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# Optimal Operation of Heat Exchanger Networks

Knut W. Mathisen*, Manfred Morari
Chemical Engineering 210-41, California Institute of Technology
Pasadena, CA 91125, USA
Sigurd Skogestad†
Chemical Engineering, University of Trondheim - NTH
N-7034 Trondheim, Norway

**Abstract** Most research on heat exchanger networks focuses on design, whereas operational and control issues have largely been neglected. In this paper a simple, parameter-independent method to ensure steady-state optimal operation is presented. It is shown that for several important classes of heat exchanger networks the sign of the transfer matrix elements between manipulated inputs and controlled outputs may be determined from structure only. This "sign matrix" may be utilized to select the heat exchangers to be bypassed in order to minimize utility consumption. The applicability of the proposed procedure is explained through several examples including networks with loops, splits and multi-bypasses over several exchangers. Manipulation of stream splits are shown to be preferred to bypass manipulations, and single bypasses are preferred to multi-bypasses.

## 1 Introduction

In this paper we discuss steady-state operation of existing heat exchanger networks (HENs) or proposed HEN designs. Thus, we assume that both the HEN structure and heat exchanger areas are fixed.

The controlled variables or outputs in HENs are usually the stream outlet temperatures. The control objective is to keep these outputs at their setpoints or targets. At steady-state HENs have one degree of freedom per exchanger, the heat load. During operation, the heat load on the utility exchangers (i.e., heater and coolers) are manipulated indirectly by adjusting the utility flowrate. The heat load on matches (i.e., process heat exchangers) are usually manipulated by adjusting a bypass flow on the hot or the cold side, but liquid level in flooded condensers or a recycle flow may also be used. Since we only discuss stationary effects, these alternative methods are equivalent, and *by bypassing a match we mean in this paper to reduce the heat load in a general sense.* We will also discuss multi-bypasses (i.e., bypasses over several matches in series) and split fractions, which will change the heat load on multiple matches.

Utility exchangers are assumed to be the final exchangers if present. Target temperatures downstream of utility exchangers are controlled by manipulating the utility flowrate, and we call them utility-controlled outputs. The remaining target temperatures must be controlled by manipulating match heat loads through bypass and split fractions, and we denote them bypass-controlled outputs. The operation objective is to fulfill all target temperatures with minimum utility consumption, and this may be achieved by minimizing the hot temperatures upstream the coolers or by maximizing the cold temperatures upstream the heaters.

### Problem definition

The optimal steady-state operation problem or network optimization problem (Marselle *et al.*, 1982) may be formulated as (Mathisen *et al.*, 1992) :

$$\min_{u} \quad (T_{t-1}^{Hj} - r_t^{Hj})w^{Hj} \quad \text{(minimize hot utility)} \qquad (1)$$

subject to:

$$\begin{array}{lcll}
T_t^{Hi} - r_t^{Hi} & = & 0 & \text{(hot and} \\
T_t^{Cj} - r_t^{Cj} & = & 0 & \text{cold target temperatures)} \\
r_t^{\hat{H}i} - T_{t-1}^{\hat{H}i} & \leq & 0 & \text{(positive or zero heat load coolers} \\
T_{t-1}^{\hat{C}j} - r_t^{\hat{C}j} & \leq & 0 & \text{and heaters)} \\
-u & \leq & 0 & \text{(bypass and split fractions above 0} \\
u - 1 & \leq & 0 & \text{and below 1)}
\end{array}$$

where $w$ means heat capacity flowrate, $T_t$ stream temperatures (controlled outputs), $r_t$ the reference values for the controlled outputs (setpoints) and $u$ split or bypass fractions (manipulated inputs). Supercript $Hi$ ($Cj$) denotes the set of hot (cold) streams, and $\hat{H}j$ ($\hat{C}j$) the subset of the hot (cold) streams that are utility-controlled, and subscript $(t-1)$ means the stream upstream the final utility exchangers. Energy balances for the exchangers and mixers, and mass balances for the splitters and the mixers yield additional equality constraints. Note that only hot utility is included in the cost function as the cold utility will be given from an energy balance.

*Assumptions:* It is assumed that a single hot and a single cold utility is used (e.g., steam and cooling water) and that the problem has a pinch (i.e., both hot and cold utility is needed). We also assume that the target temperatures to be equality constraints.

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[*] Present address: Institute for Energy Technology (IFE), Box 173, N-1751 Halden, Norway

[†] Address correspondence to this author. Fax: 47-73 59 40 80, E-mail: skoge@kjemi.unit.no.

In this paper, we present a procedure that may be used to solve the optimal operation problem by hand, i.e, it may be used to

- replace numerical optimization (e.g., Model Predictive Control, MPC)
- check and understand the solution from a numerical optimization
- decide where to install bypasses (i.e. control configuration design)
- understand interactions between design and operation control

Note that we only consider steady-state optimal operation and do not take dynamic considerations like speed of response into account. One may need additional bypass flows to allow for feedback control.

As mentioned, Marselle *et al.*, (1982) first defined and discussed the optimal operation problem for HENs. They state that "the direction of change of the decision variables (i.e., manipulated inputs) to yield an improvement of the objective is not apparent and has to be established through on-line experiments, a lengthy procedure ...". The main contribution of our paper is to show that this is possible for many different types of HENs. Our approach is to determine the sign of the gain from the alternative manipulated inputs to all the outputs. We then use this "sign matrix" to determine what matches that should be bypassed prior to the implementation.

