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Dynamics and Control of Unstable Distillation Columns

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Abstract

The paper addresses dynamics and control of distillation columns operated at open-loop unstable operating points. The fact that ideal two-product distillation columns may have multiple steady states and right half plane poles has only recently been recognized. This paper discusses the dynamics and control implications. It is shown that with reflux and boilup as independent variables the operating points become unstable if the internal flows are sufficiently large. An open-loop unstable operating point may be stabilized by use of one-point control, i.e., feedback control of a column composition or temperature. If the control is not sufficiently tight, the column may go into a stable limit cycle. Finally, it is shown that with distillate flow and boilup as independent variables the operating points may become unstable if the level control is not sufficiently tight. The column may also in this case go into a stable limit cycle.

1 Introduction

Distillation is undoubtly the most studied unit operation in the process control literature. However, in all previous studies the column dynamics have been assumed to be asymptotically stable (with level and pressure loops closed). The main reason is that most authors have considered dynamic models with constant molar flows (neglected energy balance) and in addition assumed the inputs (e.g., reflux and boilup) to be given on a molar basis.

For the case of molar inputs there exists several papers on uniqueness and asymptotic stability of the operating points in homogeneous distillation. Most papers treat the constant molar flow case (neglected energy balance), e.g., [5], [1], [6], [7]. Doherty and Perkins [2] provide a review of results published in this area, and conclude that, for constant molar flows, multiplicity and instability is impossible for single-staged "columns" and any multistage column separating a binary mixture. Sridhar and Lucia [8] include the energy balance in the model and conclude under certain assumptions that also in this general case binary distillation columns will exhibit unique and stable solutions. They do, however, only study a limited set of specifications, namely $Q_D Q_B$ and LB.

However, in a recent paper Jacobsen and Skogestad [3] report two kinds of multiplicity which may occur in distillation. 1) Jacobsen and Skogestad argue that columns under operation only in rare cases have all the manipulated inputs on a molar basis. For instance, fixing the valve position will normally correspond closely to fixing the geometric average of mass and volumetric flow-rate. As they show, the transformation from mass- or volume flows to molar flows is nonlinear due to the composition dependence and may in some cases become singular, even for the binary case with constant molar flows. A singularity in the input transformation will imply that there exist multiple solutions in terms of outputs (compositions) for a given set of inputs (flows). One of the solutions will be unstable. 2) In addition, Jacobsen and Skogestad show that when the energy balance is included in the model, even molar inputs may yield multiple solutions. Both types of multiplicity and instability may be experienced in industrial columns operated with inputs on a mass- or volume basis.

Jacobsen and Skogestad [3] treat the multiplicity from a steady-state point of view only. In this paper we study the dynamics and control of columns with multiple solutions. We limit ourselves to discuss mainly one control configuration (set of specifications), namely the case where mass reflux L_w and molar boilup V are used as independent variables. This is the most widespread configuration in industry, and is the configuration for which multiplicity and instability is most likely to occur [3]. At the end of the paper we consider the $D_w V$-configuration, and show that in this case there may exist solutions corresponding to stable limit cycles.

We will in this paper only consider the multiplicity and instability caused by singularity in the input transformation. The multiplicity and instability that may be caused by including the energy-balance will have similar implications for the dynamics and control of distillation columns.

2 Results on Steady-State Multiplicity in Ideal Distillation

We give here a brief review of the results on multiplicity caused by singularity in the input transformation presented in [3].

Consider the two-product distillation column in Fig.1. If the feed to the column is given there are at least four flows that may be specified: reflux L, boilup V, distillate D and bottoms flow B. However, for a given column there are only two degrees of freedom at steady-state, that is, only two of these flows may be specified independently. A specific choice of two independent variables is denoted a "configuration".

Jacobsen and Skogestad [3] provide an example of steady-state multiplicity in a column separating a mixture of methanol and n-propanol. The column has mass reflux and molar boilup as independent variables, i.e., $L_w V$-configuration. Data for the column are given in Table 1. Note that the energy balance is excluded, i.e., constant molar flows are assumed. Some steady-state solutions are given in Table 2, and we see that for a specification of mass reflux $L_w = 50.0$ kg/min and molar boilup $V = 2.0$ kmol/min we have three possible solutions II, III and IV in terms of compositions. The multiplicity is graphically illustrated in Fig.2.

