12th IFAC World Congress, Sydney, 18-23 July 1993

On the Structure of the Robust Optimal Controller for a Class of Problems

Morten Hovd* Richard D. Braatz† Sigurd Skogestad‡
Chemical Engineering, University of Trondheim, NTH,
N-7034 Trondheim, Norway.

Abstract. In this paper we investigate the structure of the robust optimal controller for a class of control problems investigated by many researchers. The robust optimal controller for a problem in this class is an SVD controller. This finding may be used to simplify the controller synthesis (K) part of the D-K iteration procedure used for synthesizing μ-optimal controllers.

Conditions for when the optimal controller in general has the structure of an SVD controller are discussed, focusing on the issues of realizability of the transformed interconnection matrix and whether the transformation makes the structure of the perturbation block (Δ) more conservative.

1 Introduction

In this paper we investigate the structure of the robust optimal controller for a class of control problems investigated by many researchers. An example of a control problem in this class is given by the robust controller design problem for a distillation column studied previously by Skogestad et al. [19].

The nominal plant for this problem is given by

$$G(s) = \frac{1}{75s+1}\begin{bmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{bmatrix} \tag{1}$$

which has a condition number of 141.7 and a RGA-value of 35.5 at all frequencies. This model is an excellent example for demonstrating the problems with ill-conditioned plants and has been studied by many researchers [14, 3, 21].

For this problem, the relative magnitude of the uncertainty in each of the manipulated variables is given by $w_I(s) = 0.2(5s+1)/(0.5s+1)$. The robust performance specification is that $\|w_P\hat{S}\|_\infty < 1$, where $w_P = 0.5(10s+1)/10s$ and $\hat{S}$ is the worst sensitivity function possible with the given bounds on the uncertainty in the manipulated variables.

This robust controller design problem is easily captured in the framework of the structured singular value, μ [5]. The resulting μ condition for Robust Performance (RP) becomes:

$$\text{RP} \iff \mu_\Delta(M) < 1 \quad \forall\omega \tag{2}$$

$$M = \begin{bmatrix} -W_I KSG & W_I KS \\ W_P SG & -W_P S \end{bmatrix}; \quad \Delta = \text{diag}\{\Delta_I, \Delta_P\} \tag{3}$$

where Δ_I is a *diagonal* 2×2 perturbation block, Δ_P is a full 2×2 perturbation block, $W_I = w_I I_2$ and $W_P = w_P I_2$.

Skogestad et al. [19] designed a controller giving a value of $\mu = 1.067$. Freudenberg [9] used another design method to find a controller with $\mu = 1.054$. Lundström et al. [14] used the latest state-space H_∞ software [1] to design a controller with $\mu = 0.978$. In a somewhat altered form, this robust controller design problem has been considered by Yaniv and Barlev [21], and was used as a benchmark for the 1991 CDC [3].

Engstad [8] showed that the controller obtained by Lundström et al. [14] has the structure of an SVD controller. We prove that the μ-optimal controller is an SVD controller for this robust controller design problem. This suggests that the controller obtained by Lundström et al. [14] is very near μ-optimal. The resulting analysis suggests how the design problem can be simplified prior to applying D-K iteration (or H_∞-synthesis) for finding the controller. The simplified design problem is equivalent to the original design problem provided this contains only full and/or multiplicative repeated scalar perturbation blocks. We then describe the class of problems for which the optimal controller is an SVD controller.

2 Background

Robust Performance The goal of any controller design is that the overall system is stable and satisfies some minimum performance requirements. These requirements should be satisfied at least when the controller is applied to the *nominal* plant, that is, we require nominal stability and nominal performance.

In practice the real plant $\hat{G}$ is not equal to the model G. The term *robust* is used to indicate that some property holds for a set Π of possible plants $\hat{G}$ as defined by the uncertainty description. In particular, by *robust stability* we mean that the closed loop system is stable for all $\hat{G} \in \Pi$. By *robust performance* we mean that the performance requirements are satisfied for all $\hat{G} \in \Pi$. Performance is commonly defined in robust control theory using the H_∞-norm of some transfer function of interest.

