Identification of Ill-Conditioned Plants — A Benchmark Problem*

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Abstract:
This note provides a simple process example from chemical engineering which is proposed as a challenge problem for multivariable identification. The process considered is a simple heat-exchanger with two inputs and two outputs. It is strongly interactive and also ill-conditioned. A single slow pole, resulting from the interactions, is dominating all the individual open-loop responses. Attempting to identify a model based on fitting the individual transfer-matrix elements will usually result in a multivariable model which incorrectly has this dominant pole repeated. Such a model, although a reasonable model for the open-loop dynamics, yields a poor prediction of the process behavior under feedback control, in particular when considering partial control.

The note includes a description of the process, a file for generating open-loop "experimental" data and an example demonstrating that classical identification employing an ARMAX-type of model yields a model which is poor for feedback control studies of the process.

1 Introduction

Most published work on the identification of dynamic models from experimental data has been concentrated on the single-input-single-output (SISO) case. This is also reflected in the literature on process dynamics and control, where linear dynamic models usually are obtained by fitting input-output data from a plant or nonlinear simulation to a low-order transfer-function. In cases where the process is multivariable, the transfer-matrix is usually obtained by fitting the transfer-matrix elements *independently*. However, obtaining reasonable models for the individual transfer-function elements does not guarantee a reasonable multivariable model. This is in particular true for ill-conditioned processes which is the subject of this note. Ill-conditioned processes are commonplace in the chemical process industry and include, for example, high-purity distillation columns (Skogestad et al. [6]).

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Skogestad and Morari [5] argue that fitting the transfer-matrix elements independently may easily lead to poor models for ill-conditioned processes unless one explicitly takes into account the coupling between the gains of the different elements. In particular, one is not able to obtain a good model of the low-gain direction of the plant, and the model will easily have the wrong sign of the determinant of the steady-state gain matrix, and the model will be useless for control studies. This problem may, however, usually be corrected as the sign of the determinant and its approximate value in many cases is known a priori (Kapoor and McAvoy [3], Jacobsen et al. [1]).

Another, and more fundamental problem in the identification of ill-conditioned processes, is the fact that such plants often have a *single* "slow" pole (large time-constant) which tends to dominate all responses of the plant (Jacobsen and Skogestad [2]). This dominating pole is a result of interactions in the process, and is thus shared by all the transfer-matrix elements. As shown by Jacobsen and Skogestad [2], fitting the transfer-matrix elements independently such that they all contain the dominating pole, will usually result in an inconsistent model with several poles equal to the dominating pole of the process. This inconsistency will result in a poor prediction of the process under partial feedback control, that is, with only some of the process outputs under feedback control.

The general literature on identification has so far not focused very much on multivariable issues, and the particular problems that may be encountered for ill-conditioned processes mentioned above, do not seem to have been discussed. In this note we therefore present data for an ill-conditioned process which we believe represents a "new" and difficult problem in multivariable identification.

We start the note by presenting a model and a set of input-output data of a heat-exchanger which is ill-conditioned. In addition to providing data for the process we also discuss briefly some specific process properties which are of interest for the identification problem. Having presented the problem we employ a fairly standard identification technique and show that it results in an inconsistent model which is poor for control studies of the plant. The objective of the example is to demonstrate that obtaining reasonable models for the individual transfer-matrix elements does not guarantee that the multivariable properties have been reasonably captured.

2 Process Description

The process we consider is a simple heat-exchanger where heat is transferred between a cold and a hot flow (see Fig. 1). Each side of the heat-exchanger is approximated as a single, perfectly mixed tank. Neglecting variations in liquid volume and heat accumulated in the walls yields a model with two states. The model derivation is given in Appendix 1. The linear model $y(s) = G(s)u(s)$

