Procedure for Regulatory Control Structure Selection with Application to the FCC Process

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Control structure (strategy) selection consists of the selection and pairing of manipulated and measured variables. This article outlines a procedure that uses such tools as the existence of right half plane (RHP) transmission zeros, the relative gain array, the performance relative gain array, and the closed-loop disturbance gain. The regulatory control system for the fluid catalytic cracking process is used as an example. Several authors found the Kurihara control structure to be preferable to the conventional control structure. The reason is that RHP transmission zeros limit the achievable bandwidth for the conventional control structure. Two other control structures, however, have better controllability characteristics than both the conventional and the Kurihara control structures. The sensitivity of the measurement selection and variable pairing with respect to changes in the operating point and parametric uncertainty is examined, as well as the general objectives of the regulatory control level and its interaction with the higher levels in the control hierarchy.

Introduction

In the chemical industries, the lowest level in the control system is virtually always a regulatory control level. We define the *regulatory control system* (sometimes called basic or lower-level control system) as the level in the control hierarchy which has *operation* as its main purpose, and which normally contains the control loops that must be in service in order for the operators to be able to operate the plant in an efficient manner. For a number of reasons, which are discussed below, simple decentralized control loops are usually preferred at this level. The setpoint to the regulatory control system are set by the operator or are determined by higher levels in the control hierarchy.

The higher levels in the control hierarchy are introduced to optimize the plant by coordinating the various loops or by using additional degrees of freedom. In many plants most of the "higher level" control tasks are performed by the operators. The performance of the higher levels depends critically on a regulatory control system that performs well.

The performance of the regulatory control system can be strongly affected by the control structure used, and *control structure selection* is therefore an important issue in the design of the regulatory control system. The term "control structure" is used here broadly, which corresponds to the term "control strategy" used by Downs and Doss (1991), for example. In a recent book, Rijnsdorp (1991) spends a full chapter discussing control structure selection. However, his approach is rather qualitative. In this article, we apply a more quantitative method for control structure selection to the control of the reactor-regenerator complex in a fluidized catalytic cracking (FCC) unit.

The FCC process is an important process in refineries for upgrading heavy hydrocarbons to more valuable lighter products. Both decentralized controllers and more complex model predictive controllers are used to control the FCC process. However, when model predictive control (MPC) is used, it is usually applied on top of a decentralized regulatory control level and sends setpoint changes to the individual loops. Thus, it is important also in this case that the regulatory control level is well designed.

A schematic overview of the FCC process is shown in Figure 1. Feed oil is contacted with hot catalyst at the bottom of the riser, causing the feed to vaporize. The cracking reactions occur while the oil vapor and catalyst flow up the riser. As a by-product of the cracking reactions coke is formed and is deposited on the catalyst, thereby reducing catalyst activity. The catalyst and products are separated in what for historical reasons is called the "reactor," but which nowadays is little more than a vessel housing one or two stages of cyclones. Steam is supplied to the stripper in order to remove volatile hydrocarbons from the catalyst. We shall use the term separator to

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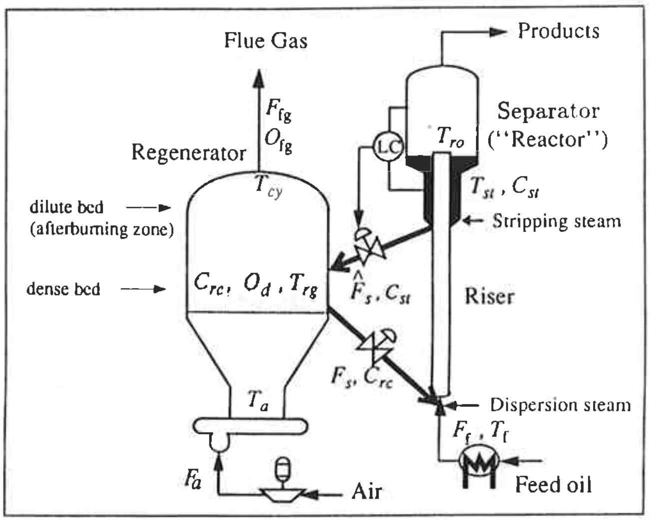

Figure 1. FCC plant.

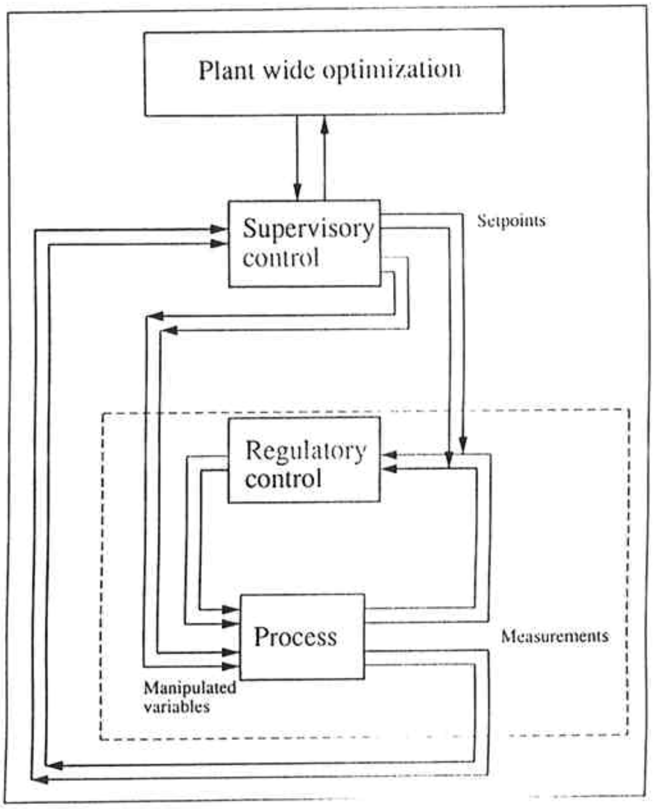

Figure 2. Hierarchical control system.

denote the combined reactor and stripper. The catalyst is then returned to the regenerator where the coke is burnt off in contact with air. The combustion of coke in the regenerator provides the heat needed for feed vaporization and the endothermic reaction in the riser.

The issue of regulatory control structure selection has been discussed by several authors, for example, Hicks et al. (1966), Kurihara (1967), Lee and Weekman (1976), Arkun and Stephanopoulos (1980), Shinnar (1981), and Lee and Groves (1985). In this article, we provide a more quantitative analysis. Specifically, the existence of right half plane (RHP) zeroes and the frequency dependent relative gain array (RGA), performance relative gain array (PRGA), and closed-loop disturbance gain (CLDG) are used for control structure selection, and we study the effect of structural and parametric uncertainty in the models and uncertainty in the manipulated variables on the choice of control structure for decentralized control. The use of frequency dependent RGA, PRGA and CLDG are explained in Hovd and Skogestad (1992). Multivariable control of FCC units has been considered by many authors, including Balchen et al. (1992) and Grosdidier et al. (1993).

Regulatory Control Problem

The *overall control objective* is to maintain acceptable operation (in terms of safety, environmental impact, work load on operators, and so on) while keeping the operating conditions close to the economically optimal conditions. This objective is commonly achieved using a hierarchical control system, with different tasks assigned to each level in the hierarchy. A schematic representation of such a control hierarchy is depicted in Figure 2. Note that we have not included functions related to logic control (startup/shutdown) and safety systems. These are of course important, but need not be considered during normal operation. Although the implementation may be done in many different ways, and even on the same control system, it is still important to distinguish between the various control levels due to their different objectives.

In this article, we consider what is typically the *lowest level* in this control hierarchy, the regulatory control level. The objective of this level is generally to facilitate smooth operation and not to optimize objectives related to profit, which is done at higher levels. Usually, this is a decentralized control system which keeps a set of measurements at given setpoints. This is a cascaded control system where the values of these setpoints are determined by the operator or by higher levels in the control hierarchy. Note that also the regulatory control system itself may include cascaded loops. For example, one often cascades the valve position to a flow measurement such that flow becomes the manipulated input rather than the valve position. In the following, the terms "regulatory control system" and "lower-level control system" will be used as synonyms.

At the *intermediate level* (supervisory control level) there may be a model-based system that uses a multivariable process model to calculate how the plant should be operated to optimize some objective. An important feature of the supervisory control level may be to take into account constraints in the controlled and manipulated variables. Specifically, it is often desirable that one does not reach saturation for the manipulated variables used by the regulatory control system. Avoiding this may involve coordination of the regulatory control loops by adjusting their setpoints or may involve direct use of additional degrees of freedom.

The *top level* in the control hierarchy is a plant-wide optimization. This optimization is usually steady state and is performed off-line at regular intervals.

The largest economic benefits are usually obtained at the higher levels, but the lower level must function properly to realize the benefits of the higher levels. Although seemingly obvious, this is often not understood.

Objectives for regulatory control

The regulatory control system should fulfill the following four general objectives:

• It should provide a sufficient quality of control to enable a trained operator to keep the plant running safely without use of the higher levels in the control system. This sharply reduces the need for providing costly backup systems for the higher levels of the control hierarchy in case of failures.

• It should be simple to understand and tune. Thus, in most cases simple decentralized control loops are used at this level. There are of course cases for which interactions are so strong that multivariable control may be needed at this level. However, very simple schemes are then preferred to compensate for interactions, such as ratios, sums, and so on.

• It should make it possible to use simple (at least in terms of dynamics) models at the higher level. We want to use relatively simple models because of reliability and the prohibitive costs involved in obtaining and maintaining a detailed dynamic model of the plant, and because complex dynamics will add to the computational burden on the higher level control system. This may be achieved by having a regulatory control level at the bottom of the control hierarchy. This may also reduce the effect of model uncertainty and provide for local linearization, for example, by using a cascade on a valve to avoid the nonlinear valve characteristics.

• It should make it possible to use longer sampling intervals at the higher levels of the control hierarchy. This will reduce the need for computing power at the higher levels. Preferably, the time scales of the lower-level and higher-level control system should be separated such that response of the lower-level control system, as seen from the higher level, is almost immediate.

As a consequence of the objectives listed above, the following four more specific objectives for the regulatory control system arise:

• It should provide for fast control when this is needed for some variables.