Definition 2.1 *The closed loop system exhibits nominal performance if*

$$\|\Psi\|_\infty \equiv \sup_\omega \bar{\sigma}(\Psi) < 1. \tag{4}$$

Definition 2.2 *The closed loop system exhibits robust performance if*

$$\|\hat{\Psi}\|_\infty \equiv \sup_\omega \bar{\sigma}(\hat{\Psi}) < 1, \quad \forall \hat{G} \in \Pi. \tag{5}$$

For example, for rejection of disturbances at the plant output, Ψ would be the weighted sensitivity

$$\begin{aligned} \Psi &= W_1 S W_2, \quad S = (I+GK)^{-1} \\ \hat{\Psi} &= W_1 \hat{S} W_2, \quad \hat{S} = (I+\hat{G}K)^{-1}. \end{aligned} \tag{6}$$

In this case, the input weight W_2 is often equal to the disturbance model. The output weight W_1 is used to specify the frequency range over which the sensitivity function should be small and to weigh each output according to its importance. The value K is the transfer function of the controller.

Doyle [5] derived the *structured singular value*, μ, to test for robust performance. To use μ we must model the uncertainty (the set Π of possible plants $\hat{G}$) as norm bounded perturbations (Δ_i) on the nominal system. Through weights each perturbation is normalized to be of size one:

$$\|\Delta_i\|_\infty \le 1. \tag{7}$$

The perturbations, which may occur at different locations in the system, are collected in the block-diagonal matrix Δ_U (the U denotes uncertainty)

$$\Delta_U = \text{diag}\{\Delta_i\} \tag{8}$$

and the system is arranged to match the left block diagram in Figure 1. The interconnection matrix M in Figure 1 is determined by the nominal model (G), the size and nature of the uncertainty, the performance specifications, and the controller (K).

For notational convenience in this section we assume M and each Δ_i are square (analogs to the definitions and theorems in this section hold in the nonsquare case [13]). We assume each Δ_i is complex. For the example studied in this paper, these assumptions hold. The definition of μ is:

Definition 2.3 *Let $M \in \mathbf{C}^{n\times n}$ be a square complex matrix and let Δ be the set of block-diagonal perturbations with the appropriate struc-*

*Present address: Fantoft Prosess A/S, P. O. Box 306, 1301 Sandvika, Norway. Phone: +47-67-540960. Fax: +47-67-546180

†currently at California Institute of Technology, Chemical Engineering, 210-41, Pasadena, CA 91125, USA

‡To whom correspondence should be addressed. Fax: +47-7-594080, e-mail: skoge@kjemi.unit.no

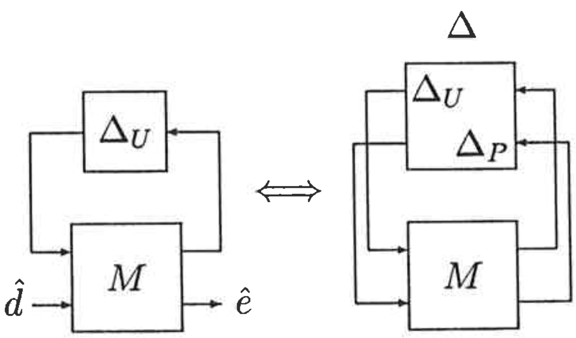

Figure 1: Robust Performance and the $M-\Delta$ block structure

ture. Then $\mu_{\mathbf{\Delta}}(M)$ (the structured singular value with respect to the uncertainty structure $\mathbf{\Delta}$) is defined as

$$\mu_{\mathbf{\Delta}}(M) \equiv \begin{cases} 0 \text{ if there does not exist } \Delta \in \mathbf{\Delta} \text{ such that} \\ \det(I+M\Delta)=0, \\ \left[\min_{\Delta\in\mathbf{\Delta}}\{\bar{\sigma}(\Delta)\,|\det(I+M\Delta)=0\}\right]^{-1} \quad \text{otherwise.} \end{cases} \tag{9}$$