The observed multiplicity is caused by the transformation between the actual flow-rates (mass) and the molar flow-rates which determine separation. For a binary mixture the transformation between mass reflux, L_w, and molar reflux, L, is given by

$$L = L_w/M; \quad M = y_D M_1 + (1 - y_D) M_2 \tag{1}$$

Here M_i denotes the molecular weight of the individual compo-

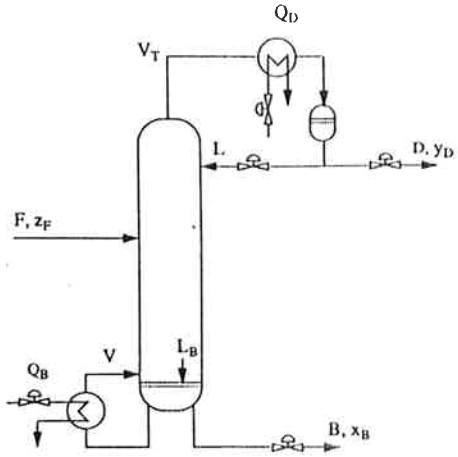

Figure 1. Two product distillation column.

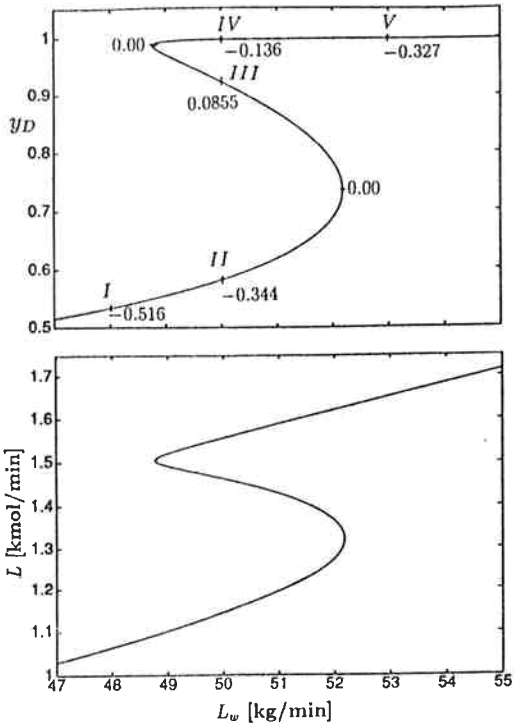

Figure 2. Methanol-propanol column: Multiple steady states for L_wV-configuration. Mass reflux L_w is varied while molar boilup V is fixed at 2.0 kmol/min. On the upper plot the corresponding maximum eigenvalue is shown at some of the steady-state solutions.

nents. One might expect the molar reflux to increase monotonically with the mass reflux, that is, $(\partial L/\partial L_w)_V > 0$. However, because M is a function of composition, y_D, and thereby of L_w, this might not be the case. Assuming molar boilup V fixed and differentiating $L_w = LM$ on both sides with respect to L yields

$$\left(\frac{\partial L_w}{\partial L}\right)_V = M + L(M_1 - M_2)\left(\frac{\partial y_D}{\partial L}\right)_V \qquad (2)$$

For $M_1 < M_2$, which is usually the case (the most volatile component has the smallest molecular weight), the second term on the right hand side of (2) will be negative and the total differential may take either sign. The transformation from L_w to L will be singular when $(\partial L_w/\partial L)_V = 0$. A singular point will correspond to a bifurcation point, and the number of solutions changes from one to three. Jacobsen and Skogestad [3] state that solutions with $(\partial L_w/\partial L)_V < 0$ (middle branch in Fig.2) correspond to unstable operating points, but they do not prove this rigorously.

3 Open-Loop Dynamics and Instability for L_wV-configuration

The maximum eigenvalue in selected operating points of the methanol-propanol column with the L_wV-configuration are shown in Fig.2. From the figure we observe that the eigenvalues at the upper and lower branches are negative, implying stability, while those at the intermediate branch (negative slope) are positive, implying instability of the operating points. Note that the unstable operating points only have a single eigenvalue in the right half plane. The eigenvalues at the singular points are zero as expected.

Table 1. Data for Methanol-Propanol Column.

z_F	F	α	N	N_F	M_1	M_2
0.50	1	3.55	8	4	32.0	60.1

Constant molar flows
Feed is saturated liquid
Total condenser with saturated reflux
Liquid holdups are $M_{Li}/F = 0.5$ min, including reboiler and condenser.

Table 2. Steady-state solutions for methanol-propanol column with V=2.0 kmol/min and L_w in the range 48 to 53 kg/min.

	L kmol/min	D kmol/min	L_w kg/min	y_D	x_B
I	1.064	0.936	48.00	0.534	$3.10e-3$
II	1.143	0.857	50.00	0.584	$3.50e-3$
III	1.463	0.537	50.00	0.9237	$7.80e-3$
IV	1.555	0.445	50.00	0.9969	0.104
V	1.650	0.350	53.00	0.9984	0.233

3.1 Conditions for instability

One-stage column Consider the simple column in Fig.3 with one theoretical stage (the reboiler) and a total condenser. Of course, such a column will never be operated in practice because the reflux is simply wasting energy and has no effect on separation. However, we analyze this column due to the simplicity of the dynamic model. As shown in [3], even such a simple column with ideal thermodynamics may have multiple steady-state solutions.

