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# Modelling and Identification for Robust Control of Ill-Conditioned Plants - a Distillation Case Study

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**Abstract**

The problem of obtaining linear models for effective control is discussed. First a good model structure is identified by comparing different models. It is demonstrated that the high-frequency behavior (initial time response) of the model is much more important for controller design than the steady-state characteristics. Finally, having determined a good model structure, the problem of obtaining models from experiments is discussed. It is shown that by combining experimental open-loop step responses with theoretically established structural properties one may obtain reasonably good models.

## 1 Introduction

When obtaining dynamic models for process control one is usually interested in simple linear models. In this paper we discuss how such models may be obtained. We investigate which effects that should be included in a model to be used for design of feedback controllers, and subsequenctly discuss how the models may be obtained from experiments.

It is stressed that we are looking for a multivariable model. The main difference between single-input-single-output (SISO) models and multivariable models is the presence of "directions" or "interactions" in the latter case. It seems likely that our model should capture these multivariable effects in a reasonable way. From the Ziegler-Nichols tuning rules, which are widely used for SISO control, we know that the plant behavior at high frequencies (at "crossover" , ie. where the plant has a phase lag of about 180°) is of primary importance for feedback control. This fact often seems to be forgotten when developing multivariable control models, and engineers often emphasize the steady-state behavior. There are probably two reasons for this: 1) Steady-state gain data are easily obtained, 2) Up to now most tools for analyzing directions and interactions, for example the Relative Gain Array [4], have been used at steady state only. In this paper we want to demonstrate that also for multivariable plants it is the high-frequency behavior, and not the steady state, which is of primary importance for feedback control. However, we shall see that it is important that the sign of the plant (experessed by the sign of the determinant or sign of the RGA-elements) is correct at steady state.

Distillation control is used as a case study throughout the paper. Composition control of distillation columns (Fig. 1) has proven difficult to implement in practice. One-point control (one composition under feedback control and the other uncontrolled) is fairly common, while two-point control (feedback control of both compositions) is rarely used. One reason for this is that on-line tuning of two composition loops on a strongly interacting distillation column is very difficult (eg., [19]). It is therefore desirable to obtain controller tunings based on some model of the column.

The linear model of the plant is written

$$\begin{pmatrix} dy_D(s) \\ dx_B(s) \end{pmatrix} = G(s) \begin{pmatrix} dL(s) \\ dV(s) \end{pmatrix} \quad (1)$$

where $G(s)$ is a $2 \times 2$ transfer matrix expressing the effect of small changes in the independent flows on the compositions. Note that we have assumed that the product flows $D$ and $B$ are used for level control such that reflux $L$ and boilup $V$ are used for composition control. This corresponds to the $LV$-configuration. This may certainly not be the best configuration for two-point control, but it is the most commonly used configuration in industry; probably because it works well for one-point control [17].

