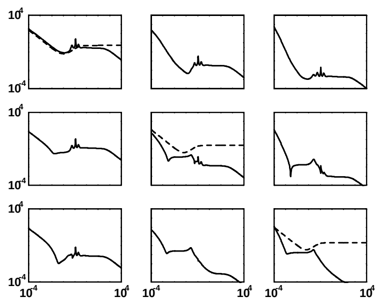

Figure 7: Gain of the control elements of the redesigned 3×3 H_∞ controller. (Local PID controllers are dashed).

We subsequently redesigned the controller to get three integrators in the control loop shape (Figure 7). The simulation in this case gives no steady-state offset. This illustrates one of the problems of the "feedforward" control block, namely the sensitivity to static uncertainty.

Simulations on the perfect model may lead the designer to believe that no integrator is necessary.

To study the feed forward effect separately, a $\mathcal{H}_\infty$ controller was designed using the measurement in tank 1, and control inputs in all tanks. The result is local control in tank 1 and feed forward from tank 1 to tanks 2 and 3. Simulation on the linear model gives the same result as for the 3×3 controller (Figure 4), whereas nonlinear simulation gives steady-state offset due to static model error and no feedback in tanks 2 and 3.

The effect of feedback from downstream tanks, i.e. the blocks above the diagonal from the discussion in section 3, is illustrated through the following simulations. We introduce a static measurement noise in tank 2 of 1 pH unit. In Figure 8 we see the response for the process with local control with PID. We can see that the pH in tank 3 relatively quickly returns to a pH of 7. The problem is the control input in tank 3, which stabilizes at a level away from the point in the middle of the range (0), which we consider as the ideal resting position. Since we really are interested in the pH in only the last tank, we get two extra degrees of freedom, which can be used for resetting the control inputs of the last two tanks. Figure 9 shows the simulation for the multivariable controller. Here we see that both the pH and the control input in tank 3 go to their desired values. The actual pH in tank 2 is risen to the correct value to obtain this. This illustrates that the elements above the diagonal in the multivariable controller give input resetting.

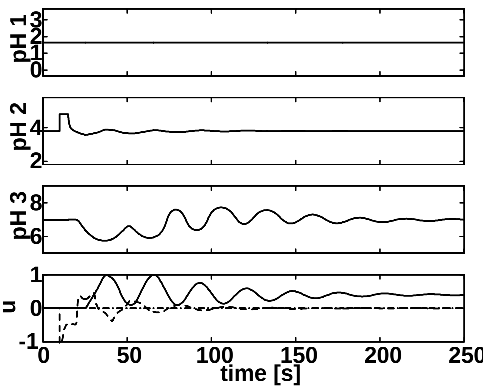

Figure 8: Steady-state measurement noise in tank 2: Local control with PID do not bring the control input for tank 3, u_3, back to the ideal resting position. (u-plot: solid line.)

To summarize the example we can say that the multivariable controller gives significant improvements compared to local control based on PID. This is especially due to the feedforward effect, and with large model errors, the feedforward may lead to worse performance. Integral action is important in the controllers, even if the feedforward effect may give no stationary deviation for the nominal case. The inputs in the last two tanks are reset to their ideal resting position with the multivariable controller, because of the feedback from downstream tanks.

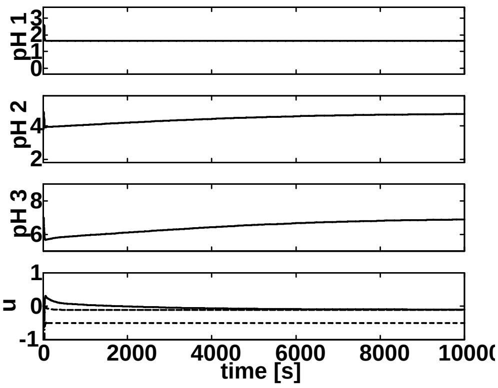

Figure 9: Steady-state measurement noise in tank 2: The multivariable controller has built in input resetting, and brings u_3 back to the ideal resting position (u-plot: solid). Note that the timescale differs from the other plots.

5 Conclusion

An example of neutralization of a strong acid with base in a series of three tanks is used to illustrate some of the ideas in the paper. This process is obviously serial. The example illustrates that the multivariable controller yields significant nominal improvements compared to local control based on PID. But this is especially due to feedforward, and with model errors, the feedforward may in fact lead to worse performance. Integral action or strong gain in the local controllers at low frequencies is important to obtain no steady-state offset, even if the feedforward effect itself may nominally give no steady-state. Feedback to upstream tanks brings the inputs to their ideal resting positions, also when a wrong pH measurement give problems in an upstream tank. The example indicates that it is possible to get a good performance with careful use of a multivariable controller or a combination of local control, feed forward from tank 1 and input resetting.

In this study we used a $\mathcal{H}_\infty$-contoller, but similar results have also been found for a MPC controller.

References

[mcmill] McMillan G.K., "pH control", Instrument Society of America, (1984)

[skoge1] Skogestad S., "A procedure for SISO controllability analysis - with application to design of pH neutralization processes", *Computers chem. Engng.*, **20**, 373-386, (1996)

[skoge2] Skogestad S., Postletwaite I. "Multivariable Feedback Control.", John Wiley & Sons, (1996)