Assume binary separation, liquid feed, constant holdup in the reboiler (M_L) and negligible holdup in the condenser. The dynamic model of the column becomes:

$$M_L\frac{dx_B}{dt} = Fz_F - Dy_D - Bx_B \qquad (3)$$

We have $D = V - L$ and $D + B = F$ and with L and V as independent variables we get

$$M_L\frac{dx_B}{dt} = F(z_F - x_B) + L(y_D - x_B) + V(x_B - y_D) \qquad (4)$$

Linearization, Laplace transformation and introduction of deviation variables assuming F, z_F and V constant yields

$$sM_L\Delta x_B(s) = -D\Delta y_D(s) - B\Delta x_B(s) + (y_D - x_B)\Delta L(s) \qquad (5)$$

Assuming constant relative volatility α yields the following relation between $\Delta y_D(s)$ and $\Delta x_B(s)$

$$\frac{\Delta y_D(s)}{\Delta x_B(s)} = \frac{\alpha}{(1+(\alpha-1)x_B)^2} = K(x_B) \qquad (6)$$

Equation (5) then becomes

$$\Delta x_B(s) = \frac{y_D - x_B}{M_L s + a}\Delta L(s) \quad ; a = KD + B \qquad (7)$$

As a is always positive, the pole $-a/M_L$ is always negative, implying that all operating points are stable when molar reflux L and molar boilup V are independent variables.

Now consider mass reflux L_w as an input instead of molar reflux $L = L_w/M$. By linearization we obtain for binary separations

$$\Delta L = \frac{1}{M}\Delta L_w + L\frac{M_2 - M_1}{M}K\Delta x_B \qquad (8)$$

Substituting (8) into (7) we obtain the following transfer-function between liquid composition, $\Delta x_B(s)$, and mass reflux $\Delta L_w(s)$:

$$\Delta x_B(s) = \frac{y_D - x_B}{M_L s + a_w}\frac{\Delta L_w(s)}{M} \qquad (9)$$

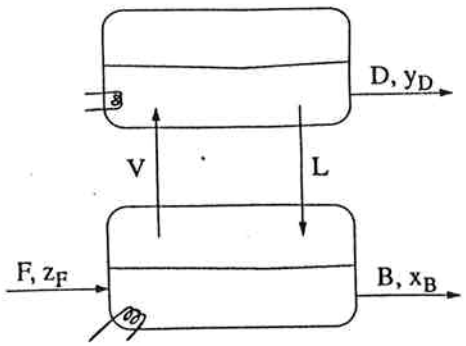

Figure 3. One-stage distillation column with total condenser.

where

$$a_w = KD + B - (y_D - x_B)\frac{M_2 - M_1}{M}KL \qquad (10)$$

The operating point is unstable for $a_w < 0$. At steady-state (6) and (7) yield

$$\left(\frac{\partial y_D}{\partial L}\right)_V = \frac{y_D - x_B}{D + B/K} \qquad (11)$$

and from (2) we find that instability ($a_w < 0$) is equivalent to a negative slope for $(dL_w/dL)_V$. This result is in accordance with numerical results.

Multistage column Jacobsen and Skogestad [4] provide evidence for the instability of multistage columns. Using certain assumptions they show that the maximum eigenvalue $\lambda_{max}^{L_wV}$ of a column operating with the L_wV-configuration is given by

$$\lambda_{max}^{L_wV} = \lambda_{max}^{LV}(1 - \frac{(\partial y_D/\partial L)_V\, L(M_2 - M_1)}{M}) \qquad (12)$$

Here M is as defined previously and λ_{max}^{LV} is the maximum eigenvalue for the LV-configuration. Jacobsen and Skogestad [4] show that instability, $\lambda_{max}^{L_wV} > 0$, corresponds to having a negative slope between mass and molar reflux. For details we refer to [4].

3.2 Effect of operating conditions on stability

Jacobsen and Skogestad [3] provide analytical results on when a negative slope between mass and molar reflux, i.e., instability, is most likely. They show that a negative slope is most likely with large internal flows (i.e., large L and V) and with intermediate purities in the top (i.e., intermediate L for given V). This corresponds to having L and $(\partial y_D/\partial L)_V$ large, and according to (12) this is the case for which instability is most likely. On this basis Jacobsen and Skogestad [3] show that there may exist three possible regimes of operation for distillation columns depending on the size of the internal flows:

- Internal flows low: Unique stable operating points.
- Internal flows intermediate: Multiple steady states, one of which is unstable.
- Internal flows high: Unique unstable operating points.