Partition M in Fig. 1 to be compatible with $\Delta = \text{diag}\{\Delta_U, \Delta_P\}$:

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}. \tag{10}$$

The following are tests for robust stability and robust performance [5]

Theorem 2.4 *The closed loop system exhibits robust stability for all $\|\Delta_U\|_\infty \le 1$ if and only if the closed loop system is nominally stable and*

$$\mu_{\Delta_U}(M_{11}(j\omega)) < 1 \qquad \forall\omega. \tag{11}$$

Theorem 2.5 *The closed loop system exhibits robust performance for all $\|\Delta_U\|_\infty \le 1$ if and only if the closed loop system is nominally stable and*

$$\mu_{\mathbf{\Delta}}(M(j\omega)) < 1 \qquad \forall\omega, \tag{12}$$

where $\mathbf{\Delta} = \text{diag}\{\Delta_U, \Delta_P\}$, and Δ_P is a full square matrix with dimension equal to the number of outputs (the subscript P denotes performance).

Multiple performance objectives can be tested similarly using block-diagonal Δ_P. Note that the issue of robust stability is simply a special case of robust performance.

It is a key idea that μ is a *general* analysis tool for determining robust performance. Any system with uncertainty adequately modeled as in (7) can be put into $M-\Delta_U$ form, and robust stability and robust performance can be tested using (11) and (12). Standard programs calculate the M and Δ [1], given the transfer functions describing the system components and the location of the uncertainty and performance blocks Δ_i.

Computation of μ μ with complex Δ is commonly calculated through upper and lower bounds. First define two subsets of $\mathbf{C}^{n\times n}$

$$\mathbf{Q} = \{Q \in \mathbf{\Delta} : Q^H Q = I_n\} \tag{13}$$

where Q^H is the conjugate transpose of Q and I_n is the $n\times n$ identity matrix, and

$$\mathbf{D} = \{\text{diag}\,[d_i I_i] : \dim(I_i) = \dim(\Delta_i), d_i \text{ positive real scalar}\}, \tag{14}$$

then [5]

$$\max_{Q\in\mathbf{Q}} \rho(QM) \le \mu_{\mathbf{\Delta}}(M) \le \inf_{D\in\mathbf{D}} \bar{\sigma}(DMD^{-1}). \tag{15}$$

A result of Doyle [5] is that the lower bound, $\max_{Q\in\mathbf{Q}} \rho(QM)$, is always equal to $\mu_{\mathbf{\Delta}}(M)$. Unfortunately, the maximization is not convex, and computing the global maximum of such functions is in general difficult. In contrast, the computation of the upper bound is convex. However, the upper bound is not necessarily equal to μ except when the number of complex Δ-blocks is ≤ 3. The upper and lower bounds are almost always within a percent or so for real problems [16], so for engineering purposes μ never has to be calculated exactly.

Controller Synthesis M is a function of the controller K. The H_∞-optimal control problem is to find a stabilizing K which minimizes $\sup_\omega \bar{\sigma}(M(K))$. The state-space approach for solving the H_∞-control problem is described in [7].

The D-K iteration method (often called μ-synthesis) is an *ad hoc* method which attempts to minimize the tight upper bound of μ in (15), i.e. it attempts to solve

$$\min_K \inf_{D\in\mathbf{D}} \sup_\omega \bar{\sigma}(DM(K)D^{-1}). \tag{16}$$

The approach in D-K iteration is to alternatingly hold K or D constant and minimize $\sup_\omega \bar{\sigma}(DM(K)D^{-1})$ with respect to the other. For fixed D, the controller synthesis is solved via H_∞-optimization. For fixed K, the quantity is minimized as a convex optimization. The resulting D as a function of frequency is fitted with an invertible stable minimum-phase transfer function and wrapped back into the nominal interconnection structure. This increases the number of states of the scaled M, which leads the next H_∞-synthesis step to give a higher order controller. The iterations stop after $\sup_\omega \bar{\sigma}(DM(K)D^{-1})$ is less than 1 or is no longer diminished. The resulting high-order controller can usually be reduced significantly using standard model reduction techniques [1]. Though this method is not guaranteed to converge to a global minimum, it has been used extensively to design robust controllers and seems to work well [6].