• It must be able to follow the setpoints set by the higher levels in the control hierarchy. The setpoints of the lower loops are often the manipulated variables for the higher levels in the control hierarchy, and we want to be able to change these variables as directly and with as little interaction as possible. Otherwise, the higher level will need a model of the dynamics and interactions of the lower level control system.

• It should provide for local disturbance rejection. This follows from the previous objective, since we want to be able to keep the controlled variables in the regulatory control system at their setpoints. As disturbances we must also include the "unused" manipulated variables (additional degrees of freedom) which are adjusted directly by the higher levels of the control system.

• It should be designed such that the remaining control problem does not contain unnecessary performance limitations such as RHP-zeros, large RGA-elements, or strong sensitivity to disturbances. The "remaining control problem" is the control problem as seen from the higher level which has as manipulated inputs the "unused" manipulated inputs and the setpoints to the lower-level control system. By "unnecessary" is meant limitations that do not exist in the original problem formulation without the lower-level control system in place.

In this article, we will primarily consider the fifth, sixth and seventh objectives (fast control, setpoint tracking, and disturbance rejection). These objectives are related to the "controllability" of the lower level control system. The second objective of simplicity is automatically fulfilled since we will only consider a fully decentralized lower level control system.

Control structure selection for regulatory control

To fulfill the objectives for the regulatory control system listed above, one must perform a *control structure (strategy) selection*. This involves making the following three structural decisions:

• Outputs y: selection of controlled variables (control objectives) for the regulatory control system. The outputs include what we denote *primary* and *secondary* controlled variables. The primary outputs are often easy to select as they are variables which are important to control in themselves, also in terms of the overall control objective. Typically, these include variables for which reasonably fast control is needed (see the fifth objective above), such as liquid levels and certain temperatures and pressures which must be kept away from given lower or upper limits. *The secondary outputs are usually easily measured variables, which in themselves may not be important to control, but which are selected to meet the first, second and fourth objectives above.* Typically, the secondary variables include temperatures and pressures at selected locations in the process. The problem of selecting the output variables for the regulatory control system is therefore closely related to the issue of measurement selection.

• Inputs u: selection of manipulated variables for the regulatory control system. These selected inputs will be a subset of all possible manipulated units, and the remaining "unused" variables will be manipulated inputs available for the operators or the higher levels in the control hierarchy.

• Pairing of the chosen controlled and manipulated variables for decentralized control. The choice of pairings will influence the effect of interactions and disturbances, as well as the system's ability to tolerate failure of one or more loops in the decentralized control system.

For the first decision, selection of outputs, the FCC example presented in this article provides a very good example. Here a relevant issue is whether to select as secondary output the regenerator temperature, T_{rg}, or the riser outlet temperature, T_{ro} (see Figure 1).

Distillation column control provides an excellent example of the importance of selecting appropriate inputs (second decision). In this case, the level control constitutes the regulatory control system, and it is well known that closing the level loops with the "LV configuration" (corresponding to having reflux L and boilup V as the remaining unused inputs for composition control) may make the remaining composition control problem difficult because of serious interactions (resulting in large RGA-values, see Skogestad et al., 1990). Note that the lower-level control system for the LV-configuration meets essentially all of the regulatory control objectives previously mentioned, with the exception of the last objective of avoiding performance limitations in the remaining problems.

Measures for Evaluating Controllability

The measures used in this article for evaluation controllability are outlined in this section. Additional measures also exist, see Wolff et al. (1992) and Skogestad and Wolff (1992).

Right half plane transmission zeros

A right half plane transmission zero of $G(s)$ limits the achievable bandwidth of the plant. This holds regardless of the type of controller used (Morari and Zafiriou, 1989). The reason is that with a RHP transmission zero the controller cannot invert the plant and perfect control is impossible. Thus plants with RHP transmission zeros within the desired bandwidth should be avoided.

In the multivariable case, a RHP transmission zero of $G(s)$ does not imply that the matrix elements, $g_{ij}(s)$, have RHP zeros. Conversely, the presence of RHP zeros in the elements does not necessarily imply a RHP transmission zero of $G(s)$. If we use a multivariable controller then RHP zeros in the elements *do not* imply any particular problem. However, if decentralized controllers are used, then we generally avoid pairing on elements with "significant" RHP zeros (RHP zeros close to the origin), because otherwise this loop may go unstable if left by itself (with the other loops open). (Usually one important objective for a decentralized control system is to allow for loops to be operated independently.)

Relative gain array

The relative gain array has found widespread use as a measure of interaction and as a tool for control structure selection for single-loop controllers. It was first introduced by Bristol (1966). It was originally defined at steady state, but it may easily be extended to higher frequencies (Bristol, 1978). Shinskey (1967, 1984) and McAvoy (1983) have demonstrated practical applications of the RGA. Important advantages with the RGA is that it depends on the plant model only and that it is scaling independent. For $n \times n$ plants, $G(s)$ the RGA matrix can be computed frequency-by-frequency ($s = j\omega$) using the formula:

$$\Lambda(s) = G(s) \times (G^{-1}(s))^T \tag{1}$$

where the $\times$ symbol denotes element by element multiplication (Hadamard or Schur product).

An important use of the RGA for decentralized control is that pairing on negative *steady-state* relative gains should be avoided (Bristol, 1966). The reason is that with integral control this yields instability of either the overall system, the individual loop, or the remaining system when the loop in question is removed (Grosdidier et al., 1985). It is also established that plants with large RGA-values, in particular at high frequencies, are fundamentally difficult to control (poor controllability) irrespective of the controller used (Skogestad and Morari, 1987b). On the other hand, if the magnitude of the RGA elements corresponding to the paired inputs and outputs are small (compared to 1) in the bandwidth region, this indicates possible stability problems when using decentralized control (Hovd and Skogestad, 1992a,b).

PRGA

One inadequacy of the RGA (McAvoy, 1983, p. 166) is that it only measures two-way interactions (for example, $\Lambda = I$ for a triangular plant), and it may therefore indicate that interactions are not a problem when significant one-way coupling exist. To overcome this problem, we introduce the frequency dependent performance relative gain array (PRGA). The PRGA-matrix is defined as (Hovd and Skogestad, 1992):

$$\Gamma(s) = \tilde{G}(s)G(s)^{-1} \tag{2}$$

where $\tilde{G}(s)$ is the matrix consisting of only the diagonal elements of $G(s)$, that is, $\tilde{G} = \mathrm{diag}\{g_{ii}\}$. The matrix Γ was originally used at steady state by Grosdidier (1990) in order to understand the effect of directions under decentralized control. The elements of Γ are given by:

$$\gamma_{ij}(s) = g_{ii}(s)[G^{-1}(s)]_{ij} = \frac{g_{ii}(s)}{g_{ji}(s)}\lambda_{ji}(s) \tag{3}$$

Note that the diagonal elements of RGA and PRGA are identical, but otherwise PRGA does not have all the algebraic properties of the RGA. PRGA is independent of *input* scaling, that is, $\Gamma(GD) = \Gamma(G)$ where D is any diagonal matrix, but it depends on output scaling. This is reasonable since performance is defined in terms of the magnitude of the outputs.

Closed-loop disturbance gain

A disturbance measure closely related to PRGA, the closed-loop disturbance gain (CLDG), was recently introduced by Skogestad and Hovd (1990). For a disturbance k and an output i, the CLDG is defined by:

$$\delta_{ik}(s) = g_{ii}(s)[G(s)^{-1}G_d(s)]_{ik} \tag{4}$$

A matrix of CLDG's may be computed from:

$$\Delta = \{\delta_{ik}\} = \tilde{G}G^{-1}G_d = \Gamma G_d \tag{5}$$

The CLDG is scaling dependent, as it depends on the expected magnitude of disturbances and outputs. Actually, this is reasonable since CLDG is a performance measure, which generally are scaling-dependent.

Use of PRGA and CLDG: performance relationships for decentralized control

The following derivation follows (Skogestad and Hovd, 1990; Hovd and Skogestad, 1992). Assume the controller $C(s)$ is diagonal with entries $c_i(s)$. (See Figure 3). This implies that after the variable pairing has been determined, the order of

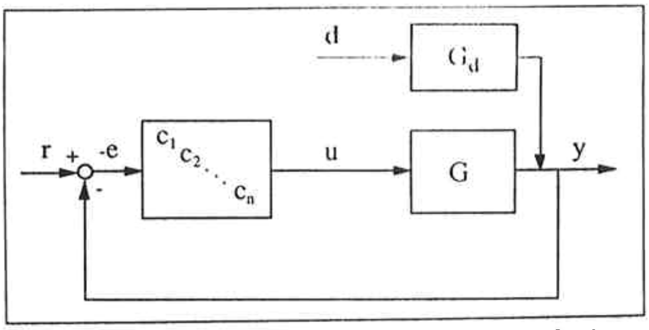

Figure 3. Block diagram of controller and plant.

the elements in y and u has been arranged so that the plant transfer matrix $G(s)$ has the elements corresponding to the paired variables on the main diagonal. Let $y(s)$ denote the output response for the overall system when all loops are closed and let $e(s)=y(s)-r(s)$ denote the output error. The closed-loop response becomes:

$$e(s)=-S(s)r(s)+S(s)G_d(s)d(s); \quad S=(I+GC)^{-1} \qquad (6)$$

where $S(s)$ is the sensitivity function for the overall system, and $d(s)$ denotes the disturbances. The Laplace variable s is often omitted to simplify notation.

At low frequencies ($\omega<\omega_B$) we usually have large controller gains and $S\approx(GC)^{-1}=C^{-1}\tilde{G}^{-1}\tilde{G}G^{-1} = (\tilde{G}C)^{-1}\tilde{G}G^{-1} \approx \tilde{S}\tilde{G}G^{-1}$ and we get:

$$e\approx-\tilde{S}\tilde{G}G^{-1}r+\tilde{S}\tilde{G}G^{-1}G_d d; \quad \omega<\omega_B \qquad (7)$$

Here we recognize the PRGA ($\tilde{G}G^{-1}$) and the CLDG ($\tilde{G}G^{-1}G_d$). When we consider the effect of a setpoint change r_j and a disturbance d_k on the offset e_i (Eq. 7) gives:

$$e_i\approx-\frac{\gamma_{ij}}{g_{ii}c_i}r_j+\frac{\delta_{ik}}{g_{ii}c_i}d_k; \quad \omega<\omega_B \qquad (8)$$

In the following, assume that G and G_d have been scaled such that the expected disturbances, $|d_k(j\omega)|$, are less than or equal to one at all frequencies, and the outputs, y_i, are such that the allowed errors, $|e_i(j\omega)|$, are less than or equal to one.