Note that the analytical treatment in [3] was based on ideal separation with constant relative volatility and constant molar flows.

4 Effect of Multiplicity and Instability on Column Operation

It is obvious that the multiplicity and instability presented here may have implications for the operation of distillation columns using the L_wV-configuration. In manual (open-loop) operation the multiplicity may be experienced as small changes in the manipulated flows may cause sudden jumps in the product compositions. Furthermore, one will experience inability to reach a desired product composition if the separation corresponds to an unstable operating point. However, we show that an unstable operating point usually may be easily stabilized by feedback control.

4.1 Manual operation

We will again consider the methanol-propanol column in Table 1. Assume that the column initially operates with reflux L_w fixed at 50.0 kg/min and boilup V fixed at 2.0 kmol/min. For these values of L_w and V there exist three possible solutions in terms of compositions according to Table 2 and Fig.2. Only two of these are stable, namely operating points II and IV. Assume that the column just has been started up and that the product compositions corresponds to solution II, i.e., $y_D = 0.584$ and $x_B = 0.0035$, while the desired operating point is operating point III in Table 2, i.e., $y_D = 0.924$. In order to increase the purity in the top the operator starts to increase reflux L_w in a stepwise fashion. The reflux is increased with 0.5 kg/min every 40 minutes. This is illustrated in Fig.4 together with the response in the top composition. In the beginning the top composition y_D increases slightly with reflux as expected. However, as the operator increases the reflux from 52 to 52.5 kg/min the top composition starts to increase drastically. The reason is that the reflux has been increased passed the lower

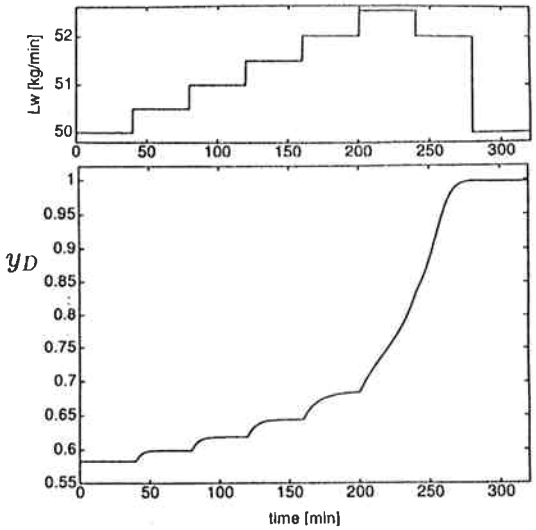

Figure 4. Nonlinear open-loop response of methanol-propanol column to changes in mass reflux L_w. Boilup V=2.0 kmol/min.

singular point in Fig.2, and the column goes trough an instability known as a "catastrophic jump". The operator observes that the top composition becomes too pure and the bottom composition too unpure, and reduces the reflux back to 52.0 kg/min. However, as seen from Fig.4 this does not have the desired effect, and the operator decides to reduce the reflux all the way back to 50.0 kg/min. However, due to the multiplicity the column now settles in operating point IV with $y_D = 0.997$, that is, hysteresis is experienced in the operation.

The simulations in Fig.4 demonstrate three different effects that may observed in columns with multiple steady states: 1) inability to reach open-loop unstable operating points by manual operation, 2) catastrophic jumps as the column goes through a singular point and 3) hysteresis in operation.

4.2 Operation with composition control

As we have seen, columns operating with mass or volume inputs may be open-loop unstable, and will require feedback control (in addition to level and pressure control) for stabilization. From control theory it is well known that unstable poles by themselves do not represent any bandwidth limitations; on the contrary they put a lower limit on allowable bandwidth of the closed-loop system. Problems will therefore only arise if there are bandwidth limitations like right half plane zeros at frequencies comparable to the right half plane pole ("The system goes unstable before we are able to observe what is happening") or if there are constraints ("we can not counteract the instability"). For plants with a RHP pole p and a RHP zero z one must in general require $p < z$ in order to be able to stabilize the column. With a limited structure of the controller, e.g., a PI controller, the distance between p and z must be larger.

Good control of distillation columns usually requires two-point control, i.e., feedback control of both product compositions. However, in order to stabilize an open-loop unstable column one-point control will suffice. This is also the way most industrial columns with composition control are operated. An unstable column operating with the L_wV-configuration may be stabilized by controlling either top or bottom composition, or any other variable related to composition, e.g. a temperature inside the column.