3 The Structure of the μ-Optimal Controller

Recall that the problem statement of Skogestad et al. [19] specifies that Δ_I be a diagonal perturbation block. Lundström et al. made the assumption that the uncertainty block Δ_I was a full block when performing the D-K iteration design method to design the controller. Even with this potentially conservative assumption, a controller with smaller μ value was obtained. This leads one to suspect that the structure of Δ_I is not important for this problem.

In Section 3.1 we give the structure of the optimal controller found by Lundström et al. in [14]. In Section 3.2 we show that the robustness of an *SVD controller* is insensitive to the structure of Δ_I. In Section 3.3 we show that the μ-optimal controller is an SVD controller. This explains why the assumption made by Lundström et al. that Δ_I is full block was nonconservative. This also allows us to derive a simplified D-K iteration design procedure in which the synthesis part (K) can be solved as two decoupled subproblems.

3.1 The Structure of the Controller Found by Lundström in [14]

The plant $G(s)$ can be decomposed into $G(s) = U\Sigma_G(s)V^H$, where

$$\Sigma_G(s) = \frac{1}{75s+1}\begin{bmatrix} 1.9721 & 0 \\ 0 & 0.0139 \end{bmatrix} \tag{17}$$

$$U = \begin{bmatrix} 0.6246 & -0.7809 \\ 0.7809 & 0.6246 \end{bmatrix};\; V = \begin{bmatrix} 0.7066 & -0.7077 \\ -0.7077 & -0.7066 \end{bmatrix}$$

U and V are unitary matrices. This is the singular value decomposition of the plant $G(s)$[1]. We define an SVD controller for the plant $G(s)$ to have the form

$$K(s) = V\Sigma_K(s)U^H \tag{18}$$

where $\Sigma_K(s)$ is a diagonal matrix. Engstad [8] found that this is indeed the structure of the controller found by Lundström et al. in [14].

3.2 The Structure of Δ_I

We now show that the structure of Δ_I is unimportant in determining robust stability provided that the controller is an SVD controller. To do this, we will need the following result, where A^H is the conjugate transpose of A.

Theorem 3.1 (μ for Normal Matrices) *Assume M is normal (i.e. $M^HM = MM^H$), then $\mu_\Delta(M) = \rho(M) = \bar{\sigma}(M)$ irrespective of the structure of Δ (provided Δ is complex).*

Proof: Result follows directly from (15) and that $\rho(M) = \bar{\sigma}(M)$ for normal matrices. QED.

This theorem states that the value for μ is independent of the structure of Δ provided that the M matrix is normal. This result has proven useful for studying the robust control of cross-directional paper manufacturing [12] and coating processes [2], and for parallel processes [11, 17].

We now apply this theorem to show that for SVD controllers the robust stability for the system under study is independent of the structure of the uncertainty block Δ_I.

The test for robust stability is given by Thm. 2.4, with $M_{11} = -W_I KSG = -W_I KG(I+KG)^{-1}$. Substitute the SVD for the plant

[1] With the slight modification that the dynamic term, $1/(75s+1)$, is multiplied into the singular value matrix Σ_G, thus giving the singular values phase.

(17) and the expression for the SVD controller (18) into the expression for M_{11} to give

$$M_{11} = -VW_I\Sigma_K\Sigma_G(I+\Sigma_K\Sigma_G)^{-1}V^H, \qquad (19)$$

where V commutes with W_I since W_I is a repeated scalar block.

Since W_I, Σ_G, Σ_K and their conjugate transposes are diagonal and commute, it is easy to show that $M_{11}M_{11}^H = M_{11}^H M_{11}$, i.e. M_{11} is normal. Applying Theorem 3.1 gives us that robust stability for this problem is independent of the structure of Δ_I. Note that this is not necessarily true when the controller is not an SVD controller. For simplicity, we would normally take the structure of Δ_I to be a full block.

