

# IMPACT OF MODEL UNCERTAINTY ON CONTROL STRUCTURE SELECTION FOR THE FLUID CATALYTIC CRACKING PROCESS

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**Abstract.** The paper demonstrates that the FCC process is very nonlinear and the control properties are very sensitive both to the structure of the process model chosen, and to uncertainties in the parameters in the model. From our analysis with RHP-zeros, RGA, and CLDG, the control of the FCC in the partial combustion mode seems to be a very difficult problem. On the other hand, the FCC process appears relatively insensitive to uncertainty in the manipulated variables. The conventional control structure is shown to be more robust with respect to changes in the operating point and parameter values than the Kurihara control structure. This is in accordance with industrial practice.

**Keywords.** Process models; Decentralized control; Parameter uncertainty; Input uncertainty; Disturbance rejection.

## 1 Introduction

The Fluid Catalytic Cracking (FCC) process is an important process in refineries for upgrading heavy hydrocarbons to more valuable lighter products. Both decentralized controllers and more complex model predictive controllers are used to control the FCC process. However, when model predictive control is used, it is often applied on top of a decentralized control system and sends setpoint changes to the individual loops. Thus it is important also in this case that the decentralized control system is well designed.

A schematic overview of the FCC process is shown in Fig. 1. Feed oil is contacted with hot catalyst at the bottom of the riser, causing the feed to vaporize. The cracking reactions occur while the oil vapor and catalyst flow up the riser. As a byproduct of the cracking reactions coke is formed and deposited on the catalyst, thereby reducing catalyst activity. The catalyst and products are separated in what for historical reasons is still known as the reactor<sup>1</sup>. The catalyst is then stripped with steam to remove any strippable hydrocarbons, before being returned to the regenerator where the coke is burnt off in contact with air. The combustion of coke in the regenerator provides the heat needed for feed vaporization and the endothermic reaction in the riser.

The objective of the control system is to maintain the desired reaction conditions in the riser. In the conventional control structure this is done by controlling the riser exit temperature, whereas the Kurihara (1967) control structure achieves this by controlling the regenerator temperature. The models used in this work assume the FCC to be operated in the partial combustion mode, where both CO and CO<sub>2</sub> is formed in the regenerator. Unreacted oxygen leaving the regenerator dense bed can then react with CO, thus causing a large temperature rise. The result can be that the temperature in the cyclones at the regenerator exit or in downstream equipment can exceed the metallurgical limits. In the partial combustion model an objective is therefore to control the flue gas temperature.

In the present work, the existence of right half plane (RHP) zeros and frequency dependent relative gain array (RGA) and closed loop disturbance gain (CLDG) are used for control struc-

ture selection, and we study the effect of structural and parametric uncertainty in the models and uncertainty in the manipulated variables on the choice of control structure for decentralized control. The use of frequency dependent RGA and CLDG are explained by Skogestad and Hovd (1990) and Hovd and Skogestad (1991).

## 2 Models of the FCC process used in this work

The models used in this work derive in the main part from the model proposed by Lee and Groves (1985). This model augments the regenerator model of Errazu and coworkers (1979) with the riser model of Shah and coworkers (1977).

### 2.1 Riser model

The riser model uses the three lump kinetic scheme of Weekman and Nace (1970). In this scheme the feed is gas oil, which can crack to gasoline or light gases/coke.

Material balance for gas oil:

$$\frac{dy_f}{dz} = -k_1 y_f^2 t_c [COR] \theta_0 \exp\left(\frac{-E_f}{RT_0(1+\Theta)}\right) \exp(-\alpha t_c [COR] z) \quad (1)$$

Material balance for gasoline:

$$\frac{dy_g}{dz} = \left( k_2 y_f^2 \exp\left(\frac{-E_f}{RT_0(1+\Theta)}\right) - k_3 y_g \exp\left(\frac{-E_g}{RT_0(1+\Theta)}\right) \right) \times t_c [COR] \theta_0 \exp(-\alpha t_c [COR] z) \quad (2)$$

In the above equations the catalyst to oil ratio [COR]<sub>i</sub>, which was omitted by Lee and Groves, is reintroduced into the second exponential factor in order to be consistent with the original model of Shah et al.

A correlation taken from Kurihara (1967) is used to estimate the amount of coke produced.

$$C_{cat} = k_c \sqrt{\frac{t_c}{C_{rc}^n} \exp\left(\frac{-E_{cf}}{RT_1}\right)} \quad (3)$$

The amount of coke on the catalyst leaving the riser is thus

$$C_{sc} = C_{rc} + C_{cat} \quad (4)$$

The energy balance is given by:

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<sup>1</sup>With less active catalysts, the residence time in the reactor was needed to perform the cracking. With modern catalysts, the catalyst and products must be separated as quickly as possible at the riser exit to prevent overcracking.

$$\frac{d\theta}{dz} = \frac{\lambda \Delta H_f F_o}{T_{id}(F_{rc}C_{p_c} + \lambda F_o C_{p_o} + (1-\lambda)F_o C_{p_D})} \frac{dy_f}{dz} \quad (5)$$

where the dimensionless temperature is defined as  $\theta = (T - T_0)/T_0$  and initial catalyst activity is given by

$$\theta_0 = 1 - mC_{rc} \quad (6)$$

## 2.2 Regenerator models

When modeling the regenerator it is common to assume that the temperature and the amount of coke on the catalyst is uniform throughout the regenerator. For the oxygen distribution we shall use two different types of models:

**Model 1.** The oxygen is assumed to be uniformly distributed in the regenerator dense bed. Errazu and coworkers (1979) found this model to describe experimental data well.