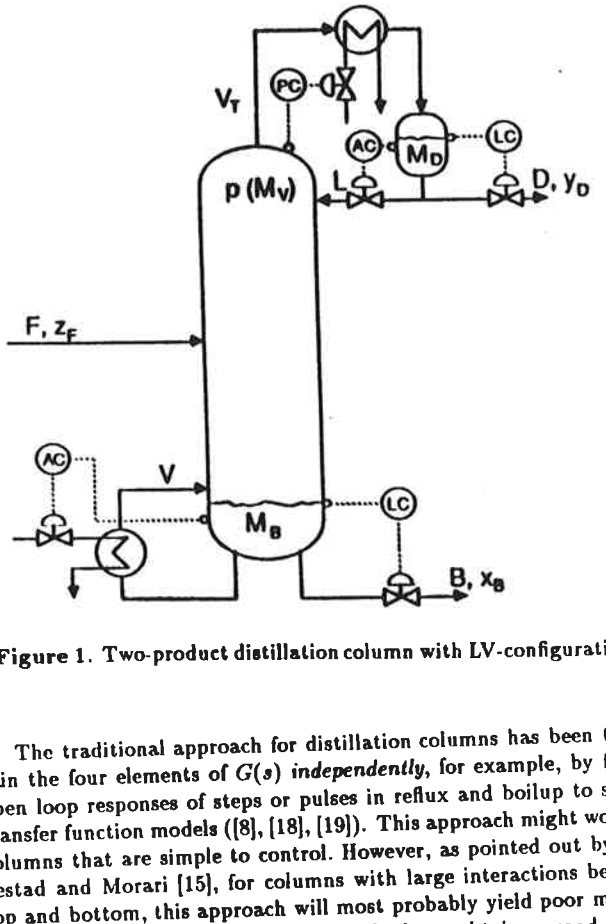


**Figure 1.** Two-product distillation column with LV-configuration.

The traditional approach for distillation columns has been to obtain the four elements of $G(s)$ *independently*, for example, by fitting open loop responses of steps or pulses in reflux and boilup to simple transfer function models ([8], [18], [19]). This approach might work for columns that are simple to control. However, as pointed out by Skogestad and Morari [15], for columns with large interactions between top and bottom, this approach will most probably yield poor models. The main reason is that it is very difficult to obtain a good model based on open-loop experiments or simulations unless one explicitly takes into account the expected couplings between the elements when formulating the model. In particular, one is not able to obtain a good model of the low-gain direction of the plant ([15], [1]). However, as we show in this paper, models obtained from fitting individual step responses may work if: 1) one uses a reasonable model structure, 2) fits only the initial part of the response and 3) corrects the obtained model according to theoretically established model properties.

We start the paper by evaluating what kind of modelling detail that is needed in a model used for controller design (part I). We then use the information gained here to fit models from experimental data (part II).

# Part I: Modelling Requirements

## 2 Nonlinear model

Data for the example column ("column A") are summarized in Table 1. The column has 40 theoretical trays (N-1 trays and a reboiler) plus a total condenser. The following modelling assumptions are used: binary separation, constant relative volatility, constant molar flows (no energy-balance), negligible vapor holdup, and vapor-liquid equilibrium as well as perfect mixing on all stages. Neglecting the vapor holdup implies immediate vapor flow responses throughout the column. Note that liquid flow-dynamics are not neglected; a simple linear relation is assumed between liquid holdup and liquid flow:

$$M_i = M_i^o + \tau_{Li}(L_i - L_i^o) \quad (2)$$

where superscript o denotes nominal steady-state value given in Table 1. $M_i^o/F^o = 0.5$ min. for all stages, including reboiler and condenser. The reboiler and condenser holdups are controlled by bottoms and distillate rate respectively. Note that the $LV$-configuration is insensitive to how tight these loops are tuned. $\tau_{Li}$=0.063 min. for all stages, except for the reboiler and condenser. The variation in liquid holdup with liquid load yields an effective deadtime $\theta_L$=2.46 min. between a change in liquid flow rate at the top and at the bottom of the column. These modelling assumptions gives for each tray two nonlinear differential equations, one for composition and one for liquid holdup, resulting in a total of 82 states for the column. A linear model, denoted "full" linear model in the following, is obtained by linearizing the nonlinear model around the nominal steady-state.

## 3 Simple linear models

### 3.1 Reduced models from physical insight

It is well known that the composition dynamics in distillation columns may be well approximated by first order responses. Skogestad and Morari [15] found that there in general will be two time-constants; one for the strong direction of the plant (change in external flows) and one for the weak direction (change in internal flows). We will use this as a model basis in the following, but will in addition introduce the flow-dynamics (2):

$$G(s) = \begin{pmatrix} \frac{k_{11}}{1+\tau_1 s} & \left(\frac{k_{11}+k_{12}}{1+\tau_2 s} - \frac{k_{11}}{1+\tau_1 s}\right) \\ \frac{k_{21}}{1+\tau_1 s} g_L(s) & \left(\frac{k_{21}+k_{22}}{1+\tau_2 s} - \frac{k_{21}}{1+\tau_1 s}\right) \end{pmatrix} \quad (3)$$

$$g_L(s) = \frac{1}{(1+(\theta_L/n)s)^n} \quad (4)$$

The numerical values used are:

$$G(0) = K = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} = \begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix} \quad (5)$$

$$\tau_1 = 194 \text{ min}; \quad \tau_2 = 15 \text{ min}; \quad \theta_L = 2.46 \text{ min}; \quad n = 5 \quad (6)$$