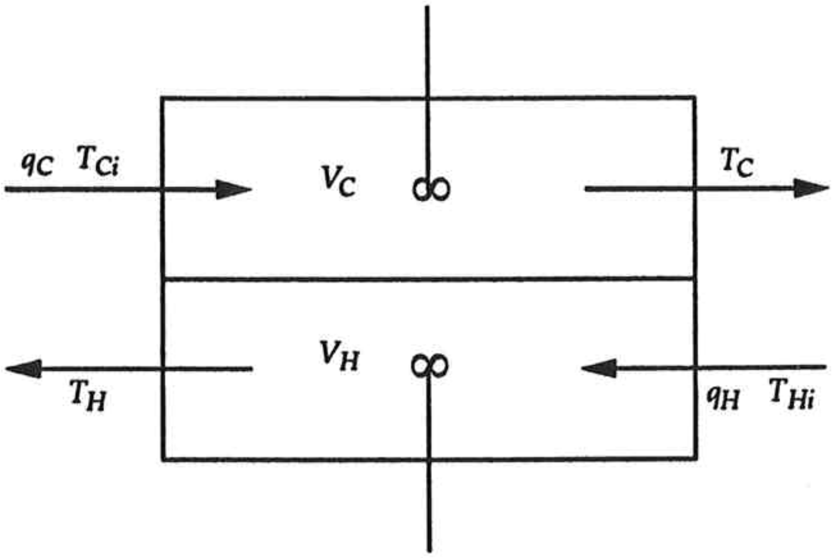

Fig. 1. Simple heat exchanger

is given by

$$G(s) = \frac{1}{(1+\tau_1 s)(1+\tau_2 s)} \begin{pmatrix} k_{11}(1+z_1 s) & k_{12} \\ k_{21} & k_{22}(1+z_1 s) \end{pmatrix}, \qquad (1)$$

where $\tau_1 = 100$; $\tau_2 = 2.44$; $z_1 = 4.76$; $k_{11} = -k_{22} = -1874$; and $k_{12} = -k_{21} = 1785$. Here $y = [T_C \quad T_H]$ is the cold and hot exit temperatures and $u = [q_C \quad q_H]$ are the cold and hot inlet flow rates. The first thing to note about the model is that there are two pole-zero cancellations such that the model contains only two and not four states. The model is also relatively ill-conditioned with a steady-state condition number of 41. The physical explanation for the ill-conditioning is simply that the two exit temperatures are almost the same (T_C =61.59 $°C$ and T_H =63.41 $°C$ in our case), and it is very difficult to change them independently. In particular, it is difficult to make them closer or further apart (this is the low-gain direction of the process) whereas we may easily make them *both* hotter or colder (this is the high-gain direction of the plant). An analysis of the model reveals that the slow pole $-1/\tau_1$ is related to the high-gain direction of the plant while $-1/\tau_2$ is related to the weak direction. The steady-state gain related to the slow pole is hence 41 times larger than the gain related to the fast pole.

The open-loop responses of the process model (1) are almost pure first-order responses with a time-constant equal to τ_1. Thus, a reasonably good fit of the individual transfer-matrix elements is obtained by first-order transfer-functions with time-constant $\tau_1 = 100$ min. However, the resulting model

$$G(s) = \frac{1}{1+\tau_1 s} \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} \qquad (2)$$

contains two poles at $-1/\tau_1$ and is thus inconsistent with the true process (1) which has only one pole at this location. The inconsistency results in a poor prediction of the process under partial control, i.e., with one feedback loop closed (Jacobsen and Skogestad [2]). To avoid the inconsistency it is at least necessary to identify also the faster pole $-1/\tau_2$. Furthermore, the identified model must be such that its minimal realization only contains a single slow pole. Of course, if one starts from a model structure where this information is included, then the identification becomes simpler. The challenge is to see if one is able to identify a good model *without* providing such information which is usually not available in a practical situation. We believe this is a problem which has not been properly addressed in identification theory, and which seems to cause problems for many classical identification methods.

2.1 The Identification Problem

In Appendix 2 we provide a Matlab file for generating open-loop "experimental" data using the linear model (1). The data are produced using a multivariable experiment, i.e., simultaneous perturbations in the two inputs. Noise is added to the inputs as well as the outputs.