**Model 2.** The oxygen is assumed to move in plug flow through the regenerator. This assumption has been made by Kurihara (1967) and Krishna and Parkin (1985).

The regenerator is described by the following equations. Balance for coke on regenerated catalyst:

$$\frac{d}{dt}(W C_{rc}) = F_{rc}(C_{st} - C_{rc}) - R_{cb} \quad (7)$$

Energy balance:

$$\frac{d}{dt}(W C_{p_c} T_{rg}) = T_{st} F_{rc} C_{p_c} + T_a F_a C_{p_a} - T_{rg}(F_{rc} C_{p_c} + F_a C_{p_a}) - \Delta H_{cb} R_{cb} / M_c \quad (8)$$

where  $\Delta H_{cb}$  depends both on the temperature and  $\sigma$ , the ratio of  $CO_2$  to  $CO$  produced (Errazu and coworkers 1979). Coke has an average composition  $CH_n$ . For Model 1 the concentration of oxygen in the regenerator dense bed is given by

$$\frac{d}{dt}(W_a cO_2) = \frac{F_a}{M_a}(cO_{2i} - cO_2) - \frac{(1+\sigma)n + 2 + 4\sigma}{4(1+\sigma)} \frac{R_{cb}}{M_c} \quad (9)$$

For Model 2 the concentration of oxygen in the gas leaving the dense bed is assumed to be pseudo-steady state, and is calculated using an equation taken from Kurihara:

$$cO_2 = cO_{2i} \exp \left[ \frac{-W P_{rg} / F_a}{[1.06 \times 10^{10} / F_a^2 + 1 / (k_{or} \exp(-E_{or} / RT_{rg}) C_{rc})]} \right] \quad (10)$$

$R_{cb}$ , the rate of coke combustion, is for Model 1 found from

$$R_{cb} = k_{cb} c_{xp} \left( \frac{-E_{cb}}{RT_{rg}} \right) cO_2 C_{rc} W \quad (11)$$

whereas for Model 2 we have<sup>2</sup>

$$R_{cb} = \frac{F_a}{M_a}(cO_{2i} - cO_2) \frac{4(1+\sigma)}{(1+\sigma)n + 2 + 4\sigma} M_c \quad (12)$$

The afterburning of  $CO$  to  $CO_2$  in the dilute phase in the regenerator was ignored by Lee and Groves (1985) in their original model, which has the concentration of oxygen leaving the regenerator dense bed as an output instead. We include the afterburning by using a simple equation taken from Kurihara (1967)

$$T_{cy} = T_{rg} + c_1 cO_2 \quad (13)$$

For Eq. (13) to be reasonable, there must be an excess of  $CO$  over  $O_2$  in the gas leaving the regenerator dense bed.

## 2.3 Reactor (stripper) model

As explained previously, no reaction occurs in the reactor, which

<sup>2</sup>It should be noted that Kurihara ignored the presence of hydrogen in the coke. Clearly, for the mass balance in (12) the presence of hydrogen in the coke, represented by the parameter  $n$ , is important. The omission of hydrogen in the mass balance is repeated by Denn (1986) in his presentation of the Kurihara model when he states that a ratio of  $CO_2$  to  $CO$  of unity results in a value of  $C_1 = 2$  in his equation (5.62a).

in modern FCC's is simply used as a stripping vessel, where volatile hydrocarbons are stripped from the catalyst using steam. Assuming the stripping is effective, the only effect of the reactor will be to introduce a lag between the riser exit and the catalyst return to the regenerator. This lag is modeled using an ideal mixing tank. The balances for coke and energy yield.

$$\frac{d}{dt}(W_{st} C_{st}) = F_{rc}(C_{sc} - C_{st}) \quad (14)$$

$$\frac{d}{dt}(W_{st} C_{p_c} T_{st}) = F_{rc} C_{p_c} (T_1 - T_{st}) \quad (15)$$

The amount of catalyst in the reactor  $W_{st}$  is assumed constant by perfect control.

## 2.4 Parameter values

The parameter values used are given in the nomenclature section. The values used for Model 1 are taken from Ljungquist (1990), who has slightly modified the values given by Lee and Groves. For eq. (10) in Model 2, the parameter values are taken from Denn (1986), except for  $k_{or}$  which has been adjusted. The value of  $c_1$  is taken from Kurihara (1967).

## 3 Measures for evaluating controllability

**Right half plane transmission zeros.** A right half plane (RHP) transmission zero of  $G(s)$  limits the achievable bandwidth of the plant. This holds regardless of the type of controller used (eg., Morari and Zafiriou, 1989). The reason is that with a RHP-zero the controller can not invert the plant and perfect control is impossible. Thus plants with RHP transmission zeros within the desired bandwidth should be avoided.

