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# Identification of Dynamic Models for Ill-Conditioned Plants
# -
# A Benchmark Problem

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## 1 Introduction

Most published work on the identification of dynamic models from experimental data has been concentrated on the single-input-single-output (SISO) case. This is also reflected in the literature on process dynamics and control, where linear dynamic models usually are obtained by fitting input- output data from a plant or nonlinear simulation to a low-order transfer-function. In cases where the process is multivariable, the transfer-matrix is usually obtained by fitting the transfer-matrix elements *independently.* However, obtaining reasonable models for the individual transfer-function elements does not guarantee a reasonable multivariable model. This is in particular true for ill-conditioned processes which is the subject of this note. Ill-conditioned processes are commonplace in the chemical process industry and include, for example, high-purity distillation columns.

Skogestad and Morari [13] argue that fitting the transfer-matrix elements independently easily may lead to poor models for ill-conditioned processes unless one explicitly takes into account the coupling between the gains of the different elements. In particular, one is not able to obtain a good model of the low-gain direction of the plant, and the model will easily have the wrong sign of the determinant of the steady-state gain matrix and therefore be useless for control studies. This problem may, however, usually be corrected as the sign of the determinant and its approximate value in many cases is known a priori [7], [6].

Another, and more fundamental problem with this identification approach, is that the model may be inconsistent in that a single physical state is repeated in the model. This issue is the main topic of this paper. Ill-conditioned plants often have a *single* dominating "slow" pole (large time constant) which is a result of interactions in the process, and is thus shared by all the transfer matrix-elements. However, by fitting the elements of an $n \times n$ process *independently,* such that they all contain the dominant pole, one may get an inconsistent model with at least $n$ poles similar to the single dominating pole of the process. As shown in this paper, the inconsistency will result in a poor prediction of the process behavior under feedback control, in particular when only some loops are closed.

The general literature on identification theory has so far not focused very much on multivariable issues, and the particular problems mentioned above that may be encountered when identifying ill-conditioned plants do not seem to have been discussed.

We start the paper with an example of an inconsistent low-order model of a heat exchanger. The model, although seemingly a good open-loop description of the plant, is shown to yield unexpected behavior when one control loop is closed ("one-point control"). The results in this example are subsequently explained using analytical results. We then briefly discuss what types of processes that are likely to be identified with an excessive number of slow poles. At the end of the paper we present "experimental" input-output data of a heat-exchanger which is ill-conditioned and show that employing a classical identification method yields a model which is poor for feedback control studies.

All the results presented in this paper are for $2 \times 2$ processes, i.e., two inputs and two outputs. However, the results are of relevance also for higher dimensional processes.

## 2 Introductory Example

*Example 1. Heat-exchanger.* Consider a heat-exchanger modeled using a single mixing tank for both the hot and cold side (see Figure 1). Neglecting the heat accumulated in the walls yields a model with two states. Data for the example we consider are given in Table 1. In the following we only use the linearized form, $y(s) = G(s)u(s)$, of the model. Here $y = [y_1 \quad y_2]^T = [T_C \quad T_H]^T$ is the cold and hot outlet temperature and $u = [u_1 \quad u_2]^T = [q_C \quad q_H]^T$ is the cold and hot inlet flow rate. The exact linear model is

$$G(s) = \frac{1}{(1+\tau_1 s)(1+\tau_2 s)} \begin{pmatrix} k_{11}(1+4.76s) & k_{12} \\ k_{21} & k_{22}(1+4.76s) \end{pmatrix} \tag{1}$$

$\tau_1 = 100 \quad ; \tau_2 = 2.44 \quad ; k_{11} = -k_{22} = -1874 \quad ; k_{12} = -k_{21} = 1785$

The model is strongly ill-conditioned and has a steady-state condition number of 41 and diagonal steady-state RGA-values of 10.8. The physical explanation for the ill-conditioning is simply that the heat transfer is very effective such that the two outlet temperatures (outputs) are almost the same (61.59°C and 63.41°C in our case), and it is very difficult to change them independently. In particular, it is difficult to make them closer (this is the "weak" or "difficult" or "low-gain" direction of the plant), whereas we may easily make them *both* hotter or colder (this is the "strong"

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**Table 1.** Steady-state data for heat-exchanger in Example 1 (see also Figure 1).

| $V_H = V_C$ $[m^3]$ | $q_C = q_H$ $[m^3/min]$ | $T_{Ci}$ $[^oC]$ | $T_{Hi}$ $[^oC]$ | $T_C$ $[^oC]$ |
|---|---|---|---|---|
| 1 | 0.01 | 25 | 100 | 61.59 |

| $T_H$ $[^oC]$ | $UA$ $[kJ/^oCmin]$ | $\rho$ $[kg/m^3]$ | $c_P$ $[kJ/^oCkg]$ |
|---|---|---|---|
| 63.41 | 300 | 500 | 3.0 |

$c_P$ and $\rho$ are equal for the hot and cold side.