We have already introduced matches, utility exchangers and bypass-controlled and utility-controlled outputs. A downstream path between an input and an output in HENs exists if the input affects the output so that the gain is structurally not zero. Two parallel downstream paths between a match and an output exist if there are downstream paths from both the hot and the cold outlet temperatures of the match to the output. Inner matches are matches with downstream matches on both the hot and the cold side.

## 2 Theory

### 2.1 Facts about input propagation

The procedure for minimizing utility consumption while maintaining all target temperatures is based on four facts on how disturbances and manipulations propagate in HENs:

*F1: A positive (negative) temperature change has a positive (negative) effect on all downstream temperatures.*

*F2: Temperature disturbances are naturally dampened.*

*F3: A flowrate increase of hot (cold) streams has a positive (negative) effect on all downstream temperatures.*

*F4: Bypass manipulations propagate as a temperature increase from the hot side and a temperature decrease from the cold side of the bypassed match.*

The facts are derived from single heat exchanger equations, see (Mathisen, 1994) for details.

### 2.2 The sign matrix

The sign matrix matrix defined below is used as part of the procedure:

**Definition:** Let $u$ denote all match heat load manipulations (bypasses), $y$ all controlled outputs, and $g_{ij}$ the transfer function between input $j$ (bypass split fractions) and output $i$ (temperatures, both utility-controlled or bypass-controlled), i.e., $y(s) = G(s)u(s)$. Furthermore, let $Q$ denote the "process heat transfer, and define sign($Q$) such that:

$$[\text{sign}(Q)]_{ij} = \begin{cases} - & \text{if } g_{ij} > 0 \, \forall G \text{ and output } i \text{ is a hot stream} \\ + & \text{if } g_{ij} > 0 \, \forall G \text{ and output } i \text{ is a cold stream} \\ + & \text{if } g_{ij} < 0 \, \forall G \text{ and output } i \text{ is a hot stream} \\ - & \text{if } g_{ij} < 0 \, \forall G \text{ and output } i \text{ is a cold stream} \\ 0 & \text{if } g_{ij} = 0 \, \forall G \\ \pm & \text{otherwise} \end{cases} \tag{2}$$

Note that the sign is defined oppositely for hot and cold outputs since the desired or positive effect of increasing process-process heat transfer (and reducing utility consumption) is achieved when hot temperatures decrease and cold temperatures increase. Deviations (errors, $e$) in the controlled outputs are defined similarly:

$$e_i = y_i - r_i \quad \text{if output } i \text{ is a hot stream}$$

$$e_i = r_i - y_i \quad \text{if output } i \text{ is a cold stream}$$

which means the deviation error is negative if the target is exceeded. ?

Depending on the kind of downstream path from the considered input (manipulated match) to the considered output (stream temperature), the sign matrix may be constructed from the following rules:

- From hot side of match to hot temperature: −
- From hot side of match cold temperature +
- From cold side of match to hot output +
- From cold side of match to cold output −
- No downstream path: 0
- From both the hot and the cold side ±

The idea is to use the sign matrix is as follows : To keep the bypass-controlled temperatures constant (i.e., deviation $e = 0$) one must manipulate some bypass (input). We prefer inputs which at the same time reduce the utility consumption, and this information is given by the sign matrix.

#### Relocating the utility exchangers

Because the sign matrix is based on information about the match structure only, it is not influenced by the existence of final utility exchangers. This means that we use the same sign matrix to optimize any set of bypass-controlled outputs. Still, the location of the utility exchangers influence both the procedure and the optimal set of bypasses.

## 3 Procedure

The optimal operation problem was defined in Eq. 1. We will here consider the following closely related problem: Given a deviation ($e$) in a bypass-controlled output, which bypass (heat load) should be adjusted and in which direction in order to reset this output to its setpoint (reference value $r$) while at the same time minimizing the utility consumption? The objective of this paper is to answer this problem based on structural information only, that is, without knowledge of temperatures, flowrates, heat heat loads etc. With an answer to this problem, one can relatively easily design a "rule-based" algorithm to optimize the operation, for example, using a simple decentralized feedback control loops with some additional logic. This could then replace a detailed numerical optimization based on solving Eq. 1, which would require a detailed steady-state model of the network (effect of all $u$'s on all $y$'s).

The suggested sequential procedure for how to find the optimal operating point by hand is presented with only a few brief comments. Additional comments and explanations are given in the examples. All selections are based on the information about the HEN structure only.

**Step 0:** Initialize inputs that have a negative or zero effect on all hot (cold) utility-controlled outputs (− or 0 in the corresponding elements of the sign matrix) to zero (i.e., minimize). Let index $h$ denote hot streams that are utility-controlled and index $c$ cold streams that are utility-controlled. Then this may be expressed as

$$\text{Set } u_j = 0 \text{ if } \quad q_{hj} \leq 0; \quad \forall h \quad or \quad q_{cj} \leq 0; \quad \forall c \qquad (3)$$

(what about the rest?)