In the multivariable case a RHP-zero of  $G(s)$  does not imply that the matrix elements,  $g_{ij}(s)$ , have RHP-zeros. Conversely, the presence of RHP-zeros in the elements does not necessarily imply a RHP-zero of  $G(s)$ . If we use a multivariable controller then RHP-zeros in the elements do not pose any particular problem. However, if decentralized controllers are used, then we generally avoid pairing on elements with "significant" RHP-zeros (RHP-zeros close to the origin), because otherwise this loop may go unstable if left by itself (with the other loops open).<sup>3</sup>

**Relative gain array.** The relative gain array (RGA) has found widespread use as a measure of interaction and as a tool for control structure selection for single-loop controllers. It was first introduced by Bristol (1966). It was originally defined at steady-state, but it may easily be extended to higher frequencies (Bristol, 1978). Shinskey (1967, 1984) and several other authors have demonstrated practical applications of the RGA. Important advantages with the RGA is that it depends on the plant model only and that it is scaling independent. The RGA matrix can be computed using the formula

$$\Lambda(s) = G(s) \times (G^{-1}(s))^T \quad (16)$$

where the  $\times$  symbol denotes element by element multiplication (Hadamard or Schur product). An important usage of the RGA is that pairing on negative steady-state relative gains should be avoided (Grosdidier and coworkers, 1985). The reason is that with integral control this yields instability of either 1) the overall system, 2) the individual loop, or 3) the remaining system when the loop in question is removed. It is also established that plants with large RGA-values, in particular at high frequencies, are fundamentally difficult to control irrespective of the controller used (poor controllability).

**Closed loop disturbance gain.** A disturbance measure related to the RGA, the closed loop disturbance gain (CLDG), was recently introduced by Skogestad and Hovd (1990). For a disturbance  $k$  and an output  $i$ , the CLDG is defined by

$$\delta_{ik}(s) = g_{ii}(s)[G(s)^{-1}G_d(s)]_{ik} \quad (17)$$

<sup>3</sup>Usually the main reason for using a decentralized control system in the first place is to allow for loops to be operated independently.

The reason for the name CLDG will become clear later. A matrix of CLDG's may be computed from

$$\Delta = \{\delta_{ik}\} = \bar{G}G^{-1}G_d \quad (18)$$

where  $\bar{G} = \text{diag}\{g_{ii}\}$  is the matrix consisting of only the diagonal elements of  $G(s)$ . The CLDG is scaling dependent, as it depends on the expected magnitude of disturbances and outputs. Actually, this is reasonable since CLDG is a performance measure, which generally are scaling dependent.

**Performance relationships** Assume the controller  $C(s)$  is diagonal with entries  $c_i(s)$ . This implies that after the variable pairing has been determined, the order of the elements in  $y$  and  $u$  has been arranged so that the plant transfer matrix  $G(s)$  has the elements corresponding to the paired variables on the main diagonal. Let  $y(s)$  denote the output response for the overall system when all loops are closed and let  $e(s) = y(s) - r(s)$  denote the output error. The closed loop response becomes

$$e(s) = -S(s)r(s) + S(s)G_d(s)z(s); \quad S = (I + GC)^{-1} \quad (19)$$

where  $S(s)$  is the sensitivity function for the overall system, and  $z(s)$  denotes the disturbances. The Laplace variable  $s$  is often omitted to simplify notation.

Consider the effect of a setpoint change  $r_j$  and a disturbance  $z_k$  on the offset  $e_i$ . (19) yields

$$e_i = -[S]_{ij}r_j + [SG_d]_{ik}z_k \quad (20)$$

For  $\omega < \omega_B$  we may usually assume  $S = (I + GC)^{-1} \approx (GC)^{-1}$ . Provided the corresponding cofactor of  $G$  is nonzero<sup>4</sup>, and  $c_i$  is sufficiently large, this approximation will also hold for individual elements

$$[S]_{ij} \approx \frac{[G^{-1}]_{ij}}{c_i}; \quad [SG_d]_{ik} \approx \frac{[G^{-1}G_d]_{ik}}{c_i}; \quad \omega < \omega_B \quad (21)$$

With this approximation, and assuming  $g_{ii}(s) \neq 0$  such that we may introduce the CLDG, (20) yields

$$e_i \approx -\frac{g_{ii}[G^{-1}]_{ij}}{g_{ii}c_i}r_j + \frac{\delta_{ik}}{g_{ii}c_i}z_k; \quad \omega < \omega_B \quad (22)$$

Using  $\bar{S} = (I + \bar{G}C)^{-1} \approx \text{diag}\{1/(g_{ii}c_i)\}$  this may be written on matrix form

$$e \approx -\bar{S}\bar{G}G^{-1}r + \bar{S}\bar{G}G^{-1}G_dz; \quad \omega < \omega_B \quad (23)$$

where  $\bar{G}G^{-1}G_d = \Delta$  is the CLDG matrix. For process control disturbance rejection is usually more important than setpoint tracking. From (22) we see that the ratio  $\delta_{ik}/(g_{ii}c_i)$  gives the magnitude of the offset in output  $i$  to a disturbance  $z_k$ . That is, the curve for  $|\delta_{ik}|$  should lie below  $|g_{ii}c_i|$  at frequencies where we want small offsets. A plot of  $|\delta_{ik}(j\omega)|$  will give useful information about which disturbances  $k$  are difficult to reject.