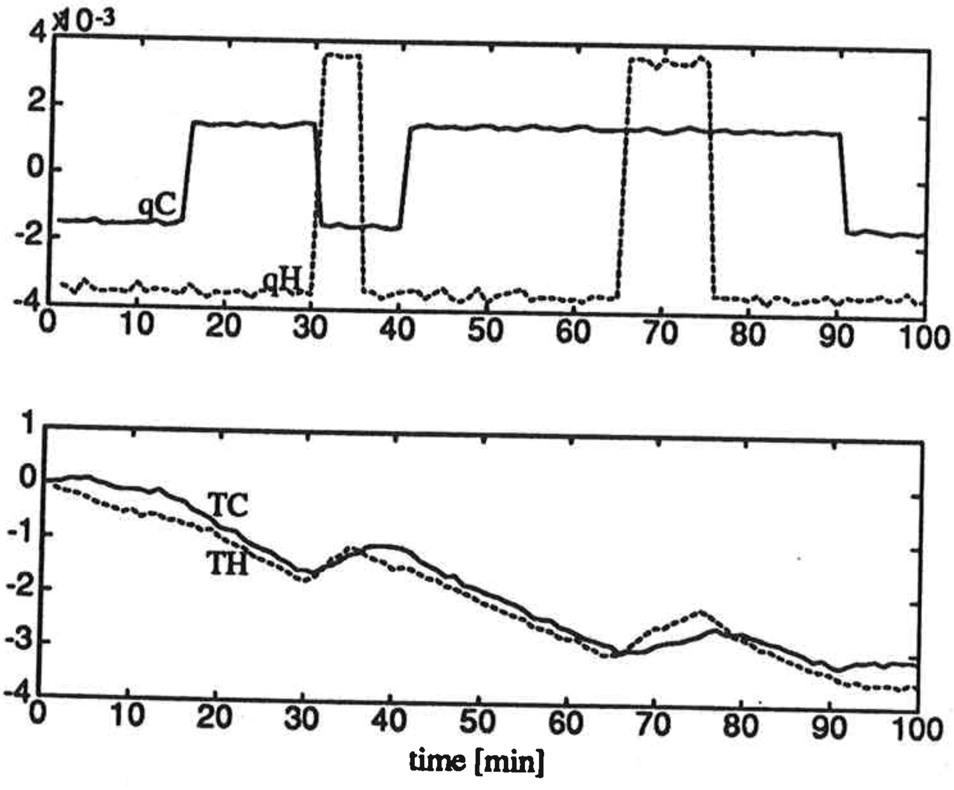

Fig. 2. Input and output data used for identification of heat-exchanger. The data were generated with the Matlab file given in Appendix 2

Figure 2 shows the 100 min. input sequence (including noise) and the resulting outputs generated using the Matlab file. The inputs to the process contain 3 % white noise, while the outputs have white noise with variance $0.03\,^{\circ}C$ (which is very small compared to practical situations).

The identification problem is to come up with a reasonable multivariable dynamic model based on these data alone, i.e., based on the noise-free inputs and the noisy measurements. One should not supply any knowledge about the special multivariable structure of the model as given by (1). The identified model is intended to be used for feedback control studies, and two different cases are of interest.

1. Partial control: Output y_1 is controlled using input u_1 while y_2 is left uncontrolled.
2. Multivariable control: Both y_1 and y_2 are controlled using both inputs.

In both cases the responses to set-point changes as well as disturbances in the inputs should be considered and compared with those of the correct model (1). The intention of the challenge problem is that one should identify the model based on open-loop data only. If one is allowed to use closed-loop data we believe the identification becomes simpler.

3 MISO-Identification using an ARMAX-type model

In this section we employ a fairly standard identification technique to the data generated using the Matlab file given in Appendix 2. We employ the Matlab System Identification Toolbox (Ljung [4]) and use MISO-identification with an ARMAX-type model structure. In the identification we fit each output with a strictly proper second order model which is the same structure as the true model (1). The model resulting from this identification is given by

$$G(s) = \begin{pmatrix} \frac{-2025(5.218s+1)}{(2.027s+1)(110.7s+1)} & \frac{1871(0.0263s+1)}{(2.027s+1)(110.7s+1)} \\ \frac{-1795(-0.0933s+1)}{(1.404s+1)(110.5s+1)} & \frac{2049(3.947s+1)}{(1.404s+1)(110.5s+1)} \end{pmatrix}. \tag{3}$$

The identified model (3) has a minimal realization with 4 states. Figure 3 compares the noise-free open-loop step responses of model (3) with those of the "true" model (1).

We see from the responses that we have obtained a reasonable identification of the individual SISO-transfer functions. Furthermore, we see from the identified model that we have been able to obtain reasonable estimates for the two poles $-1/\tau_1$ and $-1/\tau_2$. However, the multivariable interactions have not been captured as the model (3) has multivariable zeros at $-0.0217 = -1/46.1$ and $-0.426 = -1/2.35$ which do not cancel the poles. This also becomes clear if one considers the singular values of the true (1) and fitted (3) model respectively. The true model (1) has, as mentioned previously, a low-gain direction