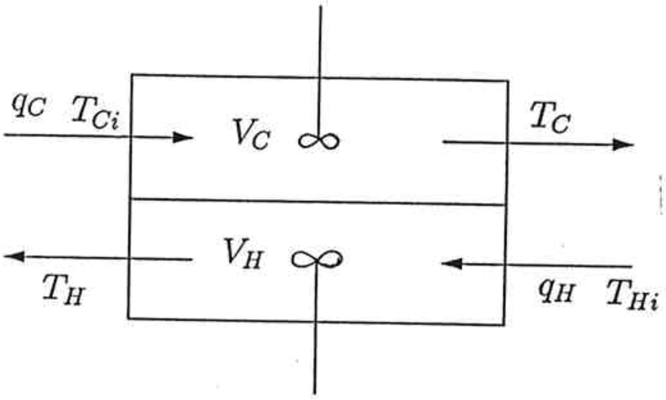


**Figure 1.** Simplified representation of heat-exchanger with one mixing tank on each side.

or "high-gain" direction of the plant).

Open-loop responses in the outlet temperatures to a 10% step change in hot inlet flow $u_2$ obtained from the model (1) are shown by the solid lines in Figure 2 (similar responses, but with opposite signs, are obtained for changes in $u_1$). From the figure we observe that the responses in both outputs are close to first-order with a time-constant around 100 minutes. We also note that the smallest time constant, $\tau_2 = 2.44$ minutes, which we later show is associated with the low-gain direction of the plant, is very difficult to observe from the open-loop responses. Indeed, as seen from the dashed lines in Figure 2, an excellent fit is obtained with the following model

$$G(s) = \frac{1}{1+\tau_1 s}\begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} \quad (2)$$

Although it may seem like this model only has a single time constant $\tau_1 = 100$ minutes, the state-space realization contains two poles at $-1/\tau_1$.

We now want to study the behavior of the process under partial ("one-point") feedback control, i.e., controlling one of the outlet temperatures. The cold outlet temperature $y_1$ is controlled with the cold inlet flow $u_1$ using a P-controller with gain $K = 0.015$ which yields a closed-loop time-constant for this loop of about 3.5 minutes. Figure 3 shows the responses to a 10% step change in hot inlet flow $u_2$ with this loop closed. The solid lines are obtained with the "full" linear model (1), whereas the dashed lines are obtained with the fitted model (2). For the response in the uncontrolled output, $y_2$, there is a significant difference between the two models. The full model yields a "fast" response in $y_2$ (similar to that of the controlled output $y_1$), whereas the fitted model yields a slow settling towards the new steady-state. The reason for the large difference in behavior is, as we shall see, the different number of slow poles in the two models.

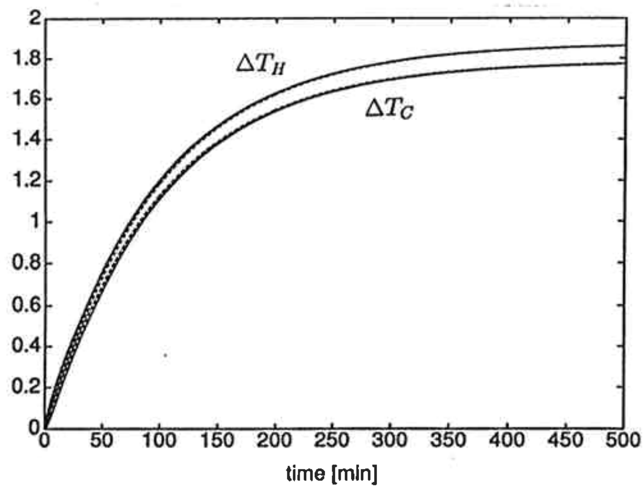


**Figure 2.** Open-loop dynamic response of heat-exchanger. Responses in outlet temperatures $T_C$ ($y_1$) and $T_H$ ($y_2$) to a 10 % step increase in hot inlet flow $q_H$ ($u_2$). Solid line: Response of full model (1). Dashed line: Response of fitted model (2).