Assume that  $G$  and  $G_d$  have been scaled such that 1) the expected disturbances,  $|z_k(j\omega)|$ , are less or equal to one at all frequencies, and 2) the outputs,  $y_i$  are such that the allowed errors,  $|e_i(j\omega)|$ , are less or equal to one. In this case the frequency where  $|\delta_{ik}(j\omega)|$  crosses one, directly corresponds to the minimum bandwidth needed in loop  $i$  to reject disturbance  $k$ . It is preferable that this frequency be low.

## 4 Analysis of FCC models

We will concentrate our attention on the two control structures that have been most used in previous work on FCC control. These are the so-called conventional control structure

$$\begin{bmatrix} T_1 \\ T_{cy} \end{bmatrix} = G_C(s) \begin{bmatrix} F_{rc} \\ F_a \end{bmatrix} \quad (24)$$

and the Kurihara control structure

$$\begin{bmatrix} T_{rg} \\ T_{cy} \end{bmatrix} = G_K(s) \begin{bmatrix} F_{rc} \\ F_a \end{bmatrix} \quad (25)$$

In the following we assume that a decentralized control system is used and that  $G_C(s)$  and  $G_K(s)$  have been arranged to give the paired elements on the diagonal. That is,  $F_{rc}$  is used to control  $T_1$  (conventional) or  $T_{rg}$  (Kurihara), and  $F_a$  is used to control  $T_{cy}$ . The word "RGA" or "RGA's" will refer to the diagonal elements of the steady-state RGA-matrix of  $G(s)$ , which are identical since  $G(s)$  is a  $2 \times 2$  matrix.

## 5 Effect of different model features

### 5.1 Afterburning

Decentralized control with integral action using pairings corresponding to negative steady state RGA's are known to require that a subset of the loops are unstable for the system as a whole to be stable (Grosdidier, 1985). Hovd and coworkers (1990) therefore expressed surprise over their finding that the Lee and Groves model without afterburning, gave negative steady state RGA values for both the conventional control structure and the Kurihara control structure. However, including the simple afterburning description in (13), as done in this paper, makes the RGA values corresponding to the conventional control structure positive. For the Kurihara control structure the result depends on the operating point, as the RGA values are negative for some operating points.

Thus this simple extension of the model appears to make it more applicable as a basis for understanding industrial practice. The results for the RGA do not appear to be sensitive to the value of  $c_i$  in (13). Thus, for the purpose of control structure selection, there does not appear to be any incentive for modelling afterburning in more detail.

### 5.2 Air flow pattern in the regenerator

Model 1, which assumes oxygen to be uniformly distributed throughout the regenerator, clearly favors the conventional control structure. This is because the transfer function matrix corresponding to the conventional control structure,  $G_C(s)$ , has no multivariable RHP zeros, whereas the transfer function matrix of the Kurihara control structure,  $G_K(s)$ , has a multivariable RHP zero at a frequency about  $0.2 \text{min}^{-1}$ , which is within the same order of magnitude as the achievable bandwidth.

Also for Model 2 the conventional control structure gives no multivariable RHP zeros. For the Kurihara control structure there is a RHP zero, but it is well beyond any realistically achievable bandwidth, and therefore of no practical significance. However, for Model 2 both the conventional control structure and the Kurihara control structure correspond to negative steady state RGA's at some operating points. The change in the sign of the steady state RGA occurs because of a change in the steady state gain from  $F_a$  to  $T_{cy}$ . Such a sign change can occur more easily with Model 2 than Model 1, because the assumption of plug flow of air in the regenerator makes the oxygen concentration in the gas leaving the regenerator dense bed more temperature dependent. Thus Model 1 and Model 2 leads to very different controllability characteristics of the plant, and this demonstrates the importance of validating the model structure as well as validating model parameter estimates. Fluidized beds are complex, and it is difficult to state a priori which of the underlying assumptions in Model 1 and 2 is closest to reality. It is however clear that both the perfect mixing of air assumed in Model 1 and the plug flow of air assumed in Model 2 are idealizations which need not be fulfilled in practice.

### 5.3 Reactor (stripper) model

The inclusion of the reactor model only has an effect at frequencies around and above that corresponding to the catalyst residence time in the reactor, which in our case is about 1 min. Thus equations describing the reactor need only be included in

<sup>4</sup>Actually, the approximation also holds for cofactors of  $G$  identically equal to zero, provided the same cofactor of  $(I + GC)$  is also zero for any diagonal  $C$ . For instance, for transfer matrices which can be made triangular by row and column interchanges, this approximation will hold for all elements.

the model if a relatively high bandwidth is required of the control system. This agrees with the results of previous authors (eg. Kurihara 1967).