For operating point III of the methanol-propanol column the RHP-pole is at $p = 0.086min^{-1}$ and we are unable to stabilize the column with a PI-controller when the deadtime exceeds 11 min. ($z \approx 2/\theta_d = 0.182min^{-1}$). However, composition measurements in industrial columns (GC-analysis) may typically have deadtimes up to 30 min., and one should then use faster temperature measurements in order to stabilize the column.

Nonlinear Simulations. Fig.5 shows nonlinear simulations of the methanol-propanol column using a single-loop PI-controller[1] between top composition y_D and mass-reflux L_w with a 1 minute measurement deadtime included. Molar boilup V is kept con-

stant at 2.0 kmol/min. The figure shows the responses to setpoint changes in y_D from operating point II (open-loop stable) to operating point III (open-loop unstable) and then further on to operating point IV (open-loop stable). A logarithmic measurement $Y_D = ln(1 - y_D)$ was used in the controller as this reduces the nonlinearity of the initial response between different operating points [9]. From the figure we see that the controller is able to stabilize the open-loop unstable operating point III with a RHP pole at $0.086\ min^{-1}$. The simulations also show that the same controller may be used in these three widely differing operating points. The reason is that the initial response (high-frequency dynamics) in terms of logarithmic composition Y_D is similar in all operating points. We would get instability if we used mole fractions, y_D, as is done conventionally. From the plot of mass-reflux against time we see that the steady-state change in the input is zero, showing that the three operating points are multiple solutions.

One should be careful about detuning a controller in an open-loop unstable process as the bandwidth may become lower than the minimum allowable and the operating point becomes closed-loop unstable. This is illustrated in Fig.6. where the controller gain has been reduced by a factor of two compared to the simulations in Fig.5. Operating point *III* is now closed-loop unstable, and a small setpoint change makes the system start drifting away. However, this does not imply that the column goes globally unstable in the sense that physical constraints are violated. Since there exists steady-state solutions above and below the unstable solution the column goes into a stable limit cycle. If the controller gain is reduced further the limit cycle will continue, but now with a longer period of each cycle and with higher peaks in composition. There will also exist cases where there are no solutions either above or below the unstable solution. In this case the column is likely to go globally unstable as either the condenser (missing upper branch) or reboiler (missing lower branch) would run dry.

For a discussion of the implications of open-loop instability for two-point control we refer to [4].

[1] Tuned to yield reasonably fast response. Note that Ziegler-Nichols tuning rules resulted in a closed-loop unstable system.

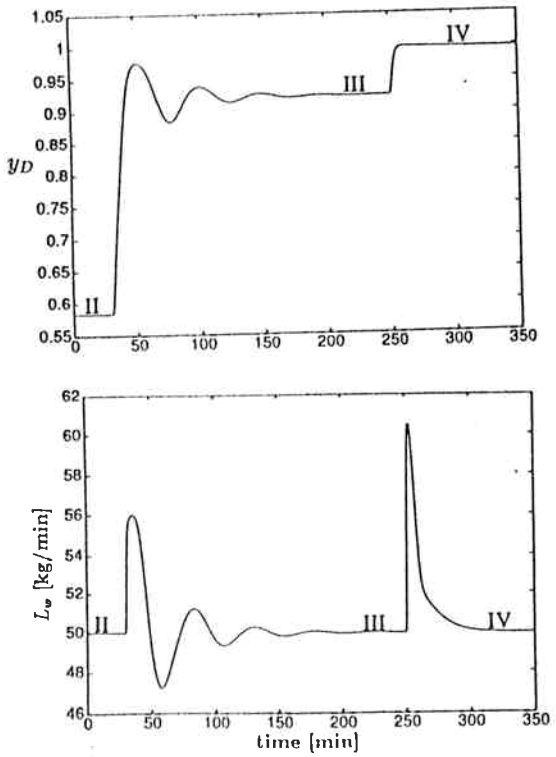

Figure 5. Nonlinear simulation of methanol- propanol column with one-point control of top-composition y_D using mass reflux L_w. Setpoint changes from operating point *II* to *III* and from *III* to *IV*. Boilup $V = 2.0$ kmol/min. Controller parameters: $k = 3.0$ and $\tau_I = 11.0$ min. Gain is for logarithmic composition, i.e., $log(1 - y_D)$.

5 Other Bifurcation Parameters

So far we have only considered the inputs, e.g. reflux L_w and boilup V, as potential bifurcation parameters. That is, in all studies we have assumed the other parameters, e.g., feed flow F, feed composition z_F, feed liquid fraction q_F, tray efficiency etc., to be fixed. However, it is clear that these parameters will vary during operation and may, similarly to the inputs, cause the column operation to go from open-loop stable to open-loop unstable.