Though this does not imply that the structure of Δ_I is unimportant when determining robust *performance*, we certainly would not be surprised if this were the case. We now show that the robust performance of the μ-optimal controller is insensitive to the structure of Δ_I for this specific example.

In Fig. 2 we give the robust performance μ plots for the controller of Lundström et al. for both when Δ_I is full block and when Δ_I consists of independent scalar blocks. The plots are indistiguishable, i.e. robust performance is independent of the structure of Δ_I for the SVD controller. Notice the flatness of the μ plots; it is well-known that the μ-optimal controller has the property that the optimal $\mu(M(j\omega))$ is constant, except at very high frequencies where μ must approach $|w_P|$ for proper controllers. Since the μ plot for the controller of Lundström et al [14] is very flat, and the controller is an SVD controller [8], we expect that this controller is very nearly μ-optimal.

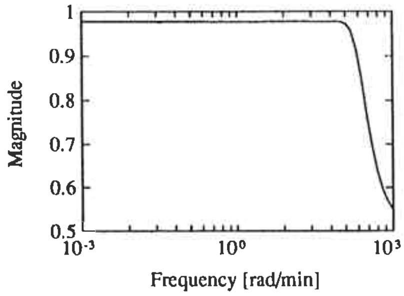

Figure 2: a) Comparision of robust performance μ plots with Δ_I equal to full block and to a diagonal block.

3.3 Analysis of the Optimal Control Problem

Here we analyze the control problem for full block Δ_I as used by Lundström et al. in [14]. We prove that the μ-optimal controller must be an SVD controller.

In Fig. 3 and we give equivalent block diagrams for the $M-\Delta$ structure in (3). Clearly, an identity matrix can be inserted anywhere in a block diagram without altering the system. In Fig. 3a we have shown M as an LFT of the controller K. Thereafter identities are inserted in four different places (e.g. $UU^H = U^HU = I$). Note that $\mathrm{diag}\{V^H, U^H\}$ commutes with $\mathrm{diag}\{W_I, W_P\}$, since both W_I and W_P are scalar times identity matrices. The blocks within each dashed box in Fig. 3a are combined to form $\tilde{\Delta}$, $\tilde{N}$, and Σ_K, which we show in Fig. 3b.

Now we consider the structure of the transformed system in Fig. 3b. The transformed controller Σ_K and all the blocks in $\tilde{N}$ in Fig. 3b are diagonal. The transformed performance block $\tilde{\Delta}_P$ is a full block. With Δ_I diagonal, the transformed uncertainty block $\tilde{\Delta}_I = V^H\Delta_I V$ in $\tilde{\Delta}$ would be a full block, with a certain structure that cannot be utilized in the μ framework. We showed in Section 3.2 that the structure of Δ_I is unimportant provided that the controller is an SVD controller, so we allow Δ_I to be full. The transformed uncertainty block $\tilde{\Delta}_I$ is then full block with no additional structure.

Now consider the robust optimal control problem, in which we desire to minimize μ:

$$\begin{aligned}\min_K \mu_\Delta(M(K)) &= \min_{\Sigma_K} \mu_{\tilde{\Delta}}(\tilde{M}(\Sigma_K)) \\ &= \min_{\Sigma_K} \inf_{D\in\mathbf{D}} \sup_\omega \bar{\sigma}\left(D\tilde{M}(\Sigma_K)D^{-1}\right) \\ &= \inf_{D\in\mathbf{D}} \min_{\Sigma_K} \sup_\omega \bar{\sigma}\left(D\tilde{M}(\Sigma_K)D^{-1}\right). \qquad (20)\end{aligned}$$

The first equality holds because the transformed system is equivalent to the original system. The second equality holds because the number of uncertainty blocks is ≤ 3. Because the perturbations are full block, the D-scales are diagonal. Since we also have that every block in $\tilde{N}$ is diagonal, the controller synthesis for the transformed system