## 6 Effect of operating point

Although a number of different operating points have been examined, our findings can be illustrated by looking at two different operating points ("cases") for each of the two Models 1 and 2. In Table 1 we show for Model 1  $G_C(0)$ , the steady state gain matrix for the conventional control structure, and  $G_K(0)$ , the steady state gain matrix for the Kurihara control structure. We also show the corresponding steady-state RGA-values, multivariable RHP-zeros, and RIIP-zeros in the individual elements. Table 2 shows the same for Model 2.

*Model 1.* We see that for Model 1, case 1, both the conventional control structure and the Kurihara control structure correspond to positive steady state RGA values. However, for Model 1, case 2, the Kurihara control structure yields a negative RGA. This is because the steady state gain from  $F_{rc}$  to  $T_{rg}$  has changed sign. This gain change may be explained as follows: An increase in  $F_{rc}$  will initially cause  $T_{rg}$  to drop, as more heat is removed from the regenerator. However, the increased  $F_{rc}$  also causes more coke to be formed, and the temperature rises again as the amount of coke in the regenerator, and therefore also the coke combustion rate increases. The sign of the steady state gain depends on which of these effects dominates. At low  $F_{rc}$  the cooling effect dominates, whereas at higher  $F_{rc}$  the heating effect dominates. It should also be noted that at operating points where the heating effect dominates at steady state, there is a RIIP zero in the transfer function from  $F_{rc}$  to  $T_{rg}$ . For the conventional control structure the change of sign does not occur for the gain from  $F_{rc}$  to the  $T_1$ , because the riser exit temperature in addition to being dependent on the regenerator temperature also is a strong function of the catalyst to oil ratio. For the same reason no RIIP zero occurs in the transfer function between  $F_{rc}$  and  $T_1$ . For Model 1 the conventional control structure is therefore more robust with respect to changes in operating point.

*Model 2.* For Model 2, case 1, both control structures have negative steady state RGA's. This is because the steady-state gain from  $F_a$  to  $T_{cy}$  is negative, and we also have a RHP-zero (inverse response) between  $F_a$  and  $T_{cy}$ : An increase in  $F_a$  initially increases  $cO_2$  and  $T_{cy}$  increases. The increase in  $F_a$  also increases  $T_{rg}$  with a resulting increase of reaction rate for oxygen.  $cO_2$  therefore decreases, giving a reduction in  $T_{cy}$ .

For Model 2, case 2, the concentration of oxygen in the gas leaving the regenerator dense bed is lower, and there is therefore less scope for further reduction of the oxygen concentration by an increase in  $T_{rg}$ . The gain from  $F_a$  to  $T_{cy}$  is therefore positive. The result is a positive steady state RGA for both control structures. For Model 2, both structures appear sensitive to changes in the operating point.

## 7 Sensitivity to parametric uncertainty

Due to the large number of parameters in the models, the sensitivity to parameter uncertainty has not been exhaustively researched. The objective of this section is to demonstrate that small errors in the parameters can have consequences for control performance, and that parametric uncertainty therefore should be considered in the process of control structure selection.

The most severe sensitivity to parametric uncertainty has been observed for the coke production rate for the Kurihara control structure: 1) For Model 2, case 2, an increase in the rate of coke production of less than 2% (changing  $k_c$  from 0.019 to 0.0193) changes the steady state gain between  $F_{rc}$  and  $T_{rg}$  from positive to negative. 2) With a further increase in the coke production rate ( $k_c = 0.0198$ ) this gain again becomes positive. 3) Similarly, for Model 1, case 1, a decrease of  $k_c$  from 0.019 to 0.0187 changes the sign of the steady state gain between  $F_{rc}$  and  $T_{rg}$  from positive to negative. 4) Similarly, for Model 1 case 1, reducing  $m$  in

(6) from 80 to 75 also changes the sign of the steady state gain between  $F_{rc}$  and  $T_{rg}$  from positive to negative. In all the cases above with a negative gain from  $F_{rc}$  to  $T_{rg}$ , the Kurihara control structure has a negative steady state RGA.

The same change in  $m$  in Model 2, case 2, makes the steady state gain from  $F_a$  to  $T_{cy}$  negative, with the result that both the conventional and the Kurihara control structures have negative steady state RGA's.

It should now be clear that the sensitivity to parameter uncertainty has implications for control structure selection. As reflected in this section, our limited numerical experience indicates that the conventional control structure is more robust to parametric uncertainty than the Kurihara control structure.

## 8 Sensitivity to input uncertainty

Skogestad and Morari (1987b) found that large RGA elements means that the system is sensitive to uncertainty in the manipulated variables. The FCC process appears to be insensitive to this type of uncertainty, as the RGA values are small at all frequencies for all operating points examined for both Model 1 and Model 2.

## 9 Disturbances

The effect of disturbances have been investigated using the closed loop disturbance gain (CLDG) explained above. The transfer functions were scaled such that output errors,  $e_i = y_i - r_i$ , of magnitude 1 corresponds to

1. Riser exit temperature,  $T_1$ , or Regenerator dense bed temperature,  $T_{rg}$ : 3 K
2. Regenerator cyclone temperature,  $T_{cy}$ : 10 K

and such that disturbances,  $z_k$ , of magnitude 1 correspond to

1. Feed oil temperature,  $T_f$ : 5 K
2. Air temperature,  $T_a$ : 5 K
3. Feed oil flowrate,  $F_o$ : 4 kg/s (ca. 10%)
4. Feed oil composition, expressed by the coke production rate factor  $k_c$ : 2.5 % relative to original value.