Repeat the remaining steps for each bypass-controlled output $i$:

**Step 1:** Find the set of possible manipulated inputs for each bypass-controlled output, that is all inputs $j$ where:

$$q_{ij} \neq 0 \qquad (4)$$

In words, find the elements with +, − or ± in the corresponding row (row $i$) of the sign matrix. what does this mean about the value of $q_j$

**Step 2:** Prefer the manipulations that has the most desirable side-effect on the utility consumption. Compare both hot and cold utility consumption. Assume that undetermined (±) effects may be both positive and negative. *Prefer in the following order:*

a) Manipulations that have positive effect on hot (cold) utility consumption. Mathematically

OUTPUT $i$ (
INPUT $j$
given column

$$q_{ij}q_{hj}e_i \geq 0; \quad \forall h \quad \text{(and at least one is positive)}$$

$$q_{ij}q_{cj}e_i \geq 0; \quad \forall c \quad \text{(and at least one is positive)} \qquad (5)$$

Note that saturated inputs must be disregarded, for example, if the requirement to get $e_i = 0$ is to *decrease* a bypass that is already zero.

b) Manipulations that have a zero or mixed effect on hot (cold) utility consumption, i.e., all cases not included in a or c.

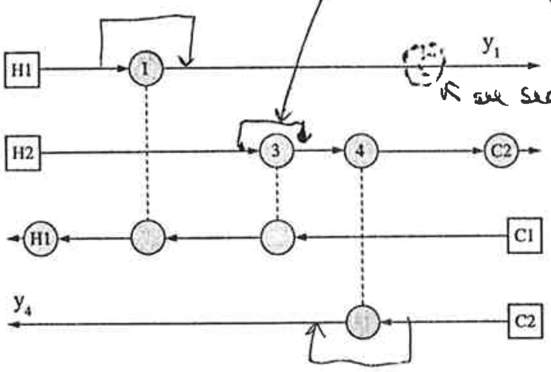
c) Manipulations that have negative effect on hot (cold) utility consumption. Mathematically

$$q_{ij}q_{hj}e_i \leq 0; \quad \forall h \quad \text{(at least one negative)}$$

$$q_{ij}q_{cj}e_i \leq 0; \quad \forall c \quad \text{(at least one negative)} \qquad (6)$$

**Step 3:** a) (Heuristic) Prefer the manipulated input closer to the bypass-controlled output (among inputs in the same group along the same downstream path). b) Disregard inputs with undetermined (±) effects on the bypass-controlled output.

Note the following: 1) Selecting the manipulated input closer to the output along the same downstream path (Step 3a), which is usually preferred from dynamic considerations, will usually have positive or no effect on utility consumption in such cases. 2) Both the effect on hot and cold utility consumption may be be used to discriminate between alternative inputs in Step 2. 3) The procedure as presented disregard multivariable effects, e.g., there must be enough degrees of freedom which means that the same manipulations cannot be used for multiple outputs. These points will be further explained through the examples.



USE IF $y_1$ OR $y_4$ ~~EXCEEDED~~ NOT REACHED) ~~(TOO MUCH HEAT TRANSF~~

see section 4.

Figure 1: Example 1 - HEN with minimum number of units.

## 3.1 Example 1 - HEN with minimum number of units

As an introductory example consider a HEN structure with minimum number of units, see Fig. 1. There are three matches in the network and the input vector may be defined as:

$$u^T = \begin{bmatrix} \text{bypass on match 1} & \text{bypass on match 3} \\ \text{bypass on match 4} \end{bmatrix}$$

whereas the output vector is:

$$y^T = \begin{bmatrix} T_t^{H1} & T_t^{H2} & T_t^{C1} & T_t^{C2} \end{bmatrix}$$

The sign matrix may be determined from the rules. Bypassing match 1 ($u_1$) affects hot output $T_t^{H1}$ ($y_1$) from its hot side, cold output $T_t^{C1}$ ($y_3$) from its cold side and has no effect on the outher outputs. The first column of sign($Q$) will therefore be

$$sign(Q_{i1}^T) = \begin{bmatrix} - & 0 & - & 0 \end{bmatrix}$$

The remaining columns of sign($Q$) are determined in the same manner, and this yields the following sign matrix:

$$sign(Q) = \begin{bmatrix} - & + & 0 \\ 0 & - & - \\ - & - & 0 \\ 0 & + & - \end{bmatrix} \begin{array}{l} y_1 \ (c) \\ h \\ c \\ y_4 \end{array} \qquad (7)$$

We now apply the sequential procedure to this HEN.

- Step 0: Utility-controlled outputs are $y_2$ (hot stream) and $y_3$ (cold stream). Initialize all inputs to zero ($u_1 = u_2 = u_3 = 0$) since the effect on the hot utility-controlled outputs are negative or zero, i.e., no +'s in row 2 the sign matrix. In this case there are no +'s in row 3 concerning the cold utility-controlled output either.
- Step 1: Bypassed-controlled outputs are $y_1$ and $y_4$. In row 1 elements $q_{11}$ and $q_{12}$ are nonzero so inputs $u_1$ and $u_2$ affect output $y_1$. In row 4 elements $q_{42}$ and $q_{43}$ are nonzero so inputs $u_2$ and $u_3$ affect output $y_4$.

THIS SHOULD THEN BE SET TO ZERO

THESE ARE USED IN PRACTICE.

| | | Primary $u$ | Secondary $u$ |
|---|---|---|---|
| $y_1$ | $T_t^{H1}$ exceeded $e_1 < 0$ | $\Delta u_2 < 0$ | $\Delta u_1 > 0$ |
| | $T_t^{H1}$ not reached $e_1 > 0$ | $\Delta u_1 < 0$ | $\Delta u_2 > 0$ |
| $y_2$ | $T_t^{C2}$ exceeded $e_4 < 0$ | $\Delta u_2 < 0$ | $\Delta u_3 > 0$ |
| | $T_t^{C2}$ not reached $e_4 > 0$ | $\Delta u_3 < 0$ | $\Delta u_2 > 0$ |

Table 1: Example 1. Priority order for the inputs (bypasses) for each of the bypass-controlled outputs.