The flow-dynamics is described by (4) where $n$ is the number of trays in the column. $n$ should equal $N-1$, but is troughout this paper chosen to be 5 do avoid models of unnecessary high order. The steady-state gains $k_{ij}$ and $\tau_1$ were obtained from the full linear model. $\tau_2$ is taken from [15] and were obtained from fitting a model without flow dynamics. When flow dynamics are included in the model, the dynamics for changes in internal flows becomes somewhat more complicated, and there should be two poles in addition to a minimum phase zero close to the imaginary axis. This is caused by the fact that there will be a temporary change in the external flows when $L$ and $V$ are changes equally, and there will be a marked overshoot in the response of the bottom composition.

To study what level of modelling detail that is needed for a good model to be used for control we will study the following models:

**N1:** $\tau_2 = \tau_1$=194 min, $\theta_L = 0$. Simplest model with only the dominant time constant $\tau_1$ and neglected flow dynamics.

**F1:** $\tau_2 = \tau_1$=194 min. One time-constant model with flow dynamics.

**F2:** Two time-constant model (Eq.3) with $\tau_1$, $\tau_2$ and $\theta_L$.

**F1X:** Same as F1, but with gains and time-constant reduced by a factor of 10.

Here "N" denotes no flow-dynamics, "F" denotes flow-dynamics, and "1" and "2" denotes one- and two-time-constant model respectively. Note that all these models, except F1X, are identical to the full linear model at steady-state, but the high-frequency dynamics differ. Model $F1X$ will be identical to $F1$ at high frequencies, but the steady-state gains are incorrect. Also note that none of the models have multivariable zeros in the right half plane.

Table 1. Steady-state data for distillation column example (Column A). Feed is liquid.

| $z_F$ | $\alpha$ | $N$ | $N_F$ | $1-y_D$ | $x_B$ | $D/F$ | $L/F$ | $V/F$ |
|---|---|---|---|---|---|---|---|---|
| 0.5 | 1.5 | 40 | 21 | 0.01 | 0.01 | 0.500 | 2.706 | 3.206 |

### 3.2 Mathematical model reduction

There exists many methods for reducing the order of a linear dynamic model. Most of the methods are based on computing the Hankel singular values[1], and then removing states corresponding to relatively small singular values. States with relatively small Hankel singular values corresponds to states having little effect on the input/output behavior of the system. In this work we applied 4 different methods to the full linear model of column A. All the methods are implemented in one of the MATLAB toolboxes:

**B1:** Balanced Truncation Approximation [9]. (Robust Control Toolbox [3]).

**B2:** Balanced Truncated Approximation without balanced minimal realization [10]. (Robust Control Toolbox).

**H1:** Hankel Norm Approximation [5]. ($\mu$-Toolbox [2]).

**H2:** Optimal Hankel Approximation without balancing [11]. (Robust Control Toolbox).

Method 1, 2 and 4 posess the same infinity-norm error bound for a reduced model of order k of and n-th order system, while method 3 guarantees half of this error. Method 1 and 2 are quite similar, as are 3 and 4, but method 1 and 3 use a balanced realization of the original model while method 2 and 4 do not.

Due to their similarities in guaranteed maximum error one might expect that all methods yields similar reduced models. However, when reducing the model of column A from 82 to 2 states[2] we found that the methods yielded very different models. Method 2 and 4 (though quite different) gave significantly better models than method 1 and 3. (see section 5 on controller design.)

The main reason why method 2 and 4 yield the best results is probably that they do not use the numerically ill-conditioned minimal realization step [11] which is used by method 1 and 3. The results we find here may of course be case dependent, but at least they demonstrate that model reduction methods should be chosen with caution.