The CLDG's consistently predict that disturbance  $k = 3$  in the feed oil flowrate,  $F_o$ , is most difficult reject, followed by disturbances 1 (in  $T_f$ ) or 4 (in  $k_c$ ). Disturbance 2 (in  $T_a$ ) appears to have very little effect. The CLDG's give a bandwidth requirement for rejecting a disturbance in  $F_o$  in the range 0.1 - 0.4 rad/min for both models and control structures. The CLDG's for Model 1, case 1 (both structures) are shown in figure 2. The curves for case 2 are very similar. It seems that disturbance rejection is somewhat easier with the Kurihara structure. However, it should be noted from Table 1 that for Model 1 the Kurihara structure has a RHP-zero at a frequency of about 0.2 rad/min, and this will limit the achievable bandwidth, and the conventional structure is preferable. The predictions based on the CLDG's have been shown to hold in closed loop simulations.

## 10 Effect of controlling the riser temperature

In almost all industrial FCC's a single-loop controller is used between the variables  $F_{rc}$  and  $T_1$ , the riser temperature (Grosdidier, 1990). Thus the Kurihara control structure does not seem to be used in practice. Based on our findings in the previous sections it also makes sense to choose the conventional control structure, as the results obtained indicate the conventional control structure to be more robust than the Kurihara control structure. In this section we study the control behavior if the FCC with the loop  $F_{rc} - T_1$  closed. We want to study the robustness of this loop and it's effect on the transfer function

from  $F_a$  to  $T_{cy}$ .

In the models used in this work, there is no fundamental limitation on the bandwidth for the control loop  $F_{rc} - T_1$ . In practice, there will obviously be various effects that we have neglected which will limit the bandwidth. We have used a controller for the loop  $F_{rc} - T_1$  which gives a closed-loop bandwidth of about 1 rad/min.

$$F_{rc}(s) = 1.23 \frac{(0.56s + 1)}{0.56s} T_1(s) \quad (26)$$

This yields a stable control loop at all the operating points studied, for both model alternatives. Closing this loop makes the steady state gain from  $F_a$  to  $T_{cy}$  positive in all cases for both models, and there are no RHP zeros in the transfer function from  $F_a$  to  $T_{cy}$ . However, for the operating points/parameters which result in a negative steady state RGA (eg., Model 2, case 1), the system will become unstable if the loop  $F_{rc} - T_1$  fails.

## 11 Conclusion

The conventional control structure is found to be more robust to changes in operating points and changes in parameters than the Kurihara control structure. This is consistent with industrial operation. Closing the loop between flow of catalyst ( $F_{rc}$ ) and riser temperature ( $T_1$ ) with a simple PI controller gave a stable system for all the operating points and parameters we have been studied. However, Model 2 predicts that the system will not be able to tolerate failure of the loop  $F_{rc} - T_1$  at all operating points. For the conventional control structure, oil feedrate disturbances at high frequencies will have some effect on the riser exit temperature. Otherwise, the bandwidth requirements for disturbance rejection are similar for the conventional and the Kurihara control structures. For the Kurihara control structure the required bandwidth can be difficult to achieve at some operating points, because of RHP transmission zeros in the same range of frequencies.

For any model to be used to study control of a real FCC process, both the model structure and parameter values need first be validated against experiments on the actual plant.

## Nomenclature

### FCC models:

- $C_{cat}$  - Catalytic coke produced in riser, mass fraction
- $C_{rc}$  - Coke on regenerated catalyst, mass fraction
- $C_{sc}$  - Coke on catalyst leaving riser, mass fraction
- $C_{st}$  - Coke on catalyst in reactor (stripper), mass fraction
- $Cp_a$  - Heat capacity of air (1.074 kJ/kgK)
- $Cp_c$  - Heat capacity of catalyst (1.005 kJ/kgK)
- $Cp_o$  - Heat capacity of oil (3.1355 kJ/kgK)
- $Cp_D$  - Heat capacity of steam (1.9 kJ/kgK)
- [COR] - Catalyst to oil ratio on a mass basis
- $cO_2$  - Concentration of oxygen in gas leaving regenerator dense bed, molefraction
- $cO_{2i}$  - Concentration of oxygen in air to regenerator (0.2136 molefraction)
- $c_t$  - Factor in Eq. (13) (5555 K/molefraction)
- $E_{cf}$  - Activation energy for coke formation (41.79 kJ/mole)
- $E_{cb}$  - Activation energy (Model 1) for coke combustion (158.59 kJ/mole)
- $E_f$  - Activation energy for the cracking of gas oil (101.5 kJ/mole)
- $E_g$  - Activation energy for the cracking of gasoline (112.6 kJ/mole)
- $E_{or}$  - Activation energy (Model 2) for coke combustion (146.4 kJ/mole)
- $F_a$  - Flowrate of air to the regenerator (kg/s)
- $F_o$  - Total feed rate, feed oil plus dispersion steam (42.05 kg/s)
- $F_{rc}$  - Flow rate of catalyst (294 kg/s)
- $k_1$  - Reaction rate constant for the total rate of cracking of gas oil ( $9.6 \times 10^5 s^{-1}$ )
- $k_2$  - Reaction rate constant for the rate of cracking of gas oil to gasoline ( $7.2 \times 10^5 s^{-1}$ )
- $k_3$  - Reaction rate constant for the rate of cracking of gasoline to light fuses/carbon ( $4.22 \times 10^5 s^{-1}$ )
- $k_c$  - Reaction rate constant for the production of coke (0.019 s<sup>-1</sup>)
- $k_{cb}$  - Reaction rate constant (Model 1) for coke combustion ( $2.077 \times 10^8 s^{-1}$ )
- $k_{or}$  - Reaction rate constant (Model 2) for coke combustion ( $58.29 m^2/(sN)$ )
- $M_a$  - Molecular weight of air (28.8544)
- $M_c$  - Bulk molecular weight of coke (14)