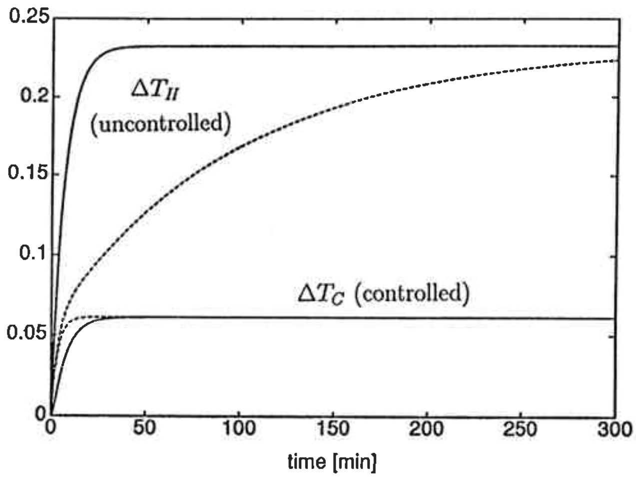


**Figure 3.** Dynamic response of heat-exchanger with one loop closed. Responses in outlet temperatures $T_C$ ($y_1$) and $T_H$ ($y_2$) to a 10 % step increase in $q_H$ ($u_2$). Cold outlet temperature $T_C$ ($y_1$) is controlled by $q_C$ ($u_1$) using a pure proportional controller with gain $K = 0.015$. Solid line: Response of full model (1). Dashed line: Response of fitted model (2).

## 3 Minimum number of states and inconsistency.

Consider a linear system described by the model

$$\dot{x} = Ax + Bu; \quad y = Cx + Du \quad (3)$$

Here $x$ denotes states, $u$ inputs, $y$ outputs and $\dot{x}$ the time derivative of $x$. Laplace transformation of (3) yields the transfer-matrix

$$G(s) = C(sI - A)^{-1}B + D \quad (4)$$

For a system with $n$ states, $m$ inputs and $p$ outputs we have $dim(A) = n \times n$, $dim(B) = n \times m$, $dim(C) = p \times n$ and $dim(D) =$

$p \times m$. The maximum rank of $G(s)$ is $r_{max} = min(p, m)$. Assume that $G(0)$ has rank $r > 1$. With $D \neq 0$ we may define a model with a single state (time-constant) by letting the dynamic part of the model, $C(sI - A)^{-1}B$, have rank equal to 1 and use $D$ to make the rank of $G(0) = r$. However, such a model yields a very poor initial response for most processes and is therefore not considered. With $D = 0$, which is more reasonable from a physical point of view, it is easily seen from (4) that we need at least $r$ states for $G(0)$ to have rank $r$.

*Example 1, continued.* In the heat-exchanger example we had a non-singular steady-state matrix $G(0)$ with rank $r = 2$, and consequently we need at least two states to describe the system using a state-space description with $D = 0$. Thus, when attempting to describe the system using only one time-constant we obtained the simplified model (2) with two poles at $-1/\tau_1$.

Some readers might believe that also the full model has two poles at $-1/\tau_1 = -1/100$ since there are two mixing tanks which isolated would have a time-constant of $V/q = 100$ minutes each. However, an analysis of the full model (1) reveals that there is a multivariable zero that cancels one of the apparent poles at $-1/\tau_1$.

The single pole at $-1/\tau_1$, which is shared by all the transfer function elements, is a result of the interactions between the two sides of the heat exchanger. In addition the full model has a significantly faster pole corresponding to a time-constant $\tau_2 = 2.44$ min. Applying one-point feedback control to the full model (1) causes the shared pole $-1/\tau_1$ to move, and also the uncontrolled response to become fast. However, this is not the case when the simplified model (2) is used, because here only one of the two poles at $-1/\tau_1$ is moved. This is shown in the next section.