Reducing the full nonlinear model of column A from 82 to 2 states with the algorithm described in [11] (method 4) yields the model $H2(s)$=

$$\frac{1}{(1.61s+1)(194s+1)} \begin{pmatrix} 0.871(2.17s+1) & -0.861(0.721s+1) \\ 1.089(0.45s+1) & -1.101(3.48s+1) \end{pmatrix} \quad (7)$$

[1]The Hankel Singular Values are square roots of eigenvalues of $PQ$, where $P$ and $Q$ are the controllability and reachability grammians of the system.

[2]At least two states are needed to describe the decoupling at high frequencies.

We see that the gains and the dominant time-constant are similar to the full linear model, but the model structure is quite different from those obtained from physical insight. The decoupling at high-frequencies due to the flow-dynamics is described by making the zeros in the off-diagonal elements larger than in the diagonal elements.

## 4 Analysis of the models

In order to evaluate the quality of the above five reduced models (N1,F1,F1X,F2,H2) we will consider: 1) Open-loop simulations, 2) The Relative Gain Array, and 3) Robust Controller design.

In the two first cases we use the model as a plant and analyze the behavior of the model. In the last case we use the reduced models for controller design and analyze how the resulting controller works on the full-order model. Since the goal is to find a model that is good for controller design, case 3 is of primary importance.

### 4.1 Open-loop simulations

#### 4.1.1 Small changes in external flows

Open-loop responses in $y_D$ and $x_B$ to a step change in $L$ (external flows) using the five linear models are shown in Fig.2. We see that the models $N1$, $F1$, $F2$ and H2 are almost indistinguishable from the full linear model. The "wrong" model $F1X$ have a correct initial response, but the steady state gains are too low. The response for a step change in $V$ is not shown, but would yield the same conclusion. The fact that the simple model $N1$ gives such a good fit implies that the responses are essentially 1.order with a time-constant of 194 min.

Most engineers would use these responses to compare the models, and would thus have concluded that only $F1X$ is a poor model.

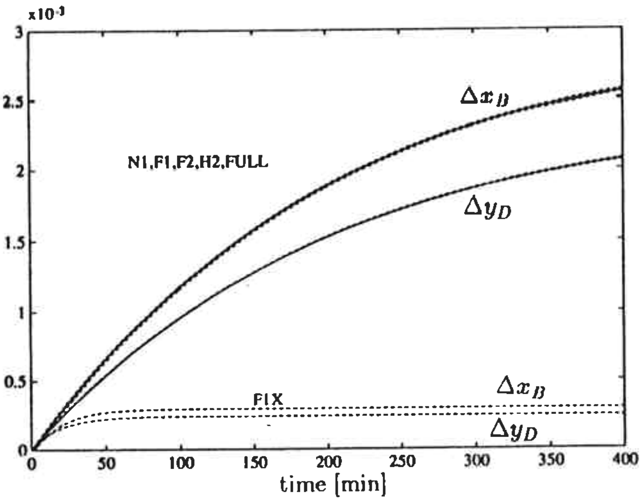


**Figure 2.** Open-loop linear responses of various models to a 0.1% increase in reflux.

#### 4.1.2 Small changes in internal flows

Figure 3 shows reponses in $y_D$ and $x_B$ to simultanous and equal changes in $L$ and $V$ (internal flows). We note that there are significant differences between the models. Model $F2$ and H2 seems to be closest to the full model, especially for the initial response. Note that the steady-state gain for changes in internal flows is about one hundreth of that for external flows. This is the reason for the ill-conditioning (high interactions) of the plant. In a real plant operating open-loop, the effect of external flows will dominate and it will not be possible to observe effects of changes in internal flows alone. However, under feedback control the effect of internal flows may be observed also in practice. The reason is that the controller in order to keep the compositions constant may have to make large changes in internal flows and thereby amplify the importance of the low-gain direction.

We rank the models from these simulations $F2, H2 > F1 > F1X > N1$.