- Step 2: First output $y_1$ is considered. When the output is exceeded ($e_1 < 0$) input $u_2$ (but not $u_1$) fulfills criteria a) since $q_{12}q_{22}e_1 > 0$ and $q_{12}q_{32}e_1 > 0$. When the output is not reached ($e_1 > 0$), input $u_1$ (but not $u_2$) fulfills criteria a) since $q_{11}q_{21}e_1 = 0$ and $q_{11}q_{31}e_1 > 0$
  Next output $y_4$ is considered. When the output is exceeded ($e_4 < 0$) input $u_2$ (but not $u_1$) fulfills criteria a) since $q_{42}q_{22}e_4 > 0$ and $q_{42}q_{32}e_4 > 0$. When the output is not reached ($e_4 > 0$), input $u_3$ fulfills criteria a) since $q_{43}q_{23}e_4 > 0$ and $q_{43}q_{33}e_4 = 0$.

The results are summarized in Table 3.1. Decreasing input $u_2$ is preferred for both outputs when they are exceeded. Only after this input saturates to zero other inputs should be used, !! but from the initialization this is likely to be the case. When output $y_1$ is not reached, input $u_1$ should be decreased. Only after this input saturates to zero, input $u_2$ may be used. When output $y_4$ is not reached, input $u_3$ should be decreased. Only after this input saturates to zero, input $u_2$ may be used.

If neither outputs are reached with zero bypass fractions, increasing input $u_2$ is preferred for both outputs. The optimal operating may then be found by increasing $u_2$ until either bypass-controlled output is met. If output $y_1$ is met first input $u_1$ may be used to control this output whereas input $u_2$ is further increased until output $y_4$ is met.

Match 1 is the final unit on a hot stream and immediately upstream a cooler, whereas match 4 is the final unit on a hot stream and immediately upstream a heater.

Additional examples are given in Section 5.

## 4 Further facts for minimizing energy

As illustrated in the above example the following facts apply about final matches and matches immediately upstream utility exchangers:

*F5: Bypassing matches immediately upstream a cooler (heater), increases the cold (hot) utility consumption.*
*Proof:* Assume the manipulated match is immediately upstream a cooler. From F3 increasing the bypass fraction will decrease the heat load on the manipulated match. To maintain the target temperature downstream the cooler, this heat load reduction must be compensated with a corresponding increase in the heat load on the cooler. If there is a downstream path from the cold side of the manipulated match to another cooler the heat load on this cooler will decrease (from F4). From F2 the hot side (negative) effect will always dominate because the downstream path from the cold side must traverse at least one additional match to reach a cooler. Analogous reasoning may be made for matches immediately upstream heaters.

*F6: Bypassing final matches on hot (cold) streams reduces or has no effect on cold (hot) utility consumption and increases or has no effect on hot utility consumption.*
*Proof:* Follows directly from F4 or the rules for setting up the sign matrix.

Since we always initialize (minimize) inputs (bypasses) with a known negative effect the process heat transfer, the saturated inputs are disregarded in the following examples. Furthermore, the reasoning are based on physical arguments rather than mathematics.

### Additional utility exchangers

Adding utility exchangers will not change the sign matrix, but 2 may affect the optimal operating point. To illustrate this point, assume that a final cooler on stream $H1$ is added to example 1.

- Step 0: Although input $u_2$ now have a mixed effect on the coolers, it has a negative effect on the only heater so all inputs may still be initialized to zero.
- Step 1: Bypasses $u_2$ and $u_3$ affect the bypass-controlled output $y_4$.
- Step 2: When output $y_2$ is exceeded input $u_2$ fulfills criteria a), but this input is saturated so input $u_3$ must be used. When output $y_2$ is not reached input $u_3$ fulfills criteria a), but this output is saturated so $u_2$ must be used. (same as before :)

Finally, assume that all streams are utility-controlled. In this case, there is no way to determine the bypass fraction around the inner match (match 3). Adjusting the bypass around this match may increase or decrease the utility consumption (+ and − to hot outputs $y_1$ and $y_2$, and − and + to cold outputs $y_3$ and $y_4$). This point about inner matches is stated in F7.

*F7: Bypassing inner matches may [increase or] even decrease the hot utility consumption.*

### Multi-bypasses

In industrial HENs, multi-bypasses or bypasses over several matches in series are sometimes used to increase the speed of response or increase the control range. To illustrate the effect of multi-bypasses, we reconsider example 1 and assume that there are multi-bypasses $31C$, i.e., bypass around matches 1 and 3 on the cold side, and $34H$, i.e., bypass around matches 3 and 4 on the hot side. The sign matrix may then be extended with another two columns:

$$u^T = \big[\ \text{match 1} \quad \text{match 3} \quad \text{match 4} \quad \text{multi-bypass 31C} \quad \text{multi-bypass 34H}\ \big]$$

Multi-bypasses decrease the heat load of both the bypassed matches, and this may be used to derive the columns in the sign matrix for the multi-bypasses:

$$sign(Q) = \begin{bmatrix} - & + & 0 & - & + \\ 0 & - & - & - & - \\ - & - & 0 & - & - \\ 0 & + & - & + & - \end{bmatrix} \qquad (8)$$