*Example 2. PI-control of Wahl and Harriot column [15].*

High-purity distillation columns operating with reflux $L$ and boilup $V$ as independent variables may be strongly ill conditioned. Furthermore, it is well known that the individual open-loop responses may be well approximated using only one dominating time-constant. This has been shown both from plant data [9] and in several theoretical papers, e.g., [4]. Due to this, first order models are commonly used in the distillation control literature.

Wahl and Harriot [15] use a simple low-order model to study the behavior of a high-purity column under one-point control. Their low-order model is somewhat more complicated than the pure first-order transfer-function matrix as given in (2), but the minimal realization of their model contains two time-constants equal to 365 min, while the full model only has one time-constant at 365 min.

The dashed lines in Figure 4 show the response in top composition $y_D$ ($y_2$) of the Wahl and Harriot low-order model to a step change in feed composition with the composition on plate 4 ($y_1$) under feedback control. The controller tuning (PI-controller) used here is somewhat different than the one used by Wahl and Harriot, but the responses resemble closely the ones shown in [15][1], i.e., a fast response in the composition on plate 4 ($y_1$) with a slow settling towards steady-state for the uncontrolled top composition ($y_2$). The slow settling in $y_2$ is noticed by Wahl and Harriot, but they assume it to be a property of the process. However, the slow settling to steady-state is simply a result of a modeling error, that is, the model has an excessive slow pole. This is seen from the solid lines in Figure 4 which show the responses obtained using the full linear model. The full model yields a fast response in both compositions.

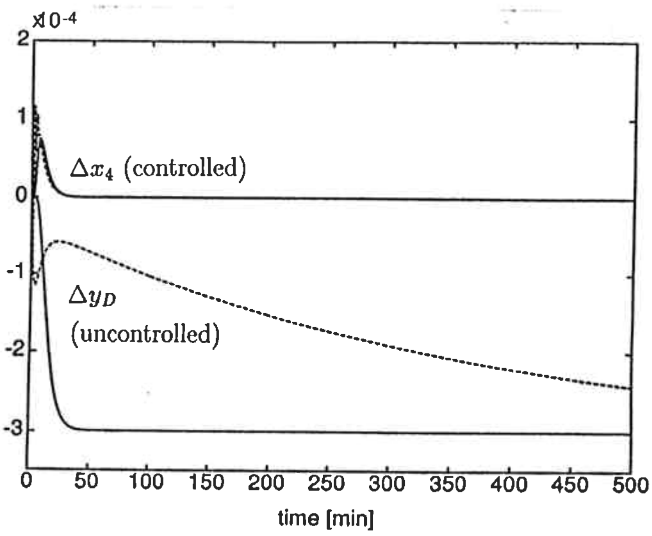


**Figure 4.** Wahl and Harriot column. Response in top-composition $y_D$ ($y_1$) and $x_4$ ($y_2$)to a disturbance in feed composition with $x_4$ controlled by reflux. Dashed lines: Simulations with low-order model given by Wahl and Harriot [15]. Solid lines: Simulations with full linear model.

Also several other authors (e.g., [14], [11]) have used inconsistent models for studies of partial feedback control in distillation. This may be seen from their figures by observing the slow settling in the uncontrolled output.

# 4 Analytical treatment of model with one loop closed.

Consider applying the control law

$$du_1 = -K(dy_1 - dy_{1s}) \tag{5}$$

to the simplified model (2) (here subscript $s$ denotes setpoint). The closed-loop transfer-matrix becomes

$$\begin{pmatrix} dy_1 \\ dy_2 \end{pmatrix} = \frac{1}{1 + \tau_{CL}s} \begin{pmatrix} \frac{Kk_{11}}{(1+Kk_{11})} & \frac{k_{12}}{(1+Kk_{11})} \\ \frac{Kk_{21}}{(1+Kk_{11})} & \frac{k_{22}(1+\tau_{CL}s) - \frac{Kk_{12}k_{21}}{1+Kk_{11}}}{(1+\tau_1 s)} \end{pmatrix} \begin{pmatrix} dy_{1s} \\ du_2 \end{pmatrix} \tag{6}$$

where

$$\tau_{CL} = \tau_1/(1 + Kk_{11}) \tag{7}$$

Thus, three of the elements are first-order with the time-constant, $\tau_{CL}$, whereas the transfer-function $g_{22}(s)$ from $u_2$ to the uncontrolled output $y_2$ is second order, as it in addition contains the open-loop dominant time-constant $\tau_1$. To see how the two time-constants contribute to the overall response in the uncontrolled output $y_2$, write $g_{22}(s)$ on the form

$$g_{22}(s) = \frac{X_1}{1 + \tau_1 s} + \frac{X_{CL}}{1 + \tau_{CL}s} \tag{8}$$

The ratio between the gains $X_1$ and $X_{CL}$ is given by

$$\frac{X_1}{X_{CL}} = (1 + Kk_{11})(\frac{1}{Y} - 1) \quad ;Y = \frac{k_{12}k_{21}}{k_{11}k_{22}} \tag{9}$$