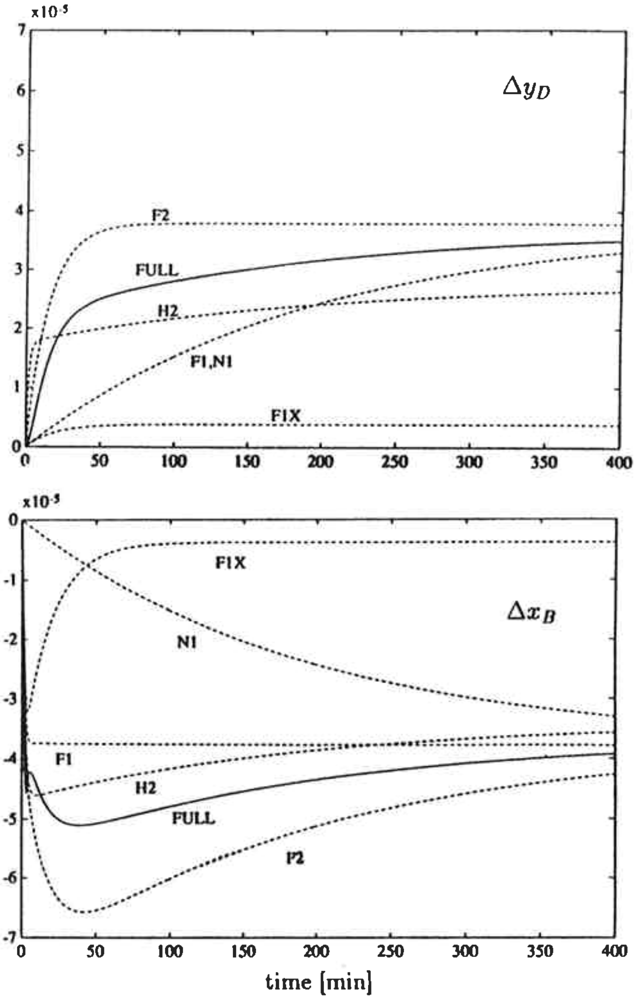


**Figure 3.** Open-loop linear responses of various models to a simultanous increase in $L$ and $V$. $\Delta L = \Delta V = 0.0027$.

### 4.2 Relative Gain Array

The 1,1 element of the RGA for 2x2 systems is given by

$$\lambda_{11}(j\omega) = \left(1 - \frac{g_{12}(j\omega)g_{21}(j\omega)}{g_{11}(j\omega)g_{22}(j\omega)}\right)^{-1} \quad (8)$$

The RGA has traditionally been evaluated at steady-state only [4], but more recently the usefulness of the RGA plotted as a function of frequency has become clear. For example, Skogestad and Morari [13] argue that the value of the RGA at frequencies close to the expected closed-loop bandwidth is a good indicator of expected control performance and that large values indicate a plant that is fundamentally difficult to control. One important property of the RGA is that it is scaling independent. This means that that the RGA-elements are unchanged if actuator (valve) or measurement dynamics are included. For example, the RGA would be unchanged if we added the same time delay to each column or row of $G(s)$.

The magnitude of $\lambda_{11}$ is shown as a function of frequency in Figure 4. At low frequenies the correct value is 35, and it falls down to 1 at high frequency where the responses becomes decoupled due to the flow-dynamics. Models $N1$, $F1$, $F1X$ and $F2$ yields a correct steady-state value, $\lambda_{11}(0) = 35$, while model H2 yields a steady-state value of 43. Model $N1$ has a constant value of 35 at all frequencies which clearly is incorrect. Model H2, while being incorrect at steady-state, yields a reasonably correct value of the RGA at high frequencies. Models $F1$, $F1X$ and $F2$ gives a reasonably correct value of the RGA at all frequencies. Note that $F1$ and $F1X$ yields identical RGA-values as they only differ by a scaling.