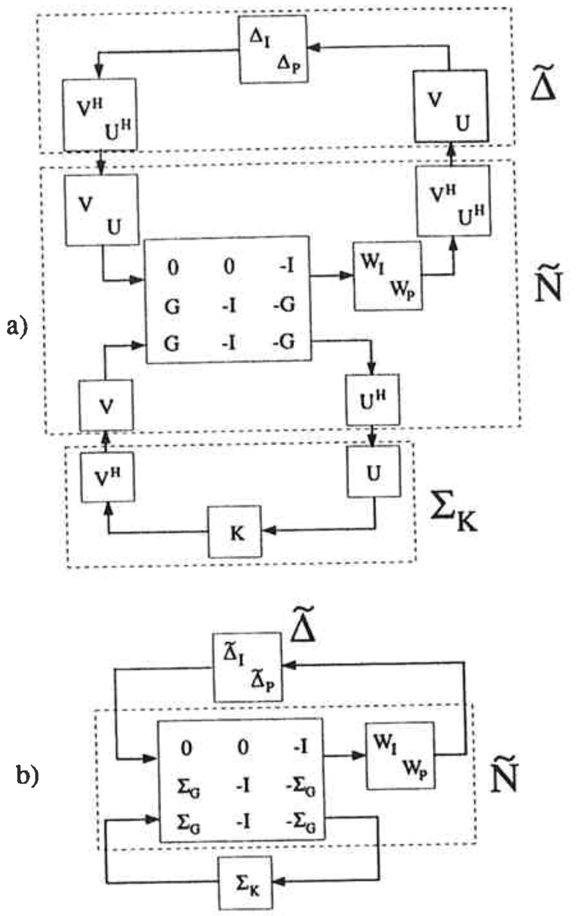

Figure 3: The $M-\Delta$ structure of the synthesis problem. a) Expressing M as a linear fractional transformation of the controller K, with identities inserted at four places in the block diagram. b) The feedback system after transformation, $\tilde{M}-\tilde{\Delta}$.

$$\min_{\Sigma_K} \sup_\omega \bar{\sigma}\left(D\tilde{M}(\Sigma_K)D^{-1}\right) \qquad (21)$$

consists of two *completely decoupled synthesis subproblems, each subproblem involving a SISO "plant"*. The resulting robust optimal controller Σ_K must be diagonal, therefore the original controller K is indeed an SVD controller.

Now consider performing the D-K iteration design procedure to the transformed system to try to determine the robust optimal controller. The controller synthesis part (K) of D-K synthesis consists of two completely decoupled synthesis subproblems, each subproblem involving a SISO "plant". This holds also after applying the D-scales from the robustness analysis (D) part of D-K synthesis, since the D-scales also consist of diagonal blocks. When using (18) to find the controller K from the diagonal Σ_K, we see that the resulting controller will have the structure of an SVD controller.

However, since $\tilde{\Delta}$ contains full blocks, the same D-scales must be applied to both synthesis subproblems. The robustness analysis (D) part of D-K iteration must therefore be performed simultaneously for both subproblems, i.e. we must consider the diagonal matrix Σ_G for robustness analysis, and not its diagonal elements separately.

Performing D-K iteration on the transformed system will converge faster and is numerically better conditioned than on the original system. This is both because the H_∞ subproblems are smaller than the original problem, and because the algorithm will be initialized with a controller which has the correct (optimal) directionality.

4 Generalization of the Results.

It is of interest to determine for which class of controller synthesis problems the μ-optimal controller has the structure of an SVD controller. It is easier to consider when the "μ-upper bound" optimal controller has the structure of an SVD controller. Since the D-K iteration procedure uses the upper bound when designing the controller, and the upper bound is within 1-2% of μ for all practical problems to date, considering the optimally in terms of the upper bound is not restrictive.