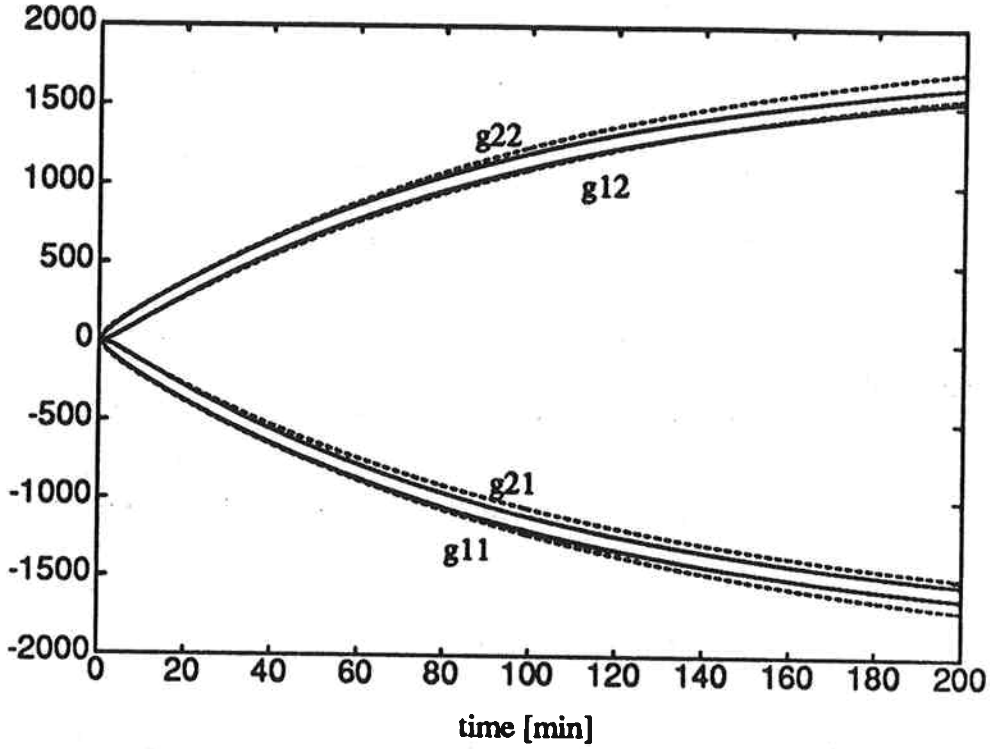

Fig. 3. Open-loop step responses of identified model (3) (dashed lines) and correct model (1) (solid lines). Labels g_{ij} denotes corresponding transfer-matrix element

with a single fast pole $-1/\tau_2$. However, the low-gain direction of the fitted model (3) has a significant part of its dynamics related to a slow pole around $-1/\tau_1$.

Figure 4 compares the closed-loop responses of the correct model (1) and the identified model (3) when output 1 is controlled with input 1 using the proportional feedback law $u_1 = K_c y_1$ with $K_c = 0.015$. We see that the identified model yields a good prediction for the controlled output y_1. However, for the uncontrolled output y_2 there is a large discrepancy between the process represented by (1) and the identified model (3). For the correct model the single slow pole is moved by the feedback controller and the response in the uncontrolled output y_2 is as fast as for y_1, while the identified model (3) contains an excessive slow pole which is left in the partially controlled model and results in a slow settling in output y_2.

4 Discussion

The noise levels of the data provided in this note are relatively small compared to what one should expect in a practical situation. Increasing the noise levels will mainly change the results obtained in a qualitative manner, that is, the excessive slow pole in the identified model will become even more marked.