- $m$  - Factor for the dependence of the initial catalyst activity on  $C_{rc}$  (80)
- $n$  - Number of moles of hydrogen per mole of carbon in the coke (2)
- $R$  - Universal gas constant
- $R_{cb}$  - Rate of coke combustion (kg/s)
- $P_{rg}$  - Regenerator pressure (172000 N/m<sup>2</sup>)
- $T_0$  - Temperature at riser entrance
- $T_1$  - Temperature at riser exit
- $T_a$  - Temperature of air to the regenerator
- $T_{cy}$  - Regenerator cyclone temperature
- $T_{st}$  - Temperature in reactor (stripper)
- $t_c$  - Residence time in riser (9.6s)
- $W$  - Holdup of catalyst in regenerator (176000 kg)
- $W_a$  - Holdup of air in the regenerator (20 kmol)
- $W_{st}$  - Holdup of catalyst in reactor (17500 kg)
- $y_f$  - Mass fraction of gas oil
- $y_g$  - Mass fraction of gasoline
- $z$  - Dimensionless distance along riser
- $\alpha$  - Catalyst deactivation constant (0.12 s<sup>-1</sup>)
- $\Delta H_{cb}$  - Heat of combustion of coke (kJ/kmol)
- $\Delta H_f$  - Heat of cracking (506.2 kJ/kg)
- $\eta$  - Exponent for the dependence of  $C_{cat}$  on  $C_{rc}$  (0.4)
- $\lambda$  - Mass fraction of gas oil in feed (0.9662)
- $\sigma$  - Molar ratio of  $C_2O_2$  to  $CO$  in the regenerator dense bed
- $\theta_0$  - Initial catalyst activity at riser entrance
- $\Theta$  - Dimensionless temperature in riser

## Frequency analysis:

- $C(s)$  - Diagonal controller transfer function matrix
- $c_i(s)$  - Controller element for output  $i$
- $e(s) = y(s) - r(s)$  - Vector of output errors
- $G(s)$  - Process transfer function matrix
- $G_C(s)$  - Transfer function matrix for the conventional control structure
- $G_K(s)$  - Transfer function matrix for the Kurihara control structure
- $G_d(s)$  - Disturbance transfer function matrix
- $g_{ij}(s)$  -  $ij$ 'th element of  $G(s)$
- $g_{dik}(s)$  -  $ik$ 'th element of  $G_d(s)$
- $r(s)$  - Reference signal for outputs
- $S(s)$  - Sensitivity function  $S = (I + GC)^{-1}$
- $y(s)$  - vector of outputs
- $\Delta(s)$  - Closed loop disturbance gain matrix
- $\delta_{ik}(s)$  -  $ij$ 'th element of  $\Delta(s)$
- $\Lambda(s)$  - Relative gain matrix
- $\lambda_{ij}(s)$  -  $ij$ 'th element of  $\Lambda(s)$
- $\omega$  - Frequency
- $\omega_B$  - Closed loop bandwidth

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TABLE 1 Operating Points used in Section 6 for Model 1

	Case 1	Case 2
$T_{rg}$	965.4 K	966.6 K
$T_1$	776.9 K	770.6 K
$T_{cy}$	988.1 K	997.4 K
$C_{rc}$	$5.207 \times 10^{-3}$	$3.578 \times 10^{-3}$
$G_C(0)$	$\begin{pmatrix} 0.5587 & 10.16 \\ -0.5577 & 10.35 \end{pmatrix}$	$\begin{pmatrix} 0.3893 & 10.83 \\ -0.7606 & 8.22 \end{pmatrix}$
RGA	0.505	0.280
RHP zeros ( $\text{min}^{-1}$ )	-	-
Multivariable	-	-
In elements	$\begin{pmatrix} - & - \\ - & - \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & - \end{pmatrix}$
$G_K(0)$	$\begin{pmatrix} 0.0124 & 18.05 \\ -0.5577 & 10.35 \end{pmatrix}$	$\begin{pmatrix} -0.1661 & 19.78 \\ -0.7606 & 8.22 \end{pmatrix}$
RGA	0.042	-0.010
RHP zeros ( $\text{min}^{-1}$ )	-	-
Multivariable	0.19	0.23
In elements	$\begin{pmatrix} 0.001 & - \\ - & - \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & - \end{pmatrix}$

- denotes that no RHP-zero is present at frequencies below  $100 \text{ min}^{-1}$ .