[1] Actually Wahl and Harriot have the wrong sign on the change in top composition

$Y$ is the ratio between the off-diagonal and diagonal steady-state gains, and is a well known measure of interactions (e.g., [2], [10]). It is also related to the 1,1–element of the Relative Gain Array [3] for $2 \times 2$ systems

$$\lambda_{11} = \frac{1}{1-Y} \tag{10}$$

The model is ill-conditioned when $Y$ is close to one which corresponds to a large value of $\lambda_{11}$.

Consider $Y$ in the range 0 to 1. For cases with $Y = 1$ ($\lambda_{11} = \infty$) we see from (9) that $X_1$ becomes zero, i.e., there is no gain related to $\tau_1$, and only $\tau_{CL}$ remains in $g_{22}(s)$. This is as expected since $Y = 1$ implies that the model is singular at all frequencies and the minimal realization of (2) will only contain one state. On the other hand, if $Y = 0$ ($\lambda_{11} = 1$) we see from (9) that the gain related to $\tau_{CL}$ will be zero and only $\tau_1$ will be left in $g_{22}(s)$. This is also as expected since $Y = 0$ implies that the steady-state matrix is triangular or diagonal, in which case it is likely that the identified process actually contains two poles at $-1/\tau_1$ (see discussion below). For values of $Y$ between 0 and 1 ($\lambda_{11} > 1$), both poles will be present in $g_{22}(s)$.

From (9) we see that the ratio $X_1/X_{CL}$ also depends on the gain $K$ used in the controller. The higher the gain is, the larger is the ratio $X_1/X_{CL}$. This means that the faster the response in the controlled output is, the more marked is the large time-constant $\tau_1$ in the uncontrolled output, $y_2$.

*Example 1, continued.* For the heat-exchanger example we have $Y = 0.907$ and $Kk_{11} = 28.1$ which yields $X_1/X_{CL} = 2.98$ for the simplified model (2). That is, a major part of the response in the uncontrolled output $y_2$ is related to $\tau_1$, which is confirmed by the slow settling for $y_2$ (dashed line) in Figure 3. For the full model (1) the single time-constant $\tau_1$ is affected by the feedback control, and $y_2$ has no slow settling (solid line in Figure 3).

Our analysis of equation (9) seems to suggest that it is for weakly interactive processes we get the largest error when an inconsistent model with excessive slow poles is used. However, this conclusion is misleading as it is for ill-conditioned processes we most likely will identify a model with too many slow poles. To see this consider a $2 \times 2$ model which is reduced to have two states. The two poles left should be the ones with the largest effect on the input-output behavior of the full model. Each of the two poles will have an input direction related to them, that is, a set of inputs that cancels the other pole. A similarity transformation of the state-space model, so that the $A$-matrix becomes diagonal, will reveal these directions in the rows of the transformed $B$-matrix. Changes in one input at the time, i.e., the input vectors $[1 \quad 0]^T$ and $[0 \quad 1]^T$, will span the input space. If one of the poles dominates the responses to both these input perturbations, it means that the gain related to the "hidden" pole must be small compared to the gain related to the dominating pole. This implies that the system has two directions with widely differing gains, i.e., the system is ill-conditioned[2]. From this we conclude that it is only for ill-conditioned systems that the open-loop responses are likely to be well approximated using an inconsistent model with a single time-constant. A diagonal or triangular $2 \times 2$ process which has $Y = 0$ ($\lambda_{11} = 1$) and is well described using only one time-constant $\tau_1$ is thus likely to actually contain two poles at $-1/\tau_1$.