To illustrate this consider Fig.7 which shows steady-state solutions for the methanol-propanol column with $L_w = 50.0$ kg/min, $V = 2.0$ kmol/min and feed composition z_F in the range 0.40 to 0.60. From the figure we see that there are multiple solutions for z_F in the range 0.46 to 0.54. This implies that disturbances in the feed composition may cause the column to go through a singular point and thereby "jump" to another solution branch. This is illustrated in Figure 8 which shows the response in top composition y_D to a change in feed composition z_F from 0.50 (operating point *IV* in Table 2) to 0.46. The figure illustrates how the top composition "jumps" to the lower solution branch and settles in operating point *VI*. When the feed composition returns to $z_F = 0.50$ the solution remains on the lower branch and settles in operating point *II*.

6 Instability with the D_wV-Configuration

We have so far only considered using reflux and boilup as independent variables, i.e., the L_wV-configuration. This is also the most widespread configuration in industry. However, there are many different configurations that may be used. For instance, changing condenser level control from using distillate D_w to using reflux L_w results in the D_wV-configuration. For all the examples we have have studied this configuration yields a unique steady-state solution in terms of compositions. Furthermore, we have assumed perfect level control, in which the operating point is found to be

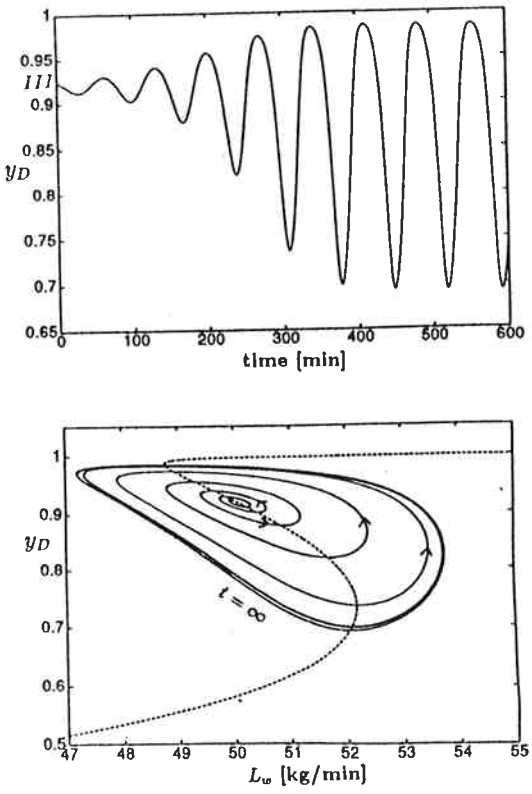

Figure 6. Nonlinear simulation of methanol- propanol column with one-point control of top-composition y_D using mass reflux L_w. Controller gain reduced by a factor of 2 compared to Fig.5. Upper plot: Time as independent variable. Lower plot: Phase-plane plot. Dashed line shows steady-state solutions.

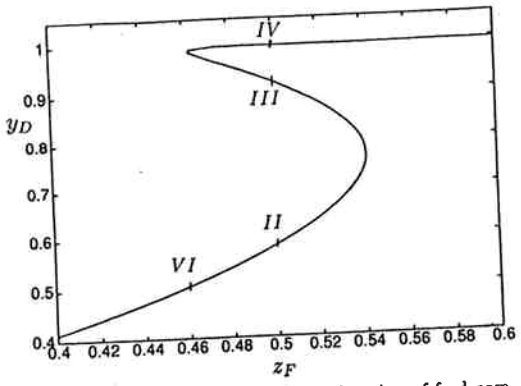

Figure 7 Steady-state solutions as a function of feed composition z_F for methanol-propanol column. Reflux $L_w = 50$ kg/min, Boilup $V = 2.0$ kmol/min.

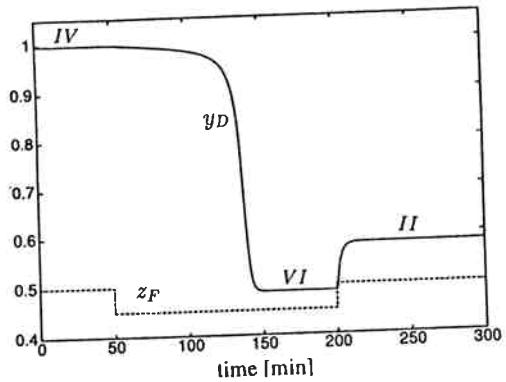

Figure 8 Nonlinear open-loop simulation of methanol-propanol column for changes in z_F. Reflux $L_w = 50$ kg/min and boilup $V = 2.0$ kmol/min. Roman numbers $II - IV$ refer to Table 2 with $z_F = 0.50$. Operating point VI corresponds to $z_F = 0.46$.

asymptotically stable. However, here we show that without the assumption of perfect level control the operating point may become unstable also with the D_wV-configuration. We start by considering an example and will then explain the results thereof using analytical results.