From Eq. 8, we see that the ratio $\gamma_{ij}/(g_{ii}c_i)$ gives the magnitude of the offset in output i to a unit setpoint change for output j. This ratio should preferably be small. That is, on a conventional magnitude Bode plot (log-log), the curve for the PRGA element $|\gamma_{ij}|$ should lie below $|g_{ii}c_i|$ at frequencies where we want small offsets. From Eq. 8, we see that the ratio $\delta_{ik}/(g_{ii}c_i)$ gives the magnitude of the offset in output i to a unit disturbance d_k. That is, the curve for the CLDG element $|\delta_{ik}|$ should lie below $|g_{ii}c_i|$ at frequencies where we want the offsets less than 1 in magnitude. A plot of $|\delta_{ik}(j\omega)|$ will give useful information about which disturbances k are difficult to reject. Of particular interest is the frequency where $|\delta_{ik}(j\omega)|$ crosses one, because this directly corresponds to the minimum bandwidth needed in loop i to reject disturbance k. It is preferable that this frequency is low in order to avoid stability problems for the individual loops.

Summary of controllability rules

Based on the above discussion, let us summarize some rules we shall use in this article:

Rule 1. Avoid plants (control structures) with RHP transmission zeros within the desired bandwidth (that is, RHP transmission zeros at low frequencies are bad).

Rule 2. Avoid plants (control structures) with large RGA values, in particular at frequencies near crossover. This rule applies for any controller, not only to decentralized control (Skogestad and Morari, 1987a,b).

Rule 3. Avoid *pairings ij* with negative values of the steady-state RGA, $\lambda_{ij}(0)$ (Grosdidier et al., 1985), and thus avoid plants (control structures) for which this rule forces one to pair on variables "far apart" from each other (thus violating rule 4).

Rule 4. Prefer *pairings ij* where $g_{ij}(s)$ puts minimal restrictions on the achievable bandwidth for this loop. That is, avoid pairings with RHP-zeros in $g_{ij}(s)$ and avoid pairings where $g_{ij}(s)$ is small (otherwise input constraints will cause problems). The rule follows from Eq. 8 above in order to satisfy performance and at the same time have stability of the individual loop. Rule 4 is the conventional rule of pairing on variables "close to each other."

Rule 5. Avoid plants (control structures) with large values of PRGA $|\delta_{ik}|$ or CLDG $|\gamma_{ij}|$ in the crossover region, and in particular if the achievable bandwidth for the corresponding loop i is restricted (because of $g_{ii}(s)$, see rule 4) (the rule follows from Eq. 8).

Rule 6. Prefer *pairings* with RGA values close to 1 in the crossover region. This rule is closely related to rule 4.

FCC Process

FCC operating modes

The coke on the catalyst is burnt in the regenerator, both to regenerate the catalyst and to supply heat for the cracking reaction which takes place in the riser. Depending on the coke-producing tendency of the feed, the FCC process can be operated in two distinct modes; the partial combustion mode and the complete combustion mode. The emphasis is this article will be on the partial combustion mode which has become increasingly more important with today's heavier feedstocks.

Partial Combustion Mode. In the partial combustion mode the conversion of coke to CO_2 is not complete, which means that relatively large amounts of both CO are formed. (This CO-rich regenerator flue gas can be sent to a CO boiler for further combustion to produce high pressure steam.) However, if there are significant amounts of oxygen leaving the regenerator dense bed, this will react with the CO to form CO_2 in the zone above the regenerator dense bed, or in the regenerator cyclones and downstream piping. This "afterburning" is a strongly exothermic reaction, and since there is relatively little mass in this zone a large temperature rise may occur. It is therefore necessary to control the afterburning to avoid violating metallurgical temperature limits for the regenerator cyclones or downstream piping.

Complete Combustion Mode. In this case, little CO leaves the regenerator dense bed because excess quantities of air are supplied. Special catalysts which promote the oxidation of CO to CO_2 may also be used (Rheaume et al., 1976). When operating in the complete combustion mode afterburning is therefore not usually a concern. However, it is not always possible to operate an FCC unit in the complete combustion mode especially if the feed oil has a large coke production tendency. There is also an economic incentive for operating in the partial combustion mode, as the heat recovered in the CO boiler is valuable.

FCC Model

The models used in this work derive in the main part from the model proposed by Lee and Groves (1985) for the partial combustion mode. This model augments the regenerator model of Errazu and coworkers (1979) with the riser model of Shah and coworkers (1977). Here, we give a short description; details about the model are given in Appendix 1.

Riser model

The residence time in the riser is only a few seconds, so a static model is used. We use an ideal plug-flow model and the three lump kinetic scheme of Weekman and Nace (1970), where the feed is gas oil, which can crack to gasoline or light gases/coke. The static riser model is used to compute:

T_{ro} = temperature at the riser outlet
C_{sc} = mass fraction of coke on catalyst at riser outlet

Regenerator model

The catalyst residence time in the regenerator is about 10 min in our case. When modeling the regenerator, it is common to assume that the temperature and the amount of coke on the catalyst is uniform throughout the regenerator dense bed. In the model used in this article, oxygen is also assumed to be uniformly distributed, as Errazu and coworkers (1979) found that this assumption allows operational data to be described well.

This yields a third order model for the regenerator, with the following states:

C_{rc} = mass fraction of coke on regenerated catalyst
T_{rg} = regenerator dense bed temperature
O_d = mole fraction of oxygen in the gas leaving the dense bed

To compute the regenerator cyclone temperature T_{cy}, we represent the afterburning of CO to CO_2 in the dilute phase in the regenerator by using a simple equation taken from Kurihara (1967):

$$T_{cy} = T_{rg} + c_t O_d \tag{9}$$

From Eq. 9 to be reasonable, there must be an excess of CO over O_2 in the gas leaving the regenerator dense bed, that is, Eq. 9 is only valid in the partial combustion mode. Note that when using Eq. 9, controlling $\Delta T_{rg} = T_{cy} - T_{rg}$ (the temperature rise from regenerator dense bed to regenerator cyclones) is equivalent to controlling O_d (the amount of oxygen leaving the regenerator dense bed).

Some authors use the assumption that the oxygen moves in perfect plug flow through the dense bed (Kurihara, 1967; Krishna and Parkin, 1985). In the discussion, we will comment on how the assumptions about oxygen flow pattern affect our conclusions.

Separator model

The separator has a catalyst residence time of about 1 min. It is modeled as a mixing tank and yields two additional state variables:

C_{st} = mass fraction of coke on catalyst in the separator
T_{st} = temperature in separator

The catalyst holdup in the separator W_{st} is assumed to be kept constant by perfect control. This means that the flow rate of spent catalyst from the separator to the regenerator, $\hat{F}_s$, equals the flow rate of regenerated catalyst, F_s.

Complete combustion mode

For the complete combustion mode the same model is used, except that some of the parameter values are adjusted, as discussed in Appendix 1.

Constraints in FCC Operation

The optimal operating point for an FCC usually lies at one or several constraints. The control structure which allows operation closest to the constraints is therefore preferable. The location of the optimal operating point, and consequently the importance of the different constraints can vary depending on the feed characteristics and the desired product split. Different control structures may thus be preferable at different operating points, but it is not realistic to expect the control structure to be reconfigured when the operating conditions are changed.

Common constraints include:

- Maximum regenerator cyclone temperature (T_{cy}) constraint. This constraint is usually important in the *partial combustion mode*, and is determined by the metallurgical properties of the cyclones.
- Minimum flue gas oxygen concentration (O_{fg}) constraint. This constraint is important in the *complete combustion mode*, as a sufficient concentration of oxygen in the flue gas ensures virtually complete combustion of CO to CO_2 within the regenerator dense bed, and therefore ensures that afterburning is avoided.
- Maximum wet gas compressor capacity. The wet gas compressor is situated downstream of the FCC unit, and compresses the products produced in the FCC for transportation to the downstream gas treatment plants.
- Maximum air blower capacity (F_a). The air blower provides the air needed for the combustion in the regenerator.

Implications for Regulatory Control. Constrained outputs (measurements) should be selected as controlled outputs for the regulatory control system, thus enabling operation close to these constraints. This means that there is an argument for selecting the regenerator cyclone temperature T_{cy} as a controlled variable in the partial combustion mode, and for selecting the flue gas oxygen concentration O_{fg} as a controlled variable in the complete combustion mode.

Manipulated inputs that are prone to reach constraints should be avoided in the regulatory control system. The feed flow rate F_f strongly influences the wet gas production, and should be avoided as a manipulated variable for plants operating close to the wet gas compressor constraint. Similarly, the air blower capacity constraint is an argument for avoiding the use of the air flow rate F_a as a manipulated variable for regulatory control. Nevertheless, in articles on regulatory control of the FCC process (Pohlenz, 1963; Hicks et al., 1966) F_a is always used as a manipulated variable. For plants operating close to the air blower capacity constraint, the supervisory control level must then ensure that this constraint is not encountered, for instance, by changing reaction conditions or feed composition such that less coke is formed.

For a more complete description of the constraints encountered in the operation of FCCs, see Grosdidier et al. (1993).

Controllability Analysis for the FCC Process

Scope of the controllability analysis

The following sections will address:

- Choice of controlled variables (outputs y). How does the choice of controlled variables affect controllability?
- Pairing of controlled (y) and manipulated variables (u) for decentralized control.

• Effect of operating point. Is the control system sensitive to changes in the operating conditions?

• Sensitivity to parametric uncertainty. Do changes in parameter values lead to different conclusions?

• Sensitivity to input uncertainty. The actual moves in the manipulated variables will not be exactly equal to those calculated by the controller. Does this influence performance?

• Disturbance rejection. Using the disturbance measures, we will investigate the effect of disturbances on the FCC when decentralized control is used.

• Effect of model features. Which features of the models are important for making decisions about the control of the FCC process?

These points will be examined for both the partial combustion and complete combustion modes.

In the following, we assume that a decentralized control system is used and that the transfer function matrices have been arranged to give the paired elements on the diagonal. The word "RGA" or "RGAs" will refer to the diagonal elements of the RGA matrix of $G(s)$, which are identical when $G(s)$ is a 2×2 matrix.