*Example 1, continued.* A similarity transformation of the state-space realization of the full heat exchanger model (1) shows that the input direction cancelling $\tau_2$ is $[1 \quad -1]^T$ and the input direction cancelling $\tau_1$ is $[1 \quad 1]^T$. A singular value decomposition of the model gives a (minimized) condition number of 41 with the high-gain input direction being $[1 \quad -1]^T$ and the low-gain input direction being $[1 \quad 1]^T$. In this case we therefore have a perfect alignment of the singular input vectors and the pole-cancelling vectors, i.e., the high-gain input direction has a pole $-1/\tau_1$ and the low-gain input direction a pole $-1/\tau_2$. The gain in the direction of the slow pole $-1/\tau_1$ is consequently 41 times the gain in the direction of the fast pole $-1/\tau_2$, and the fast pole is thus only weakly visible in open-loop simulations with perturbations in single inputs. This explains why a model using only one time-constant yields an excellent fit of the open-loop responses in Figure 2.

## 5 MISO-Identification using ARMAX

In Appendix we provide a Matlab file for generating open-loop "experimental" data using the linear model (1). The data are produced using a multivariable experiment , i.e., simultaneous perturbations in the two inputs. Noise is added to the inputs as well as the outputs. Figure 5 shows the 100 min. input sequence (including noise) and the resulting outputs generated using the Matlab file. The inputs to the process contain 3 % white noise, while the outputs have white noise with variance 0.03 $°C$ (which is very small compared to practical situations).

The identification problem is to come up with a reasonable multivariable dynamic model based on these data alone, i.e., based on the noise-free inputs and the noisy measurements. One should not supply any knowledge about the special multivariable structure of the model as given by (1). The identified model is intended to be used for feedback control studies, and two different cases are of interest. 1) Partial control: Output $y_1$ is controlled using input $u_1$ while $y_2$ is left uncontrolled. 2) Multivariable control: Both $y_1$ and $y_2$ are controlled using both inputs. In both cases the responses to set-point changes as well as disturbances in the inputs should be considered and compared with those of the

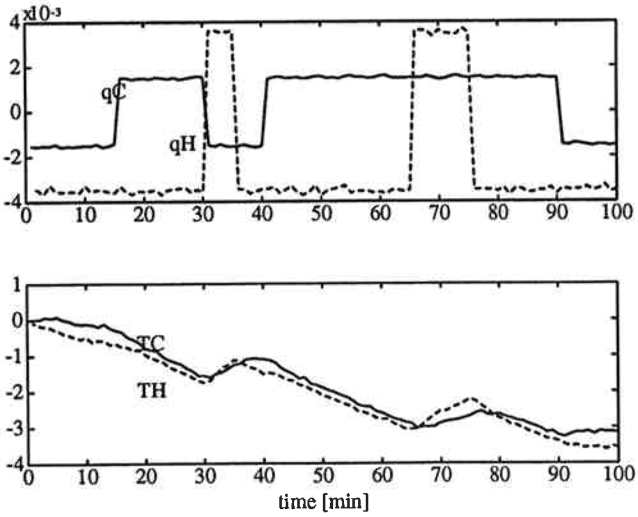


**Figure 5.** Input and output data used for identification of heat exchanger. The data were generated with the Matlab file given in Appendix.

[2]Note that some ill-conditioned systems may have the directions of the poles closely aligned with the input vectors of the perturbations. In this case both poles will show up in the simulations.

correct model (1). The intention of the challenge problem is that one should identify the model based on open-loop data only. If one is allowed to use closed-loop data we believe the identification becomes simpler.

We here employ a fairly standard identification technique to the data generated using the Matlab file given in Appendix. We employ the Matlab System Identification Toolbox [8] and use MISO-identification with an ARMAX-type model structure. In the identification we fit each output with a strictly proper second order model which is the same structure as the true model (1) (see Appendix for details). The model resulting from this identification is given by

$$G(s) = \begin{pmatrix} \frac{-2025(5.218s+1)}{(2.027s+1)(110.7s+1)} & \frac{1871(0.0263s+1)}{(2.027s+1)(110.7s+1)} \\ \frac{-1795(-0.0933s+1)}{(1.404s+1)(110.5s+1)} & \frac{2049(3.947s+1)}{(1.404s+1)(110.5s+1)} \end{pmatrix} \quad (11)$$

The identified model (11) has a minimal realization with 4 states. Figure 6 compares the noise-free open-loop step responses of model (11) with those of the "true" model (1). We see from the responses that we have obtained a reasonable identification of the individual SISO-transfer functions. Furthermore, we see from the identified model that we have been able to obtain reasonable estimates for the two poles $-1/\tau_1$ and $-1/\tau_2$. However, the multivariable interactions have not been captured as the model (11) has multivariable zeros at $-0.0217 = -1/46.1$ and $-0.426 = -1/2.35$ which do not cancel the poles. This is also becomes clear if one consider the singular values of the true (1) and fitted (11) model respectively. The true model (1) has, as mentioned previously, a low-gain direction with a single fast pole $-1/\tau_2$. However, the low-gain direction of the fitted model (11) has a significant part of its dynamics related to a slow pole around $-1/\tau_1$.