The bypass-controlled outputs are as shown in Fig. 1, i.e., $y_1$ and $y_4$. We just consider the normal case where both outputs are exceeded when the bypass fractions are zero. For output $y_1$ input $u_1$ or multi-bypass $u_4$ may be increased (−'s in the corresponding elements). Input $u_1$ is preferred to the multi-bypass because the it has no negative effect on the cooler. If output $y_4$

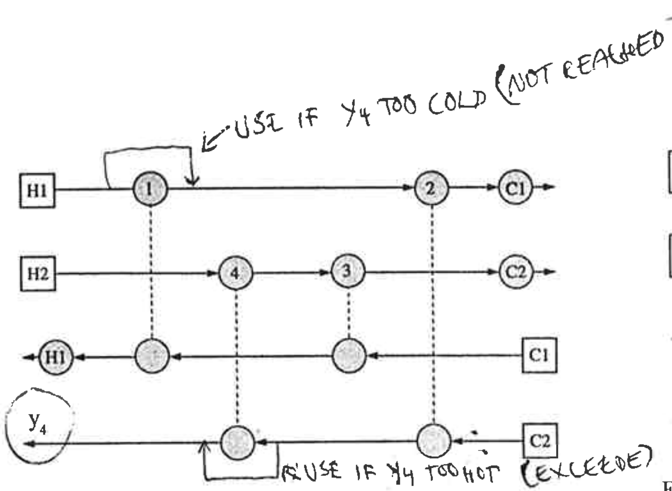


Figure 2: Example 2 - HEN with a loop

is exceeded $u_3$ or multi-bypass $u_5$ may be used. The single bypass is preferred because the multi-bypass increases hot utility consumption. The following general fact about multi-bypasses holds:

> *F8: Using multi-bypasses increases or has no effect on the utility consumption compared to using single bypasses.*

# 5 Examples

## 5.1 Example 2 - HEN with a loop

Although the network considered so far may have more than minimum number of units since we discussed adding utility exchangers, it has minimum number of *matches*. To show that the procedure may be applied to HENs with more than minimum number of matches, we consider Fig. 2. This HEN has one match more than minimum, and the extra match introduces a heat load loop that includes all 4 matches. With the input vector defined as single bypasses on the matches:

$$u^T = \left[ \text{ match 1 } \quad \text{match 2} \quad \text{match 3} \quad \text{match 4 } \right]$$

Note that there is a natural feedback loop via all four matches. This means that the matches affect the outputs from both sides, but the secondary effects may be disregarded when setting up the sign matrix as they may never dominate from facts F5 and F6.

← init. to zero!

$$sign(Q) = \begin{array}{c|l} \left[\begin{array}{cccc} - & - & + & - \\ - & + & - & - \\ - & - & - & + \\ + & - & - & - \end{array}\right] & \begin{array}{l} c \\ c \\ h \\ y_4 \end{array} \end{array}$$

"may be used" what do you mean?

- Step 0: Initialize all inputs to zero (minimize) since the effect on the hot or cold utility-controlled outputs are all negative.
- Step 1: All bypasses affect output $y_4$, too hot
- Step 2: Two different cases may be identified. 1) If the bypass-controlled output is exceeded inputs $u_2, u_3$ or $u_4$ may be used. Bypassing match 4 ($u_4$) is preferred to inputs $u_2$ and $u_3$ because it has a positive effect on stream $C1$ and the hot utility consumption. 2) If the output is not reached bypass $u_1$ must be used.

Yes, but in which direction?

Note that increasing these bypasses ($u_1$ and $u_4$) are preferred until they saturate. The corresponding match loads have then dropped to zero, which changes the match structure and the sign matrix. The procedure may then be repeated to identify that $u_2$ should be increased when $u_4$ saturates whereas there are no other alternatives when $u_1$ saturates.

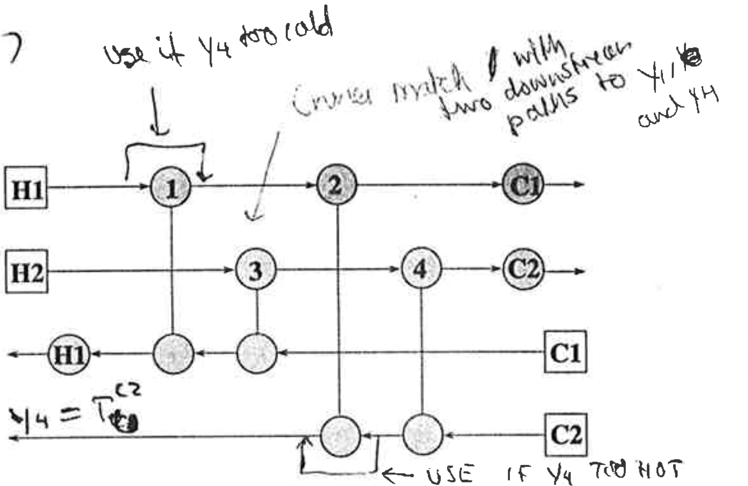


Figure 3: Example 3 - HEN with a match (match 3) with parallel downstream paths

### Additional outputs (internal temperatures)

In the previous examples, the controlled outputs are the network outlet temperatures of the process streams. Sometimes it is required to also control internal temperatures, e.g. due to material constraints on the heat exchangers. To illustrate that the procedure may handle internal constraints, assume that the hot outlet temperature of match 1 $T_t^{1h} = y_5$ must be controlled, too. The sign matrix is extended with a fifth row:

$$sign(Q) = \left[\begin{array}{cccc} - & - & + & - \\ - & + & - & - \\ - & - & - & + \\ + & - & - & - \\ - & + & + & - \end{array}\right]$$