Variable classification

Independent variables (u's and d's)

We will consider the following six independent variables:

F_s = flow rate of regenerated catalyst entering the riser (The true manipulated variable is actually the regenerated catalyst slide valve position. We use the regenerated catalyst flow rate as a manipulated variable in order to keep the model simple.)
F_a = flow rate of air to the regenerator
k_c = feed oil composition; here represented by the coke production rate factor
T_f = feed oil temperature
F_f = feed oil flow rate
T_a = air temperature

The feed oil composition may be adjusted by changing the ratio between recycled and fresh feed. There are actually a few more manipulated variables, but we have assumed that these are already used by the regulatory control level to control holdup and pressure. These variables include (see Figure 1):

$\hat{F}_s$ = spent catalyst flow rate
F_{fg} = flue gas flow rate
W_{wg} = wet gas compressor throughput (W_{wg} is not shown in Figure 1, as the wet gas compressor is situated downstream of the distillation column receiving the reaction products.)

$\hat{F}_s$ is used to control the catalyst holdup in the separator, whereas F_{fg} is used to control the regenerator pressure P_{rg}. W_{wg} indirectly controls the separator pressure P_{st}. Since we have assumed these loops to be closed, we should actually have included the pressures P_{rg} and P_{st} as disturbances for the model we are considering. However, we have not done this in order to keep the model simple. In practice the pressures P_{rg} and P_{st} may have to be adjusted when the catalyst slide valve position is changed, in order to avoid reversal of the catalyst flow (F_s). This is taken care of by the supervisory control system.

Other possible independent variables which are not included here are the fraction of dispersion steam, λ, and the stripping steam flow rate (not included in the model), as these are known to have relatively little effect provided they are above some minimum threshold values necessary for the proper operation of the plant. (A minimum amount of dispersion steam is needed for good and rapid mixing of feed and catalyst at the riser entrance. A minimum amount of stripping steam is needed for effective stripping of volatile hydrocarbons from the spe catalyst.)

All the six independent variables above may be used manipulated variables (u's) for control, but in most cases will only use two: the regenerated catalyst flow rate, F_s a the air flow rate F_a. The remaining independent variables m then be regarded as ***disturbances*** **(d's)** in the regulatory cont system. The variables k_c, T_f and F_f are all related to the feed. k_c is the coke production rate factor and depends on t feed composition. (Immediately downstream of the FCC th is a distillation column which separates the products from t cracking reactions. The heavy fraction from the distillati column, "slurry," has a large coke producing tendency. T coke production rate factor can therefore be changed indirec by changing the amount of slurry which is recycled to t riser). The air temperature T_a is generally a disturbance sin there is usually no air preheater.

Measurements

Typically, the following measurements are available:

Partial combustion mode:

T_{ro} = riser outlet temperature
T_{rg} = regenerator dense bed temperature
T_{cy} = regenerator cyclone temperature

Complete combustion mode:

T_{ro} = riser outlet temperature
T_{rg} = regenerator dense bed temperature
$O_{fg}(\approx O_d)$ = oxygen concentration in the flue gas

Controlled variables (y's)

It is not obvious what controlled variables should be us for the regulatory control system. In some implementatic only T_{rg} is controlled at this level (Grosdidier et al., 199 However, in this work we shall first consider controlling th variables at this level: T_{ro}, T_{rg}, and T_{cy} (partial combusti mode) or O_{fg} (complete combustion mode). The justificati for trying to control three variables is as follows:

The product distribution is determined by the reaction cc ditions inside the riser, which are therefore very important f the economic performance of the FCC process. There is th an incentive to control *both* T_{ro} and T_{rg}, which are direc related to the riser outlet and inlet temperature, respective

The need to control afterburning in the partial combusti mode and to avoid afterburning in the complete combusti mode should be obvious and makes it reasonable to contr T_{cy} and O_{fg}. Based on this discussion, the controlled variabl considered in this work are:

• *Partial combustion mode*:
Primary variable: T_{cy} or $\Delta T_{rg} = T_{cy} - T_{rg}$.
Secondary variables: T_{ro} and T_{rg}.

A secondary controlled variable is not necessarily less i portant than a primary controlled variable. For example, though T_{ro} is classified as a secondary controlled variat because it *in itself* is not very interesting, the importance T_{ro} as a controlled variable comes from the close connecti between T_{ro} and conversion.

• *Complete combustion mode*:
Primary variable: O_{fg}.
Secondary variables: T_{ro} and T_{rg}.

Table 1. Operating Points for the Partial Combustion Mode

	Case P1	Case P2
T_{rg}	965.4 K	966.6 K
T_{ro}	776.9 K	770.6 K
T_{cy}	988.1 K	997.4 K
C_{rc}	5.207×10^{-3}	3.578×10^{-3}
$G_H(0)$	$\begin{pmatrix} 0.5587 & 10.16 \\ -0.5577 & 10.35 \end{pmatrix}$	$\begin{pmatrix} 0.3893 & 10.83 \\ -0.7606 & 8.22 \end{pmatrix}$
RGA(0)	0.505	0.280
RHP zeros [rad/min]		
Multivariable	—	—
In elements	$\begin{pmatrix} - & - \\ - & - \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & - \end{pmatrix}$

Control related data is for the Hicks control structure (– denotes that no RHP-zero is present at frequencies below 100 [rad/min]).

Analysis of the Partial Combustion Mode

All the numerical values in this section are for Case P1 in Table 1, unless otherwise is stated.

Control structure alternatives

3×3 Control Systems. As discussed above we may want to control three outputs:

$$y^3 = (T_{ro}, T_{cy}, T_{rg})^T \tag{10}$$

We will consider the following two sets of manipulated variables:

$$u_1^3 = (F_s, F_a, k_c)^T \tag{11}$$

$$u_2^3 = (F_s, F_a, T_f)^T \tag{12}$$

Here, we assume that the unit is operating at some maximum capacity limit such that F_f is not available as a manipulated variable. In the following, we assume that the individual inputs and outputs are numbered according to their position in y^3, u_1^3, and u_2^3, and that the transfer function matrix $G(s)$ has been arranged accordingly (such that, for example, the 1, 2 element of $G(s)$ is the transfer function from F_a to T_{ro}).

When computing the RGA matrices, we find that the only pairings giving positive steady-state RGA values are those indicated by Eqs. 10–12. We thus find that when using the inputs in u_1^3, the pairing T_{ro}-F_s, T_{cy}-F_a, T_{rg}-k_c should be used. Likewise, when using the inputs in u_2^3, the pairing T_{ro}-F_s, T_{cy}-F_a, T_{rg}-T_f should be used. These RGA's are shown as functions of frequency in Figure 4a for u_1^3, and in Figure 4b for u_2^3. We see that u_1^3 gives RGA values reasonably close to 1 at steady state, but the small RGA's for loops 2 and 3 in the desired bandwidth region (approximately 1 [rad/min]) indicate control problems. The use of u_2^3 gives rise to unfavorably large steady-state RGA's, and in the desired bandwidth area the RGA's are small for loops 1 and 3. These results indicate that with the available manipulated variables, the 3×3 control problem is not well suited for high performance control with a decentralized controller. *We will therefore in the following consider the 2×2 problem.*

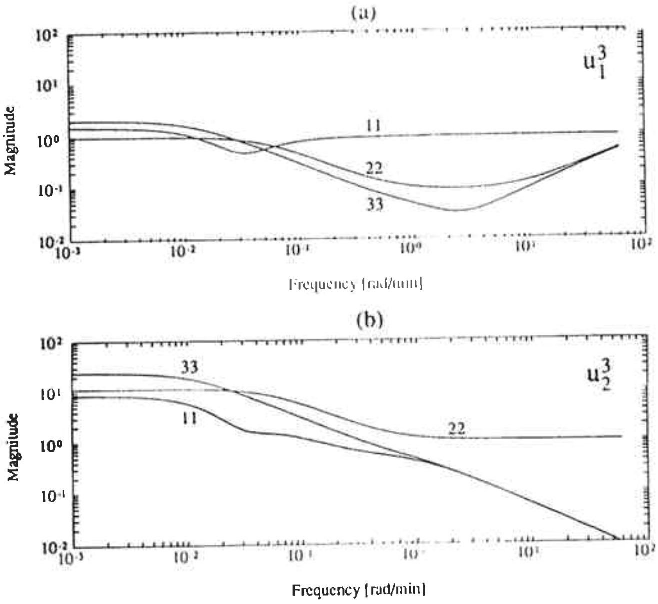

Figure 4. RGA for the 3 × 3 control problem in the partial combustion mode.

(a) $y^3-u_1^3$, with pairing T_{ro}-F_s, T_{cy}-F_a, T_{rg}-k_c; (b) $y^3-u_2^3$, with pairing T_{ro}-F_s, T_{cy}-F_a, T_{rg}-T_f (Case P1, Table 1).

2×2 Control Systems: Selection of Controlled Variables. Restricting our attention to the 2×2 control problem allows us to compare and contrast our results with the results of previous authors, and will give a good illustration of the effect measurement selection can have on plant controllability. Furthermore, the output variables T_{ro} and T_{rg} are strongly coupled, and good control of one will in many cases also limit the offset in the other variable. There is thus probably little need to control both T_{ro} and T_{rg} in order to fulfill the objectives for the lower level control system given above. In the following we will use:

$$u = (F_s, F_a)^T \tag{13}$$

as manipulated variables. These two manipulated variables are always used for 2×2 control systems for FCCs, because of their strong and direct effect on the process conditions. Wolff et al. (1992) also found from an analysis of the disturbance sensitivities that this is a good choice. The remaining independent variables k_c, T_f, F_f and T_a will in the following be considered as disturbances.

As possible controlled variables (outputs), we will consider two of the three temperatures T_{ro} (riser outlet), T_{rg} (regenerator) and T_{cy} (cyclone). In addition, we consider the temperature difference $\Delta T_{rg} = T_{cy} - T_{rg}$ which from Eq. 9 is directly proportional to O_d (concentration of oxygen in the regenerator dense bed). Of the six possible pairs of these four variables, we will consider the following five:

1. $y_C = (T_{ro}, \Delta T_{rg})^T$. This is the *conventional control structure*, which uses F_s to control T_{ro} and F_a to control ΔT_{rg}. It appears to have been described first by Pohlenz (1963), and is suggested by Hicks (1966), but the name is due to Kurihara (1967).
2. $y_K = (T_{rg}, \Delta T_{rg})^T$. This is the *Kurihara control structure*

(Kurihara, 1967), which uses F_a to control T_{rg} and F_s to control ΔT_{rg}.