TABLE 2 Operating Points used in Section 6 for Model 2

	Case 1	Case 2
$T_{rg}$	966.1 K	983.2 K
$T_1$	777.3 K	783.7 K
$T_{cy}$	987.3 K	993.1 K
$C_{rc}$	$5.039 \times 10^{-3}$	$4.409 \times 10^{-3}$
$G_C(0)$	$\begin{pmatrix} 2.915 & 36.73 \\ -5.571 & -49.10 \end{pmatrix}$	$\begin{pmatrix} 0.684 & 12.72 \\ -1.121 & 3.44 \end{pmatrix}$
RGA	-2.32	0.142
RHP zeros ( $\text{min}^{-1}$ )	-	-
Multivariable	-	-
In elements	$\begin{pmatrix} - & - \\ - & 0.20 \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & - \end{pmatrix}$
$G_K(0)$	$\begin{pmatrix} 4.056 & 63.31 \\ -5.571 & -49.10 \end{pmatrix}$	$\begin{pmatrix} 0.262 & 22.95 \\ -1.121 & 3.44 \end{pmatrix}$
RGA	-1.30	0.034
RHP zeros ( $\text{min}^{-1}$ )	-	-
Multivariable	-	-
In elements	$\begin{pmatrix} - & - \\ - & 0.20 \end{pmatrix}$	$\begin{pmatrix} 0.006 & - \\ - & - \end{pmatrix}$

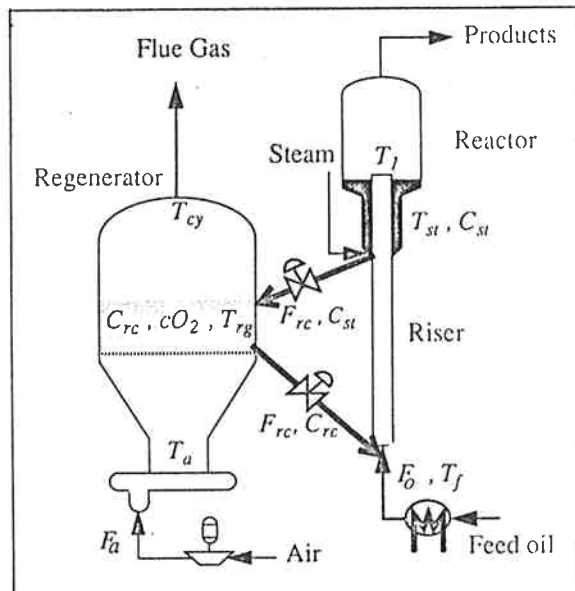


Fig. 1. Schematic overview of FCC plant.

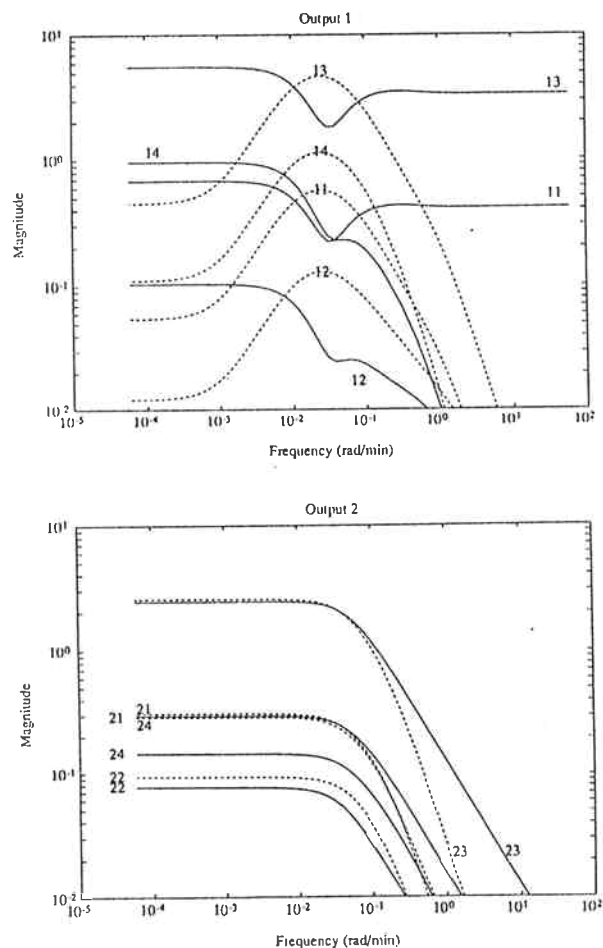


Fig. 2. Closed loop disturbance gains,  $\delta_{ik}$ , of FCC process for Model 1 case 1. A solid line is used for the conventional control structure and a dashed line for the Kurihara control structure.  $i$  denotes output and  $k$  disturbance. The variables are scaled such that the frequency where the curve crosses magnitude 1 gives the approximate bandwidth requirement,  $\omega_B$ , for rejecting this disturbance.