Now, assume that output $y_1$ is controlled with $u_4$, and that all other bypass fractions are zero. If output $y_5$ is exceeded, bypass match 1. If output $y_5$ is not reached, a bypass on match 2 or 3 may be used. Bypassing match 3 is preferred because match 2 affect the output via match 3. The following rules holds:

> *F9: The bypass closer to the output is preferred among two bypasses that affect the bypass along the same downstream path.*
> *Proof:* Fact F9 is a consequence of fact F2 since heat load changes are dampended like temperature changes.

Note that the hot utility consumption increases for both cases. There is a penalty for adding constraints.

## 5.2 Example 3 - HEN with parallel downstream paths

Although the network in Fig. 2 has more than minimum number of matches and include a loop, there is no match with two parallel downstream paths to the same output. Such parallel downstream paths may exist when the number of matches is one above minimum. To illustrate this point, we use the HEN in Fig. 3. This network include a match (match 3), which has parallel downstream paths to some of the outputs. With same input and output vector as in the previous example, the sign matrix is:

$$sign(Q) = \begin{array}{c|l} \left[\begin{array}{cccc} - & - & \pm & + \\ 0 & 0 & - & - \\ - & 0 & - & 0 \\ + & - & \pm & - \end{array}\right] & \begin{array}{l} c \\ c \\ h \\ \phantom{x} \end{array} \end{array}$$

The $\pm$ elements in column 3 indicate that there are downstream paths from both the hot and the cold side of match 3 to outputs $T_t^{H1}$ and $T_t^{C2}$ which make the sign matrix dependent on problem parameters.

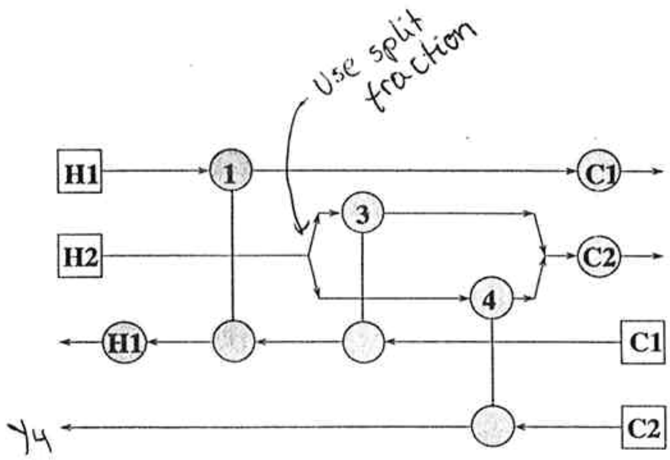


Figure 4: Example 4 - HEN with a split stream

Let us apply the proposed procedure to this network.

- Step 0: Set all bypass fractions to zero.
- Step 1: All inputs affect the bypass-controlled output.
- Step 2: Two different cases: 1) If the bypass-controlled output $y_4$ is exceeded, match 2 or match 4 may be bypassed. Manipulating match 3 may also give the desired effect, increasing input $u_3$ has a negative effect on hot utility consumption whereas inputs $u_2$ and $u_4$ have no effect on the heater. 2) If the output is not reached, bypass $u_1$ may be increased. Manipulating match 3 may also give the desired effect, and the sign matrix cannot be used to discriminate between the two possible inputs.
- Step 3: If output $y_4$ is exceeded, prefer bypass $u_2$ to bypass $u_4$ since it is closer to the output (from a). If output $y_4$ is not reached, prefer bypass $u_1$ to bypass $u_3$ since $u_3$ has an undetermined effect on the output (from b).

Note that bypassing match 2 or match 4 is equivalent from an energy point view since neither match influence the heater on stream $C1$.

### 5.3 Example 4 - HEN with a split stream

Usually streams that are split in HENs are remixed before the network outlet. When controlling such mixed streams, there are two (parallel) downstream paths from the splitter to the controlled output, see Mathisen *et al*, 1992. Therefore, the proposed procedure may not be applied to split-designs in general. Certain special situations may still be handled, see Fig. 4 where stream $H2$ is split to enable parallel heat exchange of matches 3 and 4. The input vector is defined as

$$u^T = \left[ \begin{array}{cccc} \text{match 1} & \text{match 3} & \text{match 4} & \text{split H2} \end{array} \right]$$

The sign matrix for this split structure is:

$$sign(Q) = \left[ \begin{array}{cccc} - & + & 0 & - \\ 0 & - & + & \pm \\ - & - & 0 & + \\ 0 & 0 & - & - \end{array} \right]$$

where the split fraction is defined as the fraction of the flowrate to the upper of the two branches. The ± entry in $[\text{sign}(Q)]_{24}$ indicates that increasing the split fraction may have a positive or a negative effect on the heat load of the cooler depending on the problem parameters. We try to apply the procedure to this network:

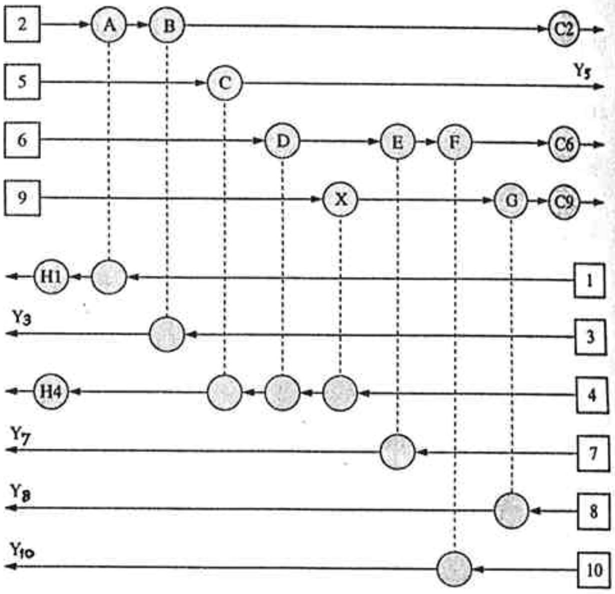


Figure 5: Example 5 - Aromatic plant. Design from Linnhoff *et al.*, 1982

- Step 0: Set all bypass fractions to zero, and the split fraction to 0.5
- Step 1: Two different cases: 1) If output $y_4$ is exceeded, bypass $u_4$ or the split fraction may be increased. 2) If output $y_4$ is not reached, the split fraction may be decreased. If output $y_4$ is exceeded, increasing the split fraction is preferred because it has a positive effect on the heater on stream $C1$.

In both cases, manipulating the split fraction is the preferred choice and the following fact holds:

*F10: Manipulating the split fraction of split streams are preferred to increasing the bypass fraction of any one of the matches on the stream branches.*

Note that the procedure may not be applied to all possible sets of utility-controlled streams.

### 5.4 Example 5 - aromatic plant

To illustrate that the suggested procedure may be applied on industrial HENs, we will use the well-known aromatic plant example from Linnhoff *et al.* (1982). Fig. 5 is the design presented in Fig. 4.11 in their book. Stream 11, which has no matches, is excluded for simplicity. The match between hot stream 9 and cold stream 4 (undenoted by Linnhoff *et al.*) is denoted X.

The input vector may be defined as (single) bypasses on the matches in the following order:

$$u^T = \left[ \begin{array}{cccccccc} A & B & C & D & E & F & G & X \end{array} \right]$$

and the output vector is numbered according to the stream numbers (1 through 10). The sign matrix for this HEN struc-

| | Primary $u$ | Secondary $u$ |
|---|---|---|
| $T_t^3$ | Bypass on B | Bypass on A |
| $T_t^5$ | Bypass on C | Bypass on D or X |
| $T_t^7$ | Bypass on E | Bypass on D |
| $T_t^8$ | Bypass on G | Bypass on X |
| $T_t^{10}$ | Bypass on F | Bypass on E or D |

Table 2: Aromatic plant design from Linnhoff *et al.*, 1982. Priority order for the alternative manipulated inputs (single bypasses) for each of the bypass-controlled outputs.

ture is:

init = 0

NOT init = 0 ?

$$sign(Q) = \begin{array}{cccccccccc} & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \swarrow & \\ \left[\right. & - & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \left.\right] c \\ & - & - & 0 & 0 & 0 & 0 & 0 & 0 & \\ & + & - & 0 & 0 & 0 & 0 & 0 & 0 & c \\ & 0 & 0 & - & - & 0 & 0 & 0 & - & c \\ & 0 & 0 & - & + & 0 & 0 & 0 & + & c \\ & 0 & 0 & 0 & - & - & - & 0 & + & h \\ & 0 & 0 & 0 & + & - & 0 & 0 & - & \\ & 0 & 0 & 0 & 0 & 0 & 0 & - & + & h \\ & 0 & 0 & 0 & 0 & 0 & 0 & - & - & \\ & 0 & 0 & 0 & + & + & - & 0 & - & \end{array}$$

Note the following: 1) There are 10 streams; 5 streams may be utility-controlled as they have downstream utility exchangers whereas the remaining 5 streams must be controlled by bypassing matches. 2) The design has 13 units, 2 above the global minimum. 2) The sign matrix is sparse, 54 of 80 elements are zero. This makes it easier to select the matches to be bypassed as there will be fewer alternatives. 3) The sign matrix include no ± elements. This means that none of the matches has downstream paths from both the hot and the cold side to one of the controlled outputs.

We apply the suggested procedure on this HEN structure.

- Step 0: Set all inputs to zero (What about bypass G)
- Step 1: We first consider bypass-controlled outputs $T_t^5 = y_5$. Two different cases: 1) If output $y_5$ is exceeded, match C should be bypassed. 2) If output $y_5$ is not reached, match D or X should be bypassed.
- Step 2: Bypass match D or match X

Similarly, one may find the possible single bypasses for the other 4 bypass-controlled outputs, and the priority order for all the outputs that ensures minimum utility consumption are shown in Table 5.2. The first column gives the preferred manipulated inputs during normal operation, i.e., when all bypass-controlled outputs would be exceeded if all bypass fractions were set to zero. The second column gives the secondary or alternative manipulated inputs when the primary input saturates to zero. The following information may be extracted from the table: 1) Bypasses on matches A and B are only manipulated to control output $y_2$, and $y_2$ cannot be controlled with any other bypasses. The reason is simply that the design consists of two subnetworks. This is easy to determine by inspection or mathematical manipulation of the sign matrix, too. 2) Bypasses on the six other matches are manipulated to control the four other bypass-controlled outputs. 3 ) During normal operation a bypass around the final match is used as manipulated input. This means that the preferred bypass combination for energy coincides with the best bypass combination for control. This very desirable feature frequently occurs for structures with minimum number of matches. 4) If one of the bypasses around the final matches saturates (to zero bypass fraction), an upstream match must be bypassed. This results in a energy penalty. The new bypass must of course introduce an additional degree of freedom, i.e., it cannot already be used to control one of the other outputs. 5) Selecting between alternative bypasses when the bypass-controlled output is not reached, (e.g., between a bypass on D and E for output $y_1$), has no effect on the utility consumption.