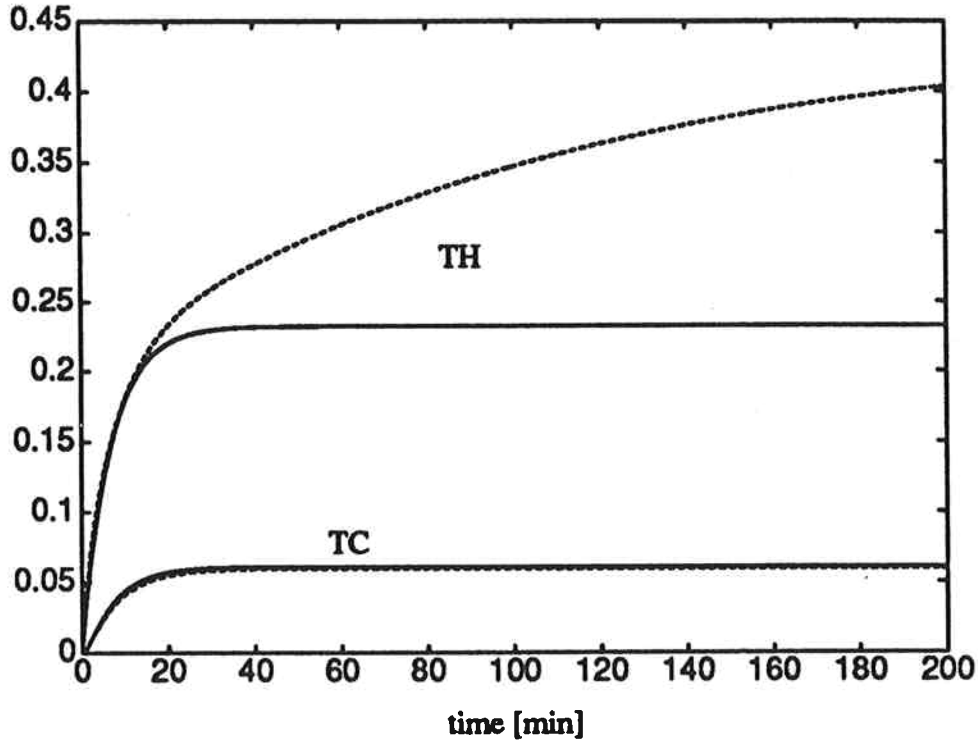

Fig. 4. Closed-loop responses of identified model (3) (dashed lines) and correct model (1) (solid lines) to step disturbance of magnitude 0.001 in the hot flow u_2. Output y_1 controlled by u_1 q_C using proportional controller with gain $K_c = 0.015$

An additional problem which may be encountered at higher noise levels is that of obtaining the correct sign of the determinant of the steady-state model. However, as mentioned in the introduction, this is usually a less crucial problem as the sign and approximate value of the determinant in many cases is known a priori.

The input sequence used to generate the "experimental" data in Appendix 2 are based on low-pass filtered PRBS signals with a minimum time between changes of 5 minutes, an experiment time of 100 minutes, and a sampling rate of 1 minute. Prolonging the time for the experiment with this set of input sequence does not seem to improve the identification.

It is worth noting that although we used a low-pass filtered input sequence, the main model error was at rather low frequencies, while the high frequency behavior of the process was reasonably well captured in the identified model. This may indicate that an input sequence with even more emphasis on low frequencies would yield better results. Indeed, with 500 minutes of experiments and a minimum time between changes of 25 minutes, we obtained better results as we have more information in the low-frequency region.

Even if the experiment time is fixed to 100 minutes, we do not rule out the possibility that a different input sequence may yield better results. The problem is how to determine the best possible input sequence when the process dynamics and multivariable interactions are largely unknown.

5 Conclusions

- We have presented a model and input-output data for an ill-conditioned process which we believe represents a "new" problem in the identification of multivariable dynamic models.
- The application of a standard identification technique (MISO ARMAX) to the process data yielded an inconsistent model with an excessive number of slow poles compared to the process, and hence a poor model for feedback control studies.

Nomenclature

A - heat transfer area (m^2)
c_P - heat capacity ($kJ/^oCkg$)
$G(s)$ - process transfer-matrix for effect of inputs u
$g_{ij}(s)$ - transfer matrix element i,j
q_C - cold inlet flow (m^3/min)
q_H - hot inlet flow (m^3/min)
T_C - cold outlet temperature (^oC)
T_H - hot outlet temperature (^oC)
U - heat transfer coefficient ($kJ/m^2\ ^oCmin$)
V_C - liquid volume cold side (m^3)
V_H - liquid volume hot side (m^3)

Greek symbols
τ_1 - dominant (largest) process time-constant (min.)
τ_2 - smaller process time-constant (min.)

Subscripts
s - setpoint change

References

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Appendix 1. Simple model of heat exchanger

Consider a simplified heat exchanger with one mixing tank on each side as shown in Fig. 1. Assume constant volumes, V, on each side, and constant values of ρ and c_P. A heat balance for the cold and hot side then yields

$$\tau_C \frac{dT_C}{dt} = \frac{q_C}{q_C^*}(T_{Ci} - T_C) + \alpha_C(T_H - T_C) \tag{4}$$

$$\tau_H \frac{dT_H}{dt} = \frac{q_H}{q_H^*}(T_{Hi} - T_H) - \alpha_H(T_H - T_C) \tag{5}$$

where q^* denotes the nominal (steady-state) flow, and

$$\tau_C = \frac{V_C}{q_C^*}; \quad \alpha_C = \frac{UA}{\rho_C q_C^* c_{PC}} \tag{6}$$

$$\tau_H = \frac{V_H}{q_H^*}; \quad \alpha_H = \frac{UA}{\rho_H q_H^* c_{PH}} \tag{7}$$