Example. We will again consider the methanol-propanol column in Table 1. The holdups in the reboiler and condenser are increased to $M_D/F = M_B/F = 5.0$ min. We consider the case with constant molar flows, and use distillate flow D_w and boilup V as independent inputs, i.e., D_wV-configuration. With this configuration the condenser level is controlled by reflux L_w and the reboiler level is controlled by bottoms flow B_w. The nominal operating point we consider has D_w=18.36 kg/min and V=2.0 kmol/min. For these specifications we obtain y_D=0.9237 and x_B=0.0078, and the steady-state is unique. Note that the operating point corresponds to solution III for the L_wV configuration in Table 2 and Fig.2, that is, the operating point is unstable with reflux and boilup as independent variables.

We now consider the stability of the operating point for different gains K_{M_D} in the condenser level controller. A pure proportional controller is used, i.e., $dL_w(s) = K_{M_D} dM_{Dw}(s)$. We assume perfect level control in the reboiler. Fig 9a shows the response in top composition to a small increase in D_w, keeping V constant, with level control gain K_{M_D}=0.05. We see that the response is oscillatory but asymptotically stable, i.e., a stable spiral. Fig.9b shows the corresponding response with K_{M_D} reduced to 0.03, and we see that the steady state now is an unstable spiral. However, the response settles into a stable periodic solution, that is, a stable limit cycle.

As the steady-state changes from a stable spiral to an unstable spiral as the level control gain is reduced implies that a pair of complex conjugate eigenvalues cross the imaginary axis. The fact that a stable limit cycle appears as the steady-state becomes unstable, implies that the system goes through a dynamic bifurcation known as the Hopf bifurcation.

6.1 Analytical treatment

To understand why the steady-state for the D_wV-configuration becomes unstable, consider the transfer function $(\partial y_D/\partial D_w)_V(s)$ which may be written

$$\left(\frac{\partial y_D}{\partial D_w}\right)_V(s) = \left(\frac{\partial y_D}{\partial L_w}\right)_V(s)\left(\frac{\partial L_w}{\partial D_w}\right)_V(s) \tag{13}$$

Here the transfer function $(\partial y_D/\partial L_w)_V(s)$ expresses the effect of reflux on top composition with the L_wV-configuration, and we have seen that it may be unstable with a single RHP pole. For simplicity we consider only the largest pole in the transfer function

$$\left(\frac{\partial y_D}{\partial L_w}\right)_V(s) = \frac{k}{s-a} \tag{14}$$

Here a denotes the maximum eigenvalue for the L_wV-configuration. The transfer function $(\partial L_w/\partial D_w)_V(s)$ may be computed from a material balance around the condenser

$$dL_w(s) = \frac{K_{M_D}}{s}(dV_{Tw}(s) - dL_w(s) - dD_w(s)) \tag{15}$$

Differentiation of (15) yields

$$\left(\frac{\partial L_w}{\partial D_w}\right)_V(s) = \frac{K_{M_D}}{K_{M_D}+s}(V_T(M_1-M_2)\left(\frac{\partial y_T}{\partial D_w}\right)_V(s) - 1) \tag{16}$$

Here y_T denotes the composition of V_T. We assume negligible condenser holdup so that $(\partial y_T/\partial D_w)_V(s) = (\partial y_D/\partial D_w)_V(s)$. Inserting (14) and (16) into (13) yields

$$\left(\frac{\partial y_D}{\partial D_w}\right)_V(s) = \frac{K_{M_D}k}{s^2 + (K_{M_D}-a)s - K_{M_D}(a + kV_T(M_1-M_2))} \tag{17}$$

The poles of the transfer function (17) become

$$\lambda_{1,2} = -\frac{1}{2}(K_{M_D}-a) \pm \frac{1}{2}\sqrt{(K_{M_D}-a)^2 + 4K_{M_D}(a + kV_T(M_1-M_2))} \tag{18}$$

Let us now use (18) to consider the stability of the D_wV-configuration for the two cases when the pole a of the L_wV-