3. $y_{AK} = (T_{rg}, T_{cy})^T$. We have chosen to call this the *alternative Kurihara* control structure, as the elements of y_{AK} are linear combinations of the elements of y_K.

4. $y_H = (T_{ro}, T_{cy})^T$. We denote this the *Hicks control structure* as it was first presented by Hicks et al. (1966) who called it the "more or less typical control technique." In this case, F_s is used to control T_{ro} and F_a to control T_{cy}.

5. $y_R = (T_{ro}, T_{rg})^T$. We will name this the *riser-regenerator control structure*. Here, F_s is used to control T_{ro} and F_a to control T_{rg}.

Scaling of variables

The PRGA in Eq. 2 and CLDG in Eq. 5 depend on scaling, and the variables must therefore be scaled appropriately to allow easy interpretation of these measures. The PRGA depends on the scaling of the outputs, and the CLDG depends on the scaling of outputs and disturbances.

The chosen scalings are based on an analysis of acceptable variations on the output temperatures and on expected normal variations in the disturbances. For the partial combustion mode, the transfer functions are scaled such that output errors, $e_i = y_i - r_i$ of magnitude 1, correspond to:

1. Riser exit temperature, T_{ro}: 3 K
2. Regenerator cyclone temperature, T_{cy}: 2 K
3. Regenerator dense bed temperature, T_{rg}: 3 K

and such that a disturbance d_k of magnitude 1 corresponds to

1. Feed oil temperature, T_f: 5 K (when considered as a disturbance)
2. Air temperature, T_a: 5 K
3. Feed oil flow rate, F_f: 4 kg/s (about 10%)
4. Feed oil composition, expressed by the coke production rate factor k_c: 2.5% relative to its original value (when k_c is considered as a disturbance)

RHP transmission zeros

Conventional Control Structure, $y_C = (T_{ro}, \Delta T_{rg})^T$. For this choice of controlled variables we get a transfer function matrix with a RHP transmission zero at 0.018 (rad/min). This RHP transmission zero will seriously limit the achievable bandwidth in certain directions, and only slow control is possible (with closed-loop time constant longer than approximately $1/0.018 = 56$ min). This is confirmed by the simulations in Balchen et al. (1992) who use a model very similar to ours together with the PI-tunings of Lee and Groves (1985). This choice of controlled variables is therefore discarded.

Kurihara Control Structure, $y_K = (T_{rg}, \Delta T_{rg})^T$. For this choice of controlled variables, we get a RHP transmission zero 0.19 [rad/min]. This is one decade higher than for the conventional control structure, and is thus a much less severe restriction on the achievable bandwidth. This result is in accordance with the results of Kurihara (1967) and Lee and Groves (1985), who found the Kurihara control structure to be preferable to the conventional control structure.

Alternative Kurihara Control Structure, $y_{AK} = (T_{rg}, T_{cy})^T$. For this choice we get the same result as for the Kurihara control structure; a RHP transmission zero at 0.19 [rad/min]. This is as expected, since the measurements for the alternative Kurihara control structure are linear combinations of the measurements for the Kurihara control structure ($T_{cy} = T_{rg} + \Delta T_{rg}$).

Hicks Control Structure, $y_H = (T_{ro}, T_{cy})^T$. For this choice of controlled variables we get no RHP transmission zero (see also Table 1). This means that there is no inherent bandwidth limitation in the model, and the bandwidth will be limited solely by (unmodeled) high-order dynamics effects and uncertainty.

Riser-Regenerator Control Structure, $y_R = (T_{ro}, T_{rg})^T$. Also, for this choice of controlled variables there is no RHP transmission zero.

The existence and location of RHP transmission zeros is a fundamental measure of controllability, as a RHP transmission zero will limit the achievable bandwidth for any type of controller. Our analysis of RHP zeros clearly indicate that one either should choose $y_H = (T_{ro}, T_{cy})^T$ or $y_R = (T_{ro}, T_{rg})^T$ as controlled variables. It may also be argued that the Hicks structure, y_H, is the best with respect to the constraints:

- Controlling T_{cy} avoids exceeding the metallurgical temperature limit in the regenerator cyclones.
- Controlling T_{ro} directly affects the amount of gas produced and therefore helps ensuring that the wet gas compressor operating limits are not exceeded. Controlling T_{rg} instead of T_{ro} would not have the same direct effect.

Based on this discussion, *we will in the following concentrate on the Hicks control structure*, but also state briefly results for the riser regenerator control structure, which also has the potential for high performance control.

Pairing of controlled and manipulated variables

For the Hicks control structure, the steady-state RGA for the pairing T_{ro}-F_s, T_{cy}-F_a, proposed by Hicks (1966), is about 0.5. In Figure 5, the PRGA for this pairing is shown. We see that the PRGA is relatively small for all frequencies, and approaches identity for frequencies higher than 0.1 rad/min. As the desired bandwidth is above 0.1 rad/min, the interaction between the loops can be expected to be small for the pairing T_{ro}-F_s, T_{cy}-F_a.

For the riser-regenerator control structure, the RGA is close to 1 at steady state for the pairing T_{ro}-F_s, T_{rg}-F_a. The frequency dependent PRGA (not shown) also indicates that this pairing gives little interaction between the loops.

Disturbances

The effect of disturbances has been investigated using the closed loop disturbance gain (CLDG) explained above. Note that it is not meaningful to use the CLDG to directly compare

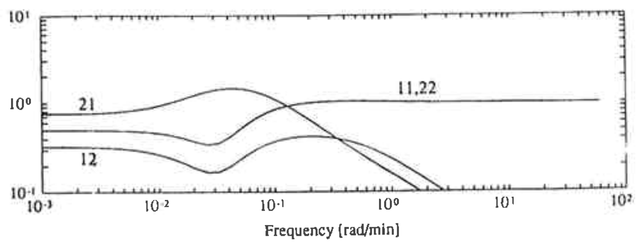

Figure 5. PRGA for the Hicks control structure with the pairing T_{ro}-F_s, T_{cy}-F_a (Case P1, Table 1).

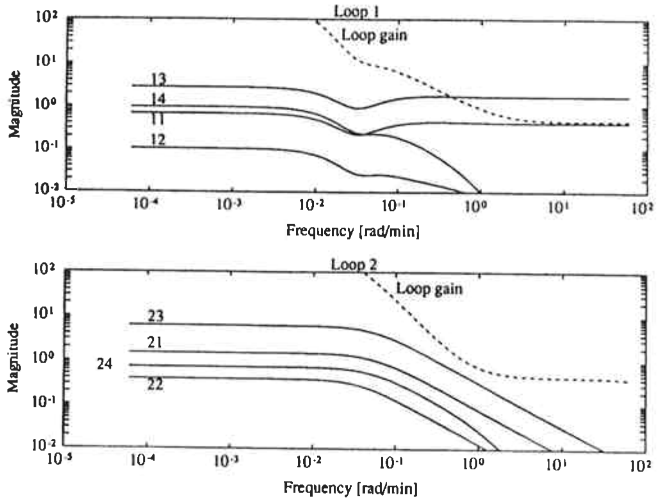

Figure 6. Closed-loop disturbance gains, δ_{ik}, for the Hicks control structure.

i denotes output and k disturbance. The loop gains resulting from the controller used in Figure 7 are shown with dashed lines (Case PI).

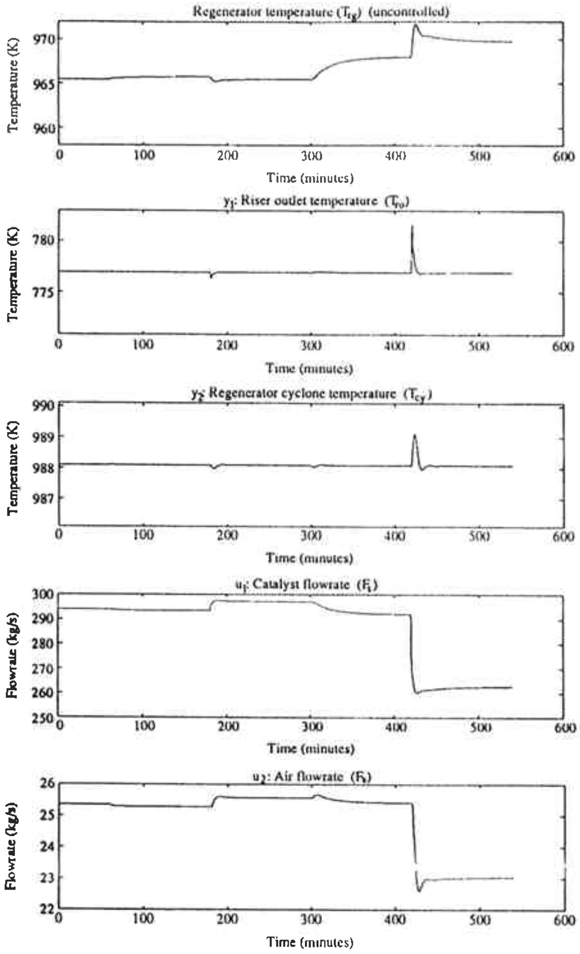

Figure 7. Closed-loop simulation of the effect of disturbances for the Hicks control structure using the PI controllers in Eqs. 14-15.

A 5 K step increase in T_a occurs at 60 min, a 5 K decrease in T_f at 180 min; a 2.5% increase in k_c at 300 min and a 4 kg/s decrease in F_f occurs at 420 min.

different choices of controlled output variables, as the CLDG contains no information about uncontrolled outputs. For this reason, we have chosen to include the CLDG's only for the Hicks control structure.