# 6 Discussion

## 6.1 Implementation

The optimal operating points may be implemented with a decentralized control system with varying configuration. Control logics must then be used to select the set of inputs for the different operating conditions. Alternatively, a decentralized control system with constant configuration may be used. To be able to maintain the same control configuration for all operating points, additional manipulated inputs must be used for constraint handling. That is, if any of the *regulatory* manipulated inputs approach saturation, an additional *supervisory* manipulated input is put into action to avoid reaching the limit. The (steady-state) utility consumption for such a decentralized control system with constant configuration will be larger than the utility consumption with varying configutration. The dynamic characteristics will however usually be superior with a constant control configuration, because the inputs often has a direct effect on the output. Thus, there is an interesting trade-off between dynamic and steady-state properties.

## 6.2 Inner matches and splits

The proposed procedure may be applied to many practically important HEN structures. The main limitations are due to inner matches and splits. For HENs with inner matches and splits, the procedure may not be applied to all combinations of utility-controlled streams. The requirement is that one of parallel downstream paths only affect hot or cold utility consumption. For the split example in Fig. 4, which may be handled with the procedure, the lower split branch only affect cold utility consumption.

The results indicate that no-split designs without inner matches should be preferred during synthesis of HENs because such designs are simple to operate optimally. Moreover, designs without inner matches and splits will the optimal bypasses for energy coincides with the optimal bypasses for control for the normal case where all bypass-controlled outputs are exceeded with zero bypass fractions.

## 6.3 Assumptions

We here discuss the three main assumptions mentioned in the introduction.

### Single utilities

The case of multiple utilities cannot be handled with the proposed procedure as is. One may be able to apply the procedure to a multiple utility problem if one gave different priority to the different utility levels, and optimized one level at a time. Consider, for example the common problem of multiple hot utility levels (i.e. two steam pressure levels) and only one cold utility type (i.e. cooling water). One may then state the optimization problem as:

1. Minimize cold utility. The total heat load of the hot utilities would then be determined by the problem parameters.
2. Minimize hottest (most expensive) hot utility.

In this way, the procedure may be applied to some practical important multiple utility level problems.

### Equality constraints

In some cases the performance specifications on the controlled temperatures should be formulated as inequality constraints rather than equality constraints. Typically this occur for hot streams where environmental regulations set an upper limit on the temperature. The procedure may still be used by simply disregarding inactive inequality constraints.

### Pinch problem

In order to get a meaningful optimization problem the problem must be a pinch problem since utility consumption is determined from problem parameters for threshold problems. Interestingly, threshold problems become meaningful optimization problems if some target temperature constraints are removed (e.g. inactive inequalities constraints instead of equality constraints). Consider Fig. 1 and assume that the target temperature constraint on stream $H1$ is removed. This gives a threshold problem because there is no point in wasting cold utility. However, a meaningful optimization problem consisting of minimizing hot utility consumption still exists. This optimization problem is equivalent to the original problem.

## 7 Conclusions

A procedure for finding the optimal operating point in terms of the input combination that minimizes the utility consumption which is based on structural information only is proposed. A sign matrix for how alternative manipulations (bypass and split fractions) affect the outputs (stream temperatures) is constructed from the network structure. The idea is to manipulate the outputs with the most positive (or least negative) effect on the temperatures upstream the heaters and coolers. The procedure may be used for all networks without inner matches or split fractions. For networks with inner matches and splits the procedure may only be used in special cases.

### NOMENCLATURE

| | | |
|---|---|---|
| $A$ | Heat exchanger area | $[m^2]$ |
| $e$ | Error | $[W/m^2K]$ |
| $G$ | Process transfer matrix | $[-]$ |
| $Q$ | "process heat transfer" (Eq. 2) | |
| $r$ | reference (setpoint) | $[K]$ |
| $T$ | Temperature | $[K]$ |
| $u$ | Manipulated input | $[-]$ |
| $y$ | Controlled output (temperature) | $[K]$ |
| $w$ | Heat capacity flowrate | $[W/K]$ |

Superscripts

| | |
|---|---|
| $c$ | cold side/fluid of heat exchanger |
| $Ci$ | set of cold streams |
| $Cj$ | set of utility-controlled cold streams |
| $h$ | hot side/fluid |
| $Hi$ | set of hot streams |
| $Hj$ | set of utility-controlled hot streams |

Subscripts

| | |
|---|---|
| $c$ | index for cold utility-controlled outputs |
| $i$ | index for outputs |
| $j$ | index for inputs |
| $h$ | index for hot utility-controlled outputs |
| $t$ | target temperature |
| $t-1$ | temperature upstream utility-controlled output |

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Linnhoff et al. (1982)