A practical requirement is that the transformed interconnection matrix (corresponding to $\tilde{N}$ in Fig. 3) must be realizable, in order to enable the use of standard state-space based H_∞-synthesis algorithms,

e.g. [7]. This will always be possible when U and V are real and the weights used commute with the matrices used to transform the interconnection matrix N (e.g. in Fig. 3 diag$\{V^H, U^H\}$ commutes with diag$\{W_I, W_P\}$). For specificity, we give the following three classes of systems for which these properties hold:
1. the plant is described by scalar dynamics multiplied by a constant matrix, with scalar times identity weights,
2. the plant is parallel, with parallel weights, and
3. the plant is symmetric circulant, with symmetric circulant weights.
The distillation column example studied in this paper is in Class 1. Distillation column models given by scalar dynamics multiplied by a constant matrix have been used by numerous researchers (for example, see the references listed in [18]).

Nominally identical units in parallel with interactions, for example flow splitters and parallel reactors with combined precooling, are described by parallel models. Hovd and Skogestad [11] has studied the robust optimal control of these systems in detail.

It is easy to show that such models are diagonalized by a real Fourier matrix.[2] To show how this diagonalization works on a simple example, a 2×2 parallel process has the model

$$G(s) = \begin{bmatrix} a(s) & b(s) \\ b(s) & a(s) \end{bmatrix}. \tag{22}$$

The 2×2 Fourier matrix, which diagonalizes $G(s)$, is

$$F = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{23}$$

Applying this to $G(s)$ gives

$$\Sigma_G = FG(s)F^T = \begin{bmatrix} a(s) + b(s), a(s) - b(s) \end{bmatrix}. \tag{24}$$

Paper machines [12, 20] and coating processes [2] have been approximated by symmetric circulant models. Though this class is more general than the class of parallel models, symmetric circulant models are diagonalized by the same real Fourier matrix.

Whether the μ-optimal controller is an SVD controller also depends on the location and structure of the perturbation blocks.[3] The μ-optimal controller will be a SVD controller only if the transformations of the interconnection matrix involved do not make the structure of the resulting perturbation matrix $\tilde{\Delta}$ more conservative than the structure of the original perturbation matrix Δ. If the original problem contains only full perturbation blocks and/or multiplicative (or inverse multiplicative) repeated scalar perturbation blocks, the structure of $\tilde{\Delta}$ will equal the structure of Δ. On the other hand, if the problem contains diagonal or additive repeated scalar blocks, the structure of $\tilde{\Delta}$ may be more conservative than the structure of Δ. For the special case when the plant is described by a normal transfer function matrix (such as for parallel or symmetric circulant plants), additive repeated scalar perturbation blocks do not make the structure of $\tilde{\Delta}$ any more conservative than the structure of Δ.

Even when the structure of the transformed perturbation matrix $\tilde{\Delta}$ is more conservative than the structure of the original perturbation matrix Δ, it may still be useful to perform D-K iteration on the transformed system to get initial D-scales for performing D-K iteration on the original system.

For the cases of H_2- and H_∞-optimal control there is no perturbation block (Δ) in the problem, and considerations about the structure of the perturbation block therefore do not apply. However, both the H_2 and H_∞ norms are invariant under unitary transformations, and the optimal controller will have the structure of an SVD controller provided $\tilde{N}$ is realizable, as discussed above for the μ-optimal controller.

5 Conclusions

We have shown that the robustness of an SVD controller is insensitive to the structure of the input uncertainty for the distillation control problem in [19]. We further showed that the μ-optimal controller for this problem has the structure of an SVD controller. This finding may be used to simplify the controller synthesis (K) part of the D-K iteration procedure used for synthesizing μ-optimal controllers.

Conditions for when the optimal controller in general has the structure of an SVD controller have been discussed, focusing on the issues of realizability of the transformed interconnection matrix $\tilde{N}$ (see Fig. 3b) and whether the transformation makes the structure of the perturbation block (Δ) more conservative.

[2]The standard Fourier matrix of [4] has pairs of columns which are complex conjugates of each other. A real Fourier matrix is defined by replacing each of these pairs of columns by an appropriate scaling to the addition and subtraction of the columns (see [10] for details).

[3]Recall that the performance specifications can be written in terms of performance perturbation blocks. Thus without loss of generality we can speak only of the locations of the perturbation blocks and not of the transfer functions of interest for performance.