Based on the results in the previous sections, we will here only consider the pairing T_{ro}-F_s, T_{cy}-F_a. The frequency-dependent CLDG's shown in Figure 6 predict that disturbance $k=3$ in the feed oil flow rate, F_f, is most difficult to reject, followed by disturbances 1 (in T_f) or 4 (in k_c). Disturbance 2 (in T_a) appears to have very little effect. The CLDG for the effect of F_f on T_{ro}, δ_{13}, does not roll off at high frequencies. Some high frequency effect of F_f on T_{ro} must therefore be expected. This suggests the use of feedforward control from F_f to F_s used together with feedback control for good disturbance rejection at high frequencies, unless F_f is controlled such that only slow changes in this variable can occur. The predictions based on the CLDG's, which are independent of the controller, are verified by closed-loop simulations using two PI controllers in Figure 7. Since it is mainly unmodeled effects that limit the achievable bandwidth for the Hicks control structure, we have chosen to tune each loop to give a closed-loop bandwidth for the loop of approximately one min. This results in the following PI controllers:

$$c_1(s) = 1.23\,\frac{0.56s+1}{0.56s} \quad (14)$$

$$c_2(s) = 0.5\,\frac{s+1}{s} \quad (15)$$

These controller tunings result in a sensitivity function S with essentially no peak.

Let us now consider the regenerator temperature, T_{rg}, which is uncontrolled in the Hicks control structure. From the simulation in Figure 7, we see that the effects of disturbances on T_{rg} are relatively small, and certainly much better than for the open loop system. Nevertheless, we see from Figure 7 that the disturbances combine to produce an offset in T_{rg} of about 4.5 K. Note, however, that there is no hard constraint associated with T_{rg}. Thus, if control of T_{rg} is desired, we may use for example the feed composition as an input, and tune the loop loosely, such that the control of T_{rg} does not interfere significantly with the control of T_{ro} and T_{cy}.

For the riser-regenerator control structure, the CLDG's (not shown) give similar results as for the Hicks control structure. However, we emphasize that with the riser-regenerator control structure T_{cy} is uncontrolled, and measures *must* be taken, for example, using the supervisory control system, to ensure that T_{cy} does not exceed its maximum temperature constraint.

Discussion of FCC Case Study

In this section, we discuss the effects of changes in the operating point, sensitivity to uncertainty in the parameters and the effects of model features.

Effect of changes in the operating point

The transfer function matrix G_H for the Hicks structure is defined by:

$$\begin{bmatrix} T_{ro} \\ T_{cy} \end{bmatrix} = G_H \begin{bmatrix} F_s \\ F_a \end{bmatrix} \quad (16)$$

Although a number of operating points ("cases") have been studied, our findings can be illustrated by the two cases in Table 1. Case P1 was studied above and corresponds to an operating point with a catalyst to oil ratio of 7.0, and Case P2 corresponds to a catalyst to oil ratio of 6.7.

The results on *selection of controlled variables* appear unaffected by changes in the operating point, as we have not found any operating point with a RHP transmission zero for the transfer function matrix with $y_H = (T_{ro}, T_{cy})^T$ or $y_R = (T_{ro}, T_{rg})^T$ as controlled variables. The RHP transmission zeros found for the other possible choices of controlled variables are also found when the operating point is changed, although their locations do vary somewhat.

The *choice of pairing* also appears insensitive to changes in the operating point, as the RGA at steady state is positive for the operating points studied, and is close to 1 in our desired bandwidth region for both the Hicks and riser-regenerator control structures. This is also as expected since this corresponds to pairing variables "close to each other."

Sensitivity to parametric uncertainty

The objective of this discussion is to investigate whether small errors in the parameters can have consequences for control performance, and whether parametric uncertainty therefore should be considered in the process of control structure selection. Due to the large number of parameters in the models, the sensitivity to parameter uncertainty has not been exhaustively researched.

The results on *selection of controlled variables* appear unaffected by parametric uncertainty also. We have not found any parameter change which causes a RHP transmission zero with $y_H = (T_{ro}, T_{cy})^T$ or $y_R = (T_{ro}, T_{rg})^T$ as controlled variables. Again, the RHP transmission zeros found for the other possible choices of controlled variables are also found when parameters are changed, although their locations do vary somewhat.

In general, we also have found the *choice of pairing* to be insensitive to changes in the parameters both for the Hicks and the riser-regenerator control structures. Cases have been found for which the steady-state RGA indicates that the pairing for the Hicks control structure should be changed. However, on closer scrutiny these cases appear to be unrealistic, since disturbances have such a large effect that the plant would be virtually inoperable.

Lee and Weekman (1976) claim that the control structure selection for the FCC process can be very sensitive to the model structure and the parameter values used. Our results appear relatively insensitive to errors in the parameter values for the particular model structure used. The sensitivity to the model structure will be discussed below.

Effect of model features

Afterburning. The results have proved to be relatively insensitive to the value of c_t used in the simplified afterburni model $T_{cy} - T_{rg} = c_t O_d$ in Eq. 9, and since afterburning is know to be a very fast phenomenon there appears to be little ne for modeling afterburning in more detail.

Air flow pattern in the regenerator. We have based o model on the work of Errazu and coworkers (Errazu, 197 who found that the behavior of the regenerator is well describ by a model which assumes that the oxygen is uniformly di tributed in the regenerator dense bed. However, Kurihara (196 and Krishna and Parkin (1985) assume the air to move in pl flow through the regenerator. We have studied the effect this assumption on our results.

Kurihara assumes the oxygen leaving the regenerator den bed to be at a pseudo-steady state, and provides the followin equation

$$O_d = O_{in} \exp\left[\frac{-WP_{rg}/F_a}{\{1.06\times10^{10}/F_a^2 + 1/[k_{or}\exp(-E_{or}/RT_{rg})C_{rc}]\}}\right] \quad (1$$

R_{cb}, the rate of coke combustion is found from a mass balan with the oxygen consumed:

$$R_{cb} = \frac{F_a}{M_a}(O_{in} - O_d)\frac{4(1+\sigma)}{(1+\sigma)n + 2 + 4\sigma} M_c \quad (1$$

The values of E_{or} and P_{ra} are taken from Denn (1986), where the value for k_{or} has been adjusted to give a steady-state clo to Case P1 in Table 1. (It should be noted that Kurihara ignore the presence of hydrogen in the coke. Clearly, for the ma balance in Eq. 18, the presence of hydrogen in the coke, re resented by the parameter n, is important. The omission c hydrogen in the mass balance is repeated by Denn (1986) i his presentation of the Kurihara model when he states that ratio of CO_2 to CO of unity results in a value of $C_1 = 2$ in h Eq. 5.62a. The righthand side of Eq. 5.62a also needs to b multiplied by $32/M_a$ to be correct.)

The results for the Hicks control structure in the partia combustion mode are summarized for two cases in Table 2 In Case K1, the gain from F_a to T_{cy} is negative, and we hav

Table 2. Operating Points Assuming Plug Flow of Air in th Regenerator for the Hicks Control Structure in the Partia Combustion Model

	Case K1	Case K2
T_{rg}	966.1 K	983.2 K
T_{ro}	777.3 K	783.7 K
T_{cy}	987.3 K	993.1 K
C_{rc}	5.039×10^{-3}	4.409×10^{-3}
$G_H(0)$	$\begin{pmatrix} 2.907 & 36.72 \\ -5.562 & -45.73 \end{pmatrix}$	$\begin{pmatrix} 0.695 & 12.72 \\ -1.125 & 5.306 \end{pmatrix}$
RGA(0)	−1.87	0.205
RHP zeros [rad/min]		
Multivariable	—	—
In elements	$\begin{pmatrix} - & - \\ - & 0.025 \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & - \end{pmatrix}$

– denotes that no RHP zero is present at frequencies below 100 [rad/min].

a negative RGA. The immediate effect of increasing F_a will be to increase O_d, and hence also T_{cy}. However, T_{rg} will also increase, and because O_d in Eq. 17 is a very strong function of T_{rg}, O_d eventually decreases to a value below its original value. We therefore get a negative steady-state gain from F_a to T_{cy}. In Case K2 in Table 2, the value of O_d is lower because of the higher T_{rg}. There is therefore less scope for further reduction of O_d, and the steady-state gain from F_a to T_{cy} is positive, and we get a positive RGA.

Kurihara (1967) states that the Hicks control structure is "incomplete" from a safety point of view, because it has incomplete feedback information with respect to the states T_{rg} and C_{rc}, which he states are the variables which govern FCC safety. The argument is supported by a closed-loop simulation of the Hicks control structure in which the system goes unstable after the air blower saturates. The controller tunings used in the simulations are not provided, but these results are probably explained by the negative RGA found for Case K1 in Table 2 which means that one of the control loops must be unstable for the overall closed-loop system to be stable (Grosdidier et al., 1985). In Kurihara's simulation example, it must therefore be the loop F_s-T_{ro} which is unstable, and the system therefore becomes unstable when the other input (F_a, the air blower) saturates. In our opinion, the assumption of plug flow of air in the regenerator used by Kurihara is too simplistic, as strong backmixing occurs in the regenerator, and we do not believe the sign reversal occurs in practice. These results do however demonstrate the validity of Lee and Weekmans (1976) claim that the control structure selection for the FCC is sensitive to the model structure used.

Model Reduction. All the calculations above were based on a five state model. However, the dynamics of the oxygen in the regenerator and of the temperature and coke concentration in the separator are much faster than the dynamics of the temperature and coke concentration in the regenerator. We have found that the FCC model can be reduced to only two states by setting dO_d/dt, dT_{st}/dt and dC_{st}/dt equal to zero. The error introduced by this model reduction is minor, the most important effect being that the RHP transmission zero for the Kurihara control structure appears at a slightly higher frequency for the two state model than for the five state model. The zeros for the individual model elements are also affected, but we have found no instance where this will affect the conclusions for control structure selection. To provide the reader with a simple model, a state space realization of the two state models for Case P1 in Table 1 is given in Appendix 2.

Sensitivity to input uncertainty

In this article, only decentralized controllers are considered. Decentralized controllers are known to be relatively insensitive to input uncertainty (uncertainty in the actuators). The low RGA values found indicate that more complex controllers (for example, decouplers) will also be insensitive to input uncertainty (Skogestad and Morari, 1987).

Complete combustion mode

The control structure selection for the FCC in the complete combustion mode has been studied using the same procedure as demonstrated above for the partial combustion mode. We will here only include the main results. Numerical results are for Case C1 in Table 3, unless otherwise stated.