Figure 7 compares the closed-loop responses of the correct model (1) and the identified model (11) when output 1 is controlled with input 1 using the proportional feedback law $u_1 = K_c y_1$ with $K_c = 0.015$. We see that the identified model yields a good prediction for the controlled output $y_1$. However, for the uncontrolled output $y_2$ there is a large discrepancy between the process represented by (1) and the identified model (11). For the correct model the single slow pole is moved by the feedback controller and the response in the uncontrolled output $y_2$ is as fast as for $y_1$. The identified model (11), on the other hand, contains an excessive slow pole which is left in the partially controlled model and results in a slow settling in output $y_2$.

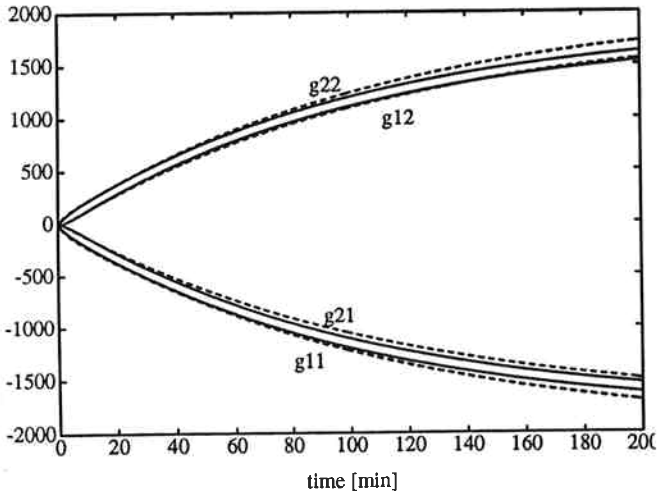


**Figure 6.** Open-loop step responses of identified model (11) (dashed lines) and model (1) (solid lines). Labels $g_{ij}$ denotes corresponding transfer-matrix elements.

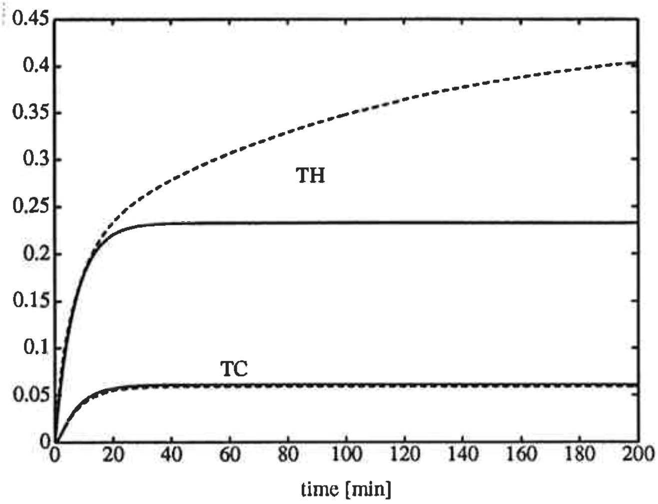


**Figure 7.** Closed-loop responses of identified model (11) (dashed lines) and correct model (1) (solid lines) to step disturbance of magnitude 0.001 in the hot flow $q_H$ ($u_2$). Output $T_C$ ($y_1$) controlled by $q_C$ ($u_1$) using proportional controller with gain $K_c = 0.015$.

The noise levels of the data provided in Appendix are relatively small compared to what one should expect in a practical situation. However, increasing the noise levels will mainly change the results obtained in a qualitative manner, that is, the excessive slow pole in the identified model will become even more marked. An additional problem which may be encountered at higher noise levels is that of obtaining the correct sign of the determinant of the steady-state model. However, as mentioned in the introduction, this is usually a less crucial problem as the sign and approximate value of the determinant in many cases is known a priori.