6 Acknowledgements

The authors thank Petter Lundström for providing Fig. 2. The second author thanks the American-Scandinavian Association of Los Angeles and the Norway-America Association in Oslo, Norway for support during the author's stay in Trondheim during the summer of 1992; and he thanks the Fannie and John Hertz Foundation for their continued support.

References

[1] Balas, G., Doyle, J. C., Glover, K., Packard, A. and Smith, R. (1991). "μ-analysis and synthesis toolbox. User's guide." MUSYN Inc., Minneapolis, USA.

[2] Braatz, R. D., Tyler, M. L., Morari, M., Pranckh, F. R. and Sartor, L. (1992). ""Identification and cross-directional control of coating processes: Theory and experiments". *Proc. American Control Conference*, Chicago, USA, pp. 3021-3025.

[3] *Proceedings of the 30th Conference on Decision and Control*, (1991), Brighton, England, Session T2-7.

[4] Davis, P. J. (1979). *Circulant Matrices*. Wiley & Suns, New York, USA.

[5] Doyle, J. C. (1982). "Analysis of feedback systems under structured uncertainty". *IEE Proc.*, **129**(D), pp. 242-250.

[6] Doyle, J. C. (1985). "Structured uncertainty in control system design". *Proc. of the 24th IEEE Conf. on Decision and Control*, Ft. Lauderdale, Florida, USA., pp. 260-265.

[7] Doyle, J. C., Glover, K., Khargonekar, P. and Francis, B. (1989). "State-space solutions to standard H_2 and H_∞ control problems". *IEEE Trans. Autom. Control*, **34**, 8, pp. 831-847.

[8] Engstad, P. K. (1991). "Comparison of robustness between different control algorithms." *Diploma Thesis*, University of Trondheim-NTH.

[9] Freudenberg, J. S. (1989). "Analysis and design for ill-conditioned plants. Part 2: Directionally uniform weightings and an example". *Int. J. Control*, **49**, 3, pp. 873-903.

[10] Hovd, M. (1992). Studies on Control Structure Selection and Design of Robust Decentralized and SVD Controllers. *Dr. Ing. Thesis*, University of Trondheim-NTH, Norway.

[11] Hovd, M. and Skogestad, S. (1992). "Robust control of systems consisting of symmetrically interconnected subsystems". *Proc. American Control Conference*, Chicago, USA, pp. 3021-3025.

[12] Laughlin, D. L., Morari, M. and Braatz, R. D. (1992). "Robust performance of cross-directional basis-weight control in paper machines". *Automatica*, preprint.

[13] Lee, J. H., Braatz, R. D., Morari, M. and Packard, A. (1992). "Screening tools for robust control structure selection". *Automatica*, preprint.

[14] Lundström, P., Skogestad, S. and Wang, Z.-Q. (1991). "Performance weight selection for H-infinity and μ-control methods". *Trans. Inst. MC*, **13**, 5, pp. 241-252.

[15] Lundström, P. (1992). Personal communication.

[16] Packard, A. (1988). "What's new with μ: Structured uncertainty in multivariable control". *PhD Thesis*, University of California, Berkeley.

[17] Skogestad, S., Lundström, P. and Hovd, M. (1989). "Control of Identical Parallel Processes". *AIChE Annual Meeting*, Paper 167Ba, San Francisco, USA.

[18] Skogestad, S. and Morari, M. (1987). "The dominant time constant for distillation columns". *Comput. Chem. Engng.*, **11**, 6, pp. 607-617.

[19] Skogestad, S., Morari, M. and Doyle, J. C. (1988). "Robust control of ill-conditioned plants: High purity distillation". *IEEE Trans. Autom. Control*, **33**, 12, pp.1092-1105.

[20] Wilhelm, R. G. Jr. and Fjeld, M. (1983). "Control algorithms for cross directional control: The state of the art". *Preprints 5th IFAC PRP Conference*, Antwerp, Belgium, pp. 139-150.

[21] Yaniv, O. and Barlev, N. (1990). "Robust non iterative synthesis of ill-conditioned plants". *Proc. American Cont. Conf.*, San Diego, USA, pp. 3065-3066.