Table 3. Operating Points Used for the Complete Combustion Mode, with Control Related Data for the Conventional Control Structure

	Case C1	Case C2*
T_{rg}	998.4 K	1,012.3 K
T_{ro}	788.0 K	797.3 K
O_d	1.038×10^{-2}	3.098×10^{-3}
C_{rc}	9.645×10^{-3}	2.527×10^{-3}
$G_{CC}(0)$	$\begin{pmatrix} 29.98 & -11.23 \\ -0.001875 & 0.01210 \end{pmatrix}$	$\begin{pmatrix} -11.23 & 84.18 \\ 0.004131 & -0.02798 \end{pmatrix}$
RGA(0)	2.38	−9.3
RHP zeros [rad/min]		
Multivariable	—	0.014
In elements	$\begin{pmatrix} - & 0.018 \\ - & - \end{pmatrix}$	$\begin{pmatrix} - & 0.0007 \\ 1 & 0.08 \end{pmatrix}$

– denotes that no RHP-zero is present at frequencies below 100 [rad/min].
*Corresponds to an unstable operating point.

Control Structure Alternatives. Analysis of the RGA's shows that the 3×3 control problem with $y^3=(T_{ro}, O_{fg}, T_{rg})^T$ as controlled variables is not well suited for decentralized control. In the following, we will therefore consider the 2×2 control problem. We use $u=(F_s, F_a)^T$ as manipulated variables, and consider the remaining independent variables k_c, T_f, F_f and T_a as disturbances. The following three choices of controlled variables will be considered:

(1) $y_{CC}=(T_{ro}, O_{fg})^T$. We term this the *conventional control structure for the complete combustion mode.*

(2) $y_{KC}=(T_{rg}, O_{fg})^T$. We will call this the *Kurihara control structure for the complete combustion mode*, although Kurihara (1967) only considered the partial combustion mode.

(3) $y_R=(T_{ro}, T_{rg})^T$. We will consider the riser-regenerator control structure also for the complete combustion mode.

Scaling of Variables. We use the same scalings as for the partial combustion mode. For the oxygen concentration in the flue gas, O_{fg}, the transfer functions are scaled such that an offset of magnitude 1 corresponds to 0.1 mole%.

RHP Transmission Zeros: Conventional Control Structure, $y_{CC}=(T_{ro}, O_{fg})^T$. For this choice of controlled variables, we obtain no RHP transmission zero.

RHP Transmission Zeros: Kurihara Control Structure, $y_{KC}=(T_{rg}, O_{fg})^T$. For this choice of controlled variables, we obtain a RHP transmission zero at the frequency 0.40 (rad/min).

RHP Transmission Zeros: Riser-Regenerator Control Structure, $y_R=(T_{ro}, T_{rg})^T$. A RHP transmission zero is found at the frequency 0.013 (rad/min) which will seriously limit the achievable control performance.

We will therefore only consider the conventional control structure in the following.

Pairing of Controlled and Manipulated Variables. The RGA at steady state for the pairing $T_{ro}-F_s$, $O_{fg}-F_a$ is positive and about 2–3 in magnitude for almost all operating points investigated. The only exception is found in a region of high regenerator temperatures and low concentration of oxygen in the flue gas. In this region, the RGA at low frequencies is negative and larger (ca. 9), whereas the RGA approaches 1 at high frequencies also in this region. This operating region should be avoided, as the low oxygen concentration in the flue gas indicates that the oxygen concentration in the dense bed is insufficient to convert all CO to CO_2 within the dense bed, and afterburning may therefore result. This problem is discussed further below.

The PRGA (not shown) also indicates that the pairing T_{ro}-F_s, O_d-F_a is preferable.

Disturbances. We find from the CLDG's (not shown) that a disturbance in F_f will have some high frequency effect on T_{ro}, and the other disturbances can be rejected provided both loops have bandwidths of around 1 (rad/min). At low frequencies, somewhat higher loop gains are required in the complete combustion mode than for the Hicks control structure in the partial combustion mode.

Effects of Changes in the Operating Point. Although a number of operating points ("cases") have been studied, our findings can be illustrated by the results in Table 3.

Case C1 in Table 3 shows a typical operating point. Case C2 shows an unstable operating point, found in a region of high regenerator temperatures and low concentration of oxygen in the flue gas. We concluded above that this operating region should be avoided. The transfer function matrix G_C has a RHP pole at 7×10^{-4} (rad/min). At this operating point, there is also a pair of complex RHP transmission zeros in G_C at a frequency of 0.14 (rad/min). The system is easily stabilized, for example, by feedback from T_{ro} to F_s, but fast control of both T_{ro} and O_{fg} will not be possible in this region. However, the drift into the unstable region is slow and a well designed control system should easily avoid this region.

Conclusion on FCC Controllability Analysis

Partial combustion mode

A favorable selection of *controlled variables* is critical for good control of the FCC process. The so-called conventional control structure, which has T_{ro} and ΔT_{rg} as controlled variables, yields poor performance due to multivariable couplings which give rise to a RHP transmission zero which seriously limits the achievable speed of response. The Kurihara control structure, which has T_{rg} and ΔT_{rg} as controlled variables, also yields a RHP transmission zero, but it is at a much higher frequency and is thus less serious. However, the Kurihara structure does not directly control the riser temperature, T_{ro}, which is the most important parameter for controlling the cracking reactions.

Fortunately, there exist structures that control T_{ro} directly, and which do not yield any RHP transmission zeros. This is the case with the Hicks control structure, which has T_{ro} and T_{cy} as controlled variables, and with the riser-regenerator control structure, which has T_{ro} and T_{rg} as controlled variables.

The Hicks control structure has the additional advantage that it controls directly T_{cy}, which often must be controlled tightly, because it is constrained.

With hindsight the choice of the Hicks control structure might appear obvious, but the fact that the conventional control structure was predominant for many years (for example, Pohlenz, 1963; Lee and Weekman, 1976) makes it clear that it is not so obvious. Also, recently Balchen et al. (1992) use the poor performance of the conventional structure as a justification to use multivariable predictive control and do not seem to be aware of the fact that much better control may be achieved with simple PI-controllers if other structures, such as the Hicks structure, are used. To our knowledge, this article is the first to demonstrate quantitatively how controllability of the FCC process is affected by the choice of controlled variables.

Complete combustion mode

The pair T_{ro}, O_d should be chosen as *controlled variables*, as this choice of controlled variables give no RHP transmission zero. In contrast, choosing T_{rg}, O_d or T_{ro}, T_{rg} as controlled variables gives RHP transmission zeros which limit the achievable bandwidth.

Disturbances

Both in the partial combustion mode and in the complete combustion, the most difficult disturbance is the effect of changes in the feed oil rate on the riser outlet temperature. Fortunately, the feed oil flow rate is usually controlled, and our results demonstrate that any deliberate change in this variable should be made slowly; otherwise feedforward control from F_f to F_s should be used.

Sensitivity to parametric uncertainty and changes in the operating point

Our results on measurement selection and variable pairing appear relatively insensitive to parametric uncertainty and changes in the operating point.

Effect of model features

A sign reversal of the steady-state RGA for the Hicks structure is obtained at some operating points in the partial combustion mode when we follow Kurihara (1967) and assume plug flow of air in the regenerator. However, in our opinion the assumption of plug flow of air is a poor one, and we do not believe this sign reversal occurs in practice. For the same reason we believe that Kurihara's (1967) criticism of the Hicks structure as being "incomplete" from a safety point of view is invalid.

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Notation

Nominal parameter values are given in parentheses. Different values are given as partial/complete combustion mode.

$c_i(s)$ = controller element for output i
c_{pa} = heat capacity of air (1.074 kJ/kg·K)
c_{pc} = heat capacity of catalyst (1.005 kJ/kg·K)
c_{pD} = heat capacity of steam (1.9 kJ/kg·K)
c_{po} = heat capacity of oil (3.1355 kJ/kg·K)
c_t = factor in Eq. 9 (5,555 K/molefraction)

C_{cat} = catalytic coke produced in riser, mass fraction
C_{rc} = coke on regenerated catalyst, mass fraction
C_{sc} = coke on catalyst leaving riser, mass fraction
C_{st} = coke on catalyst in separator, mass fraction
$C(s)$ = diagonal controller transfer function matrix
[COR] = catalyst to oil ratio on a mass basis
$d(s)$ = vector of disturbances
$e(s)$ = $y(s)-r(s)$—vector of output errors
E_{cb} = activation energy for coke combustion assuming uniformly distributed oxygen in the regenerator (158.59 kJ/mol)
E_{cf} = activation energy for coke formation (41.79 kJ/mol/20.00 kJ/mol)
E_f = activation energy for the cracking of gas oil (101.5 kJ/mol)
E_g = activation energy for the cracking of gasoline (112.6 kJ/mol)
E_{or} = activation energy for coke combustion assuming plug flow of air in the regenerator (146.4 kJ/mol)
F_a = flow rate of air to the regenerator (25.35 kg/s/28.0 kg/s)
F_f = feed oil flow rate (40.63 kg/s)
F_{fg} = flue gas flow rate
F_s = flow rate of regenerated catalyst (294 kg/s)
$\bar{F}_s$ = flow rate of spent catalyst (assumed $=F_s$)
$g_{dik}(s)$ = ikth element of $G_d(s)$
$g_{ij}(s)$ = ijth element of $G(s)$
$G_{CC}(s)$ = transfer function matrix for the conventional control structure in the complete combustion mode
$G_d(s)$ = disturbance transfer function matrix
$G_H(s)$ = transfer function matrix for the Hicks control structure in the partial combustion mode
$G(s)$ = process transfer function matrix
k_1 = reaction rate constant for the total rate of cracking of gas oil (9.6×10^5 s^{-1})
k_2 = reaction rate constant for the rate of cracking of gas oil to gasoline (7.2×10^5 s^{-1})
k_3 = reaction rate constant for the rate of cracking of gasoline to light gases/carbon (4.22×10^5 s^{-1})
k_c = reaction rate constant for the production of coke (0.019 s^{-1}/0.0093 s^{-1})
k_{cb} = reaction rate constant for coke combustion assuming uniformly distributed oxygen in the regenerator (2.077×10^8 s^{-1})
k_{or} = reaction rate constant for coke combustion assuming plug flow of air in the regenerator (58.29 m^2/sN)
m = factor for the dependence of the initial catalyst activity on C_{rc} (80)
M_a = molecular weight of air (28.8544)
M_c = bulk molecular weight of coke (14)
n = number of moles of hydrogen per mole of carbon in the coke (2)
N = exponent for the dependence of C_{cat} on C_{rc} (0.4/0.0)
O_d = concentration of oxygen in gas leaving regenerator dense bed, molefraction
O_{in} = concentration of oxygen in air to regenerator (0.2136 molefraction)
P_{rg} = regenerator pressure (172,000 N/m^2)
P_{st} = separator pressure
$r(s)$ = reference signal for outputs
R = universal gas constant
R_{cb} = rate of coke combustion (kg/s)
$S(s)$ = sensitivity function $S=(I+GC)^{-1}$
t_c = residence time in riser (9.6 s)
T_a = temperature of air to the regenerator
T_{cy} = regenerator cyclone temperature
T_f = feed oil temperature
T_0 = temperature at riser entrance
T_{ro} = temperature at riser outlet
T_{st} = temperature in separator
$T(z)$ = temperature at elevation z in the riser
y_f = mass fraction of gas oil
y_g = mass fraction of gasoline
$y(s)$ = vector of outputs
$u(s)$ = vector of manipulated inputs
W = holdup of catalyst in regenerator (176,000 kg)
W_a = holdup of air in the regenerator (20 kmol)
W_{st} = holdup of catalyst in separator (17,500 kg)
W_{wg} = wet gas compressor throughput
z = dimensionless distance along riser