The input sequence used to generate the "experimental" data in Appendix are based on low-pass filtered PRBS signals with a minimum time between changes of 5 minutes. It was assumed that 100 minutes of experiments with a sampling rate of 1 minute would provide sufficient data for the identification. However, we do not rule out the possibility that a different input sequence may yield better results. The problem is how to determine the best possible input sequence when the process dynamics are largely unknown. It is worth noting that although we used a low-pass filtered input sequence, the main model error was at rather low frequencies, while the high frequency behavior of the process was reasonably well captured in the identified model. This may indicate that an input sequence with even more emphasis on low frequencies would have yielded better results than the actual input sequence used in Appendix.

# 6 Conclusions

- The open-loop responses of ill-conditioned processes will often take the form of almost pure first-order dynamics, and the open-loop dynamics of such processes are seamingly well approximated by a low-order model containing only the dominant time-constant. However, the model will have the *single* slow pole of the process repeated and is therefore physically inconsistent. The inconsistency results in a poor prediction of the process behavior, in particular under partial feedback control.

- We believe the problem of obtaining models for ill-conditioned processes which are consistent in terms of the number of slow poles represents a "new" and challenging problem in identification.

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**APPENDIX 1. Matlab-file for generating input-output data of heat exchanger.**

```
% This file generates inputs, u, and outputs, y,
% for heat exchanger identification problem:
rand('normal');
A=[-.21 .20;.20 -.21];B=[-36.5853 0;0 36.5853];
C=eye(2);D=zeros(2);
%PRBS-signals (low-pass filtered):
q1=1.5e-3*[-1 -1 -1 1 1 1 -1 -1 1 1 1 1 1 1 1 1 1 1 -1 -1];
q2=3.5e-3*[-1 -1 -1 -1 -1 -1 1 -1 -1 -1 -1 -1 -1 1 1 1 -1 -1 -1 -1 -1];
% Inputs last for 5 minutes (sampling time 1 min.):
for i=1:length(q1),
u(1+5*(i-1):5*i,1)=q1(i)*ones(5,1);
u(1+5*(i-1):5*i,2)=q2(i)*ones(5,1);
end
%Noisy inputs for simulation:
usim(:,1)=u(:,1)+0.03*max(u(:,1))*rand(100,1);
usim(:,2)=u(:,2)+0.03*max(u(:,2))*rand(100,1);
% Obtain noise-free outputs:
t=1:100;
ysim=lsim(A,B,C,D,usim,t);
% Noise on outputs has variance 0.03 degrees centigrades:
y(:,1)=ysim(:,1)+0.03*rand(100,1);
y(:,2)=ysim(:,2)+0.03*rand(100,1);
```

Example of using MISO ARMAX-identification [8]

```
% Fit model for y1:
the1=pem([y(:,1) u],[2 2 2 2 0 0 0 1 1]);
% Fit model for y2:
the2=pem([y(:,2) u],[2 2 2 2 0 0 0 1 1]);
% Note: the1 and the2 are on discrete polynomial form.
```

**NOMENCLATURE**

$A$ - heat transfer area ($m^2$)
$c_P$ - heat capacity ($kJ/{}^oCkg$)
$G(s)$ - process transfer-matrix for effect of inputs $u$
$g_{ij}(s)$ - transfer matrix element i,j
$k_{ij}$ - steady state process gains
$I$ - identity matrix
$K$ - controller gain
$q_C$, $q_H$ - cold and hot inlet flows ($m^3/min$)
$T_C$, $T_H$ - cold and hot outlet temperatures (${}^oC$)
$U$ - heat transfer coefficient ($kJ/m^2\ {}^oCmin$)
$u_i$ - process input i
$V_C$ - liquid volume cold side ($m^3$)
$V_H$ - liquid volume hot side ($m^3$)
$Y = \frac{k_{12}k_{21}}{k_{11}k_{22}}$ - interaction measure
$y_D$ - distillate composition
$y_i$ - process output i

*Greek symbols*

$\lambda_{11}$ - 1,1 element of RGA
$\tau_1$ - dominant (largest) process time-constant (min)
$\tau_2$ - smaller process time-constant (min)
$\tau_{CL}$ - closed-loop time-constant (min)

*Subscripts*

$s$ - setpoint change