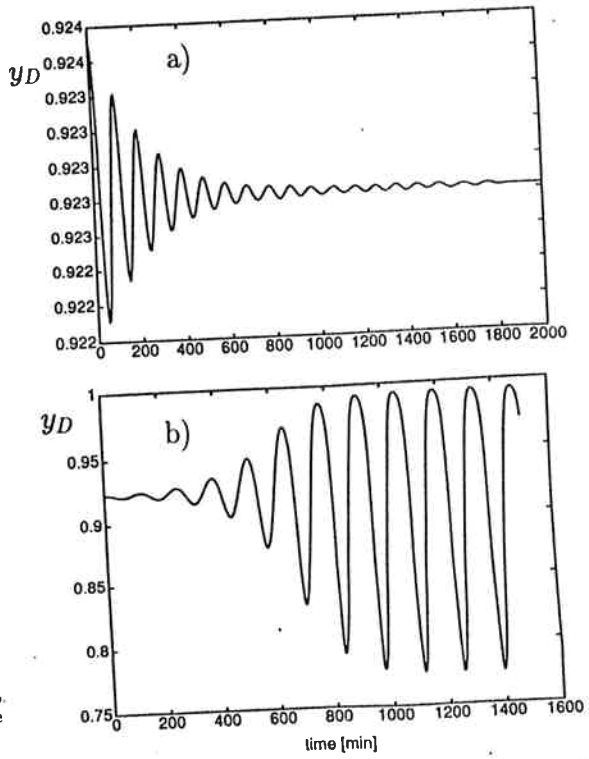

Figure 9 Nonlinear open-loop simulation of methanol-propanol column with D_wV-configuration. Responses to small increase in D_w with different gains K_{M_D} in condenser level controller. a) K_{M_D}=0.05. b) K_{M_D}=0.03. Boilup V=2.0 kmol/min.

configuration is in the LHP and RHP, respectively:
1) Stable L_wV-configuration, $a < 0$: In this case the first term in (18) is negative for all values of $K_{M_D} > 0$. Furthermore, the second term under the root in (18) is negative and the root will be real with a value less than $(K_{M_D} - a)$ or it will be imaginary. This implies that both eigenvalues in (18) are in the LHP, that is, the D_wV-configuration is stable for all values of $K_{M_D} > 0$.
2) Unstable L_wV-configuration, $a > 0$: In this case we have that the first term in (18) is positive if $K_{M_D} < a$, that is, at least one of the eigenvalues in (18) are in the RHP with $K_{M_D} < a$. The size of K_{M_D} will determine whether the root in (18) is imaginary. For $K_{M_D} = a$, i.e., the bifurcation point, we have that the root is imaginary if $kV_T(M_1 - M_2) < -a$, which is the case in all examples we have studied.

We conclude from the above analysis that a prerequisite for instability with the D_wV-configuration is that the operating point is unstable with the L_wV-configuration. This is not too surprising as the level control for the D_wV-configuration may be viewed as a feedback effect on the L_wV-configuration. If the feedback control is not tight enough, we are not able to stabilize the column, which is similar to what we found for the case of one-point control with the L_wV-configuration. With a gain K_{M_D}=0, i.e., no condenser level control, we see from (18) that there will be a RHP pole at a (in addition to a pole at 0), and we effectively have the stability properties of the L_wV-configuration.

In our example we find that the L_wV-configuration is unstable with a pole $a = 0.047$ (with $M_D/F = M_B/F = 5min.$) and from (18) we predict instability for the D_wV-configuration with $K_{M_D} < 0.047$. From the full model we find that instability occurs for $K_{M_D} < 0.043$. The deviation in predicted and computed value is explained by our assumptions of first-order response in (14) and negligible condenser holdup in the analytical treatment.

7 Conclusions

- Distillation columns operating with the L_wV-configuration may have multiple steady states and unstable operating points. The probability of instability increases with increased internal flows.
- The multiplicity and instability may cause problems for manual operation of distillation columns. Three effects may be observed: 1) catastrophic jump phenomena, 2) inability to reach certain separations and 3) hysteresis in operation.
- An open-loop unstable operating point may usually be easily stabilized by feedback control of a composition or temperature. If the bandwidth is not sufficiently tight the column may go into a stable limit cycle.
- Columns operating with the D_wV-configuration may become unstable if the level control is not sufficiently tight. The instability will correspond to a Hopf bifurcation, and a stable limit cycle appears.

NOMENCLATURE (see also Fig.1)

B - bottoms flow (kmol/min)
D - distillate flow (kmol/min)
F - feed rate (kmol/min)
L - reflux flow rate (kmol/min)
M - mole weight, usually of top product (kg/kmol)
M_1 - pure component mole weight of most volatile component (kg/kmol)
M_2 - pure component mole weight of least volatile component (kg/kmol)
N - no. of theoretical stages in column
N_F - feed stage location (1-reboiler)
V - boilup (kmol/min) (determined indirectly by heating)
x_B - mole fraction of most volatile component in bottom product
y_D - mole fraction of most volatile component in top product
z_F - mole fraction of most volatile component in feed

Greek symbols

$\alpha = \frac{y_i/x_i}{(1-y_i)/(1-x_i)}$ - relative volatility (binary mixture)
$\lambda_{max} = \max_i |\lambda_i(A)|$- maximum eigenvalue = dominant pole

Subscripts
w - flow rate in kg/min

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