Linearizing the model assuming UA and thus α constant (independent of flow and temperature), introducing deviation variables, and taking Laplace transforms yields

$$\tau_C s T_C(s) = T_{Ci}(s) - T_C(s) + (T_{Ci}^* - T_C^*)\frac{q_C(s)}{q_C^*} + \alpha_C(T_H(s) - T_C(s)) \tag{8}$$

$$\tau_H s T_H(s) = T_{Hi}(s) - T_H(s) + (T_{Hi}^* - T_H^*)\frac{q_H(s)}{q_H^*} - \alpha_H(T_H(s) - T_C(s)) \tag{9}$$

where the superscript $*$ denotes steady-state values. In the following we will assume $\tau_C = \tau_H = \tau = 100$ [min], $\alpha_C = \alpha_H = \alpha = 20$ and $q_C^* = q_H^* = q^* = 0.01$ [m^3/min] (see data in Table 1). Rearranging yields

$$\begin{pmatrix} T_C(s) \\ T_H(s) \end{pmatrix} = G(s)\begin{pmatrix} q_C(s) \\ q_H(s) \end{pmatrix} + G_d(s)\begin{pmatrix} T_{Ci}(s) \\ T_{Hi}(s) \end{pmatrix} \tag{10}$$

where

$$G_d(s) = \frac{1}{(\tau s + 1)(\tau s + 1 + \alpha)}\begin{pmatrix} \tau s + 1 + \alpha & \alpha \\ \alpha & \tau s + 1 + \alpha \end{pmatrix} \tag{11}$$

and

$$G(s) = G_d(s)\begin{pmatrix} (T_{Ci}^* - T_C^*)/q_C^* & 0 \\ 0 & (T_{Hi}^* - T_H^*)/q_H^* \end{pmatrix} \tag{12}$$

Inserting the numerical values finally yields

$$G_d(s) = \frac{0.02439}{(100s+1)(2.439s+1)} \begin{pmatrix} 21(1+4.76s) & 20 \\ 20 & 21(1+4.76s) \end{pmatrix} \quad (13)$$

and $G(s) = G_d(s) \cdot \begin{pmatrix} -3659 & 0 \\ 0 & 3659 \end{pmatrix}$

$V_H = V_C$ m^3	$q_C = q_H$ m^3/min	T_{Ci} $°C$	T_{Hi} $°C$	T_C $°C$	T_H $°C$	UA $kJ/°C min$	ρ kg/m^3	c_P $kJ/°C kg$
1	0.01	25	100	61.59	63.41	300	500	3.0

c_P and ρ are equal for the hot and cold side.

Table 1. Steady-state data for heat-exchanger (see also Fig. 1)

Appendix 2. Matlab-file for generating input-output data of heat-exchanger

```
% This file generates inputs, u, and outputs, y,
% for heat exchanger identification problem:
rand('normal');
A=[-.21 .20;.20 -.21];B=[-36.5853 0;0 36.5853];C=eye(2);D=zeros(2);
%PRBS-signals (low-pass filtered):
q1=1.5e-3*[-1 -1 -1 1 1 1 -1 -1 1 1 1 1 1 1 1 1 1 1 -1 -1];
q2=3.5e-3*[-1 -1 -1 -1 -1 -1 1 -1 -1 -1 -1 -1 -1 1 1 -1 -1 -1 -1 -1];
% Inputs last for 5 minutes (sampling time 1 min.):
for i=1:length(q1),
u(1+5*(i-1):5*i,1)=q1(i)*ones(5,1); u(1+5*(i- 1):5*i,2)=q2(i)*ones(5,1);
end
%Noisy inputs for simulation:
usim(:,1)=u(:,1)+0.03*max(u(:,1))*rand(100,1);
usim(:,2)=u(:,2)+0.03*max(u(:,2))*rand(100,1);
% Obtain noise-free outputs:
t=1:100;
ysim=lsim(A,B,C,D,usim,t);
% Noise on outputs has variance 0.03 degrees centigrades:
y(:,1)=ysim(:,1)+0.03*rand(100,1);
y(:,2)=ysim(:,2)+0.03*rand(100,1);
```