Greek letters

α = catalyst deactivation constant (0.12 s^{-1})
α_2 = fraction of the gas oil that cracks which cracks to gasoline, $k_2/k_1=0.75$
$\delta_{ik}(s)$ = ijth element of $\Delta(s)$
ΔH_{cb} = heat of combustion of coke (kJ/kmol)
ΔH_f = heat of cracking (506.2 kJ/kg)
$\Delta(s)$ = closed-loop disturbance gain matrix
Θ = dimensionless temperature at position z in riser
λ = mass flow rate of dispersion steam/mass flow rate of feed oil (0.035)
$\lambda_{ij}(s)$ = ijth element of $\Lambda(s)$
$\Lambda(s)$ = relative gain matrix
σ = molar ratio of CO_2 to CO in the regenerator dense bed
ϕ_0 = initial catalyst activity at riser entrance
ω = frequency
ω_B = closed-loop bandwidth

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Appendix A. Details of the FCC Model

Riser model

Material balance for gas oil:

$$\frac{dy_f}{dz} = -K_1 y_f^2 [\mathrm{COR}] \Phi t_c \tag{A1}$$

Material balance for gasoline:

$$\frac{dy_g}{dz} = (\alpha_2 K_1 y_f^2 - K_3 y_g)[\mathrm{COR}] \Phi t_c \tag{A2}$$

where

$$K_1(\Theta) = k_1 \exp\left(\frac{-E_f}{RT_0(1+\Theta)}\right) \tag{A3}$$

$$K_3(\Theta) = k_3 \exp\left(\frac{-E_g}{RT_0(1+\Theta)}\right) \tag{A4}$$

$$\Theta = (T(z) - T_0)/T_0 \tag{A5}$$

$$\Phi = \phi_0 \exp(-\alpha t_c [\mathrm{COR}] z) \tag{A6}$$

$$\phi_0 = 1 - mC_{rc} \tag{A7}$$

Here $K_1 y_f^2 [COR]$ represents the kinetics for the cracking of gas oil and $K_3 y_g [COR]$ the kinetics for cracking of gasoline. Φ represents the deactivation of the catalyst caused by coke deposition, of which ϕ_0 represents the reduction in catalyst activity caused by the coke remaining on the catalyst after regeneration. t_c is the residence time in the riser, and $\alpha_2 = k_2/k_1$ is the fraction of the cracked gas oil which cracks to gasoline.

The catalyst to oil ratio $[COR]$, which was omitted in Lee and Groves (1985), is reintroduced into Φ in order to be consistent with the original method of Shah and coworkers (1977). A correlation taken from Kurihara (1967) is used to estimate the amount of coke produced.

$$C_{cat} = k_c \sqrt{\frac{t_c}{C_{rc}^N} \exp\left(\frac{-E_{cf}}{RT_{ro}}\right)} \tag{A8}$$

The amount of coke on the catalyst leaving the riser is thus

$$C_{sc} = C_{rc} + C_{cat} \tag{A9}$$

The energy balance yields:

$$\frac{d\Theta}{dz} = \frac{\Delta H_f F_f}{T_0 (F_s c_{ps} + F_f c_{po} + \lambda F_f c_{pd})} \frac{dy_f}{dz} \tag{A10}$$

Regenerator model

The regenerator is described by the following equations. Balance for coke on regenerated catalyst:

$$W \frac{d}{dt} C_{rc} = F_s (C_{sc} - C_{rc}) - R_{cb} \tag{A11}$$

Energy balance:

$$Wc_{ps} \frac{d}{dt} T_{rg} = T_{st} F_s c_{ps} + T_a F_a c_{pa} - T_{rg}(F_s c_{ps} + F_a c_{pa}) - \Delta H_{cb} R_{cb}/M_c \tag{A12}$$

where ΔH_{cb} depends both on the temperature and σ, which is the ratio of CO_2 to CO produced (Erraza et al., 1979). Here n denotes the average coke composition CH_n. The concentration of oxygen in the regenerator dense bed is given by a material balance

$$W_a \frac{d}{dt} O_d = \frac{F_a}{M_a} (O_{in} - O_d) - \frac{(1+\sigma)n + 2 + 4\sigma}{4(1+\sigma)} \frac{R_{cb}}{M_c} \tag{A13}$$

The rate of coke combustion is given by

$$R_{cb} = k_{cb} \exp\left(\frac{-E_{cb}}{RT_{rg}}\right) O_d C_{rc} W \tag{A14}$$

Separator model

The flow rate of stripping steam is small compared to the flow rates of catalyst and feed oil, and the effect of the steam on the heat balance of the separator is therefore ignored. Assuming the stripping is effective, the only effect of the separator will be to introduce a lag between the riser outlet and the catalyst return to the regenerator. This lag is modeled using an ideal mixing tank, and the balances for coke and energy yield the following equations.

$$W_{st} \frac{d}{dt} C_{st} = F_s (C_{sc} - C_{st}) \tag{A15}$$

$$W_{st} c_{ps} \frac{d}{dt} T_{st} = F_s c_{ps} (T_{ro} - T_{st}) \tag{A16}$$

Parameter values

The parameter values used are given in the notation section. The values used are taken from Ljungquist (1990) (also see Balchen et al., 1992) who has slightly modified the values given by Lee and Groves. The value of c_t is taken from Kurihara (1967).

Complete combustion mode

The above model has been adjusted to describe the complete combustion mode as follows:

(1) The ratio of CO_2 to CO produced in the regenerator, σ, is fixed to a high value, such that there is an excess of O_2 over CO in the gas leaving the regenerator dense bed. Then the oxygen concentration in the flue gas, O_{fg}, will be approximately equal to the oxygen concentration in the gas leaving the regenerator dense bed, O_d.

(2) The air rate to the regenerator, the coke production rate (k_c) and the feed oil temperature have been adjusted in order to achieve energy balance in the desired operating region.

(3) Finally, the model parameters have been adjusted to obtain the same signs for the steady-state gains as observed by Grosdidier (1990a):

$$\begin{bmatrix} T_{ro} \\ O_d \end{bmatrix} = \begin{bmatrix} + & - \\ - & + \end{bmatrix} \begin{bmatrix} F_s \\ F_a \end{bmatrix} \tag{A17}$$

These signs for the steady-state gains should be obtained in the region of the following operating conditions (Grosdidier, 1990b):

Flue gas oxygen concentration: 0.4–1.04 mole%
Regenerator dense bed temperature: 989–1,009 K
In order to obtain the desired signs for the steady-state gains the activation energy for coke formation, E_{cf}, has been reduced, and the exponent expressing the dependency of the coke production on the amount of coke on regenerated catalyst, N, has been set to zero. (This seems reasonable since FCCs operating in the complete combustion mode commonly achieve very good regeneration with very small amounts of coke on regenerated catalyst (Rheaume, 1976). For these low amounts of coke on regenerated catalyst, Eq. 26 would predict both a very large coke production and a very strong dependence of the coke production on the amount of coke on regenerated catalyst. This does not appear realistic, and we have therefore set $N=0$ in our studies of the complete combustion mode of operation.)

Appendix B. Linear Two State Model for the FCC

A linear two state model for the FCC in the partial combustion mode (Case P1 in Table 1), obtained by linearization and model reduction of the five state nonlinear model in Appendix 1. Note that the time scale is in minutes.

$$\frac{dx}{dt} = Ax + Bu + Ed \tag{B1}$$

$$y = Cx + Du + Fd \tag{B2}$$

$$A = \begin{bmatrix} -2.55\times 10^{-2} & 1.51\times 10^{-6} \\ 227 & -4.10\times 10^{-2} \end{bmatrix} \tag{B3}$$

$$B = \begin{bmatrix} 3.29\times 10^{-6} & -2.60\times 10^{-5} \\ -2.80\times 10^{-2} & 7.80\times 10^{-1} \end{bmatrix} \tag{B4}$$

$$C = \begin{bmatrix} 1.32\times 10^{3} & 0.559 \\ -4.42\times 10^{3} & 0.538 \\ 0 & 1 \end{bmatrix} \tag{B5}$$

$$D = \begin{bmatrix} 0.362 & 0 \\ 0 & 0.877 \\ 0 & 0 \end{bmatrix} \tag{B6}$$

$$E = \begin{bmatrix} 6.87\times 10^{-7} & 0 & -7.06\times 10^{-6} & 3.53\times 10^{-2} \\ 2.47\times 10^{-2} & 9.24\times 10^{-3} & -2.54\times 10^{-1} & 0 \end{bmatrix} \tag{B7}$$

$$F = \begin{bmatrix} 0.246 & 0 & -0.253 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{B8}$$

$$y = \begin{bmatrix} T_{ro} \\ T_{cy} \\ T_{rg} \end{bmatrix}; \quad x = \begin{bmatrix} C_{rc} \\ T_{rg} \end{bmatrix}; \quad u = \begin{bmatrix} F_s \\ F_a \end{bmatrix}; \quad d = \begin{bmatrix} T_f \\ T_a \\ F_f \\ k_c \end{bmatrix}$$

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