We rank the models based on the RGA-plot: $F2, H2 > F1, F1X > N1$

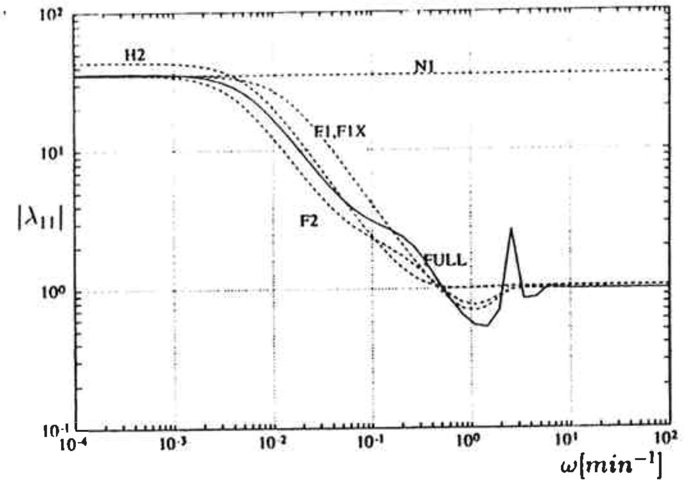


**Figure 4.** $\lambda_{11}$ as a function of frequency for various models.

## 5 Controller design

Here we shall use the various models to design single-loop PID controllers, that is, top composition $y_D$ controlled by reflux $L$ and bottom composition $x_B$ controlled by boilup $V$. Single loop controllers are a preferred choice in the industry, and seems to be a good one for the $LV$-configuration if tuned properly [16].

To allow for differences also between the full linear model and the true plant we shall include model uncertainty in the controller design. The structured singular value, $\mu$, is then a reasonable performance index. Robust performance (worst case response acceptable) is satisfied if $\mu_{RP}$ is less than 1, and designs with low $\mu_{RP}$-values are preferable. Uncertainty and performance are defined as in [14]. The uncertainty corresponds to 1 minute deadtime and 20 % uncertainty in each input. The performance requirement corresponds to a maximum peak of 2 on the sensitivity function and a closed-loop time-constant of 20 min. Single-loop PID's were tuned for the various models by minimizing $\mu_{RP}$. The results are summarized in Table 2. Comparing the results for model $N1$, $F1$ and $F2$ we see that the optimal $\mu_{RP}$-value becomes lower as flow dynamics ($\theta_L$) and internal flow dynamics ($\tau_2$) are included. This may be explained in terms of lower RGA-values at high frequency which makes control easier.

**Table 2.** $\mu_{RP}$-optimal PID-tunings for various column models. $C_{PID}(s) = k\frac{1+\tau_I s}{\tau_I s}\frac{1+\tau_D s}{1+0.1\tau_D s}$.

| Model | $\mu_{RP}$ | $k_y$ | $k_x$ | $\tau_{Iy}$ min | $\tau_{Ix}$ min | $\tau_{Dy}$ min | $\tau_{Dx}$ min |
|---|---|---|---|---|---|---|---|
| N1 | 1.32 | 438 | 130 | 178.86 | 1.87 | 0.32 | 0.23 |
| F1 | 0.91 | 85 | 38 | 7.77 | 3.61 | 0.81 | 1.11 |
| F1X | 1.12 | 96 | 93 | 2.48 | 2.23 | 0.68 | 1.10 |
| F2 | 0.80 | 38 | 36 | 6.49 | 5.80 | 1.13 | 0.91 |
| H1 | 0.95 | 2.2 | 10.8 | 0.73 | 0.92 | 1.20 | 4.98 |
| B1 | 1.23 | 6.0 | 13.9 | 2.08 | 2.53 | 1.26 | 0.23 |
| H2 | 0.78 | 32 | 34 | 4.39 | 4.14 | 1.03 | 0.41 |
| B2 | 0.80 | 36 | 35 | 5.48 | 4.52 | 1.45 | 0.40 |
| Full | 0.86 | 22 | 32 | 3.51 | 4.71 | 1.22 | 0.61 |

However, it is of course of no practical significance how well the *models* may be controlled; the interesting point is how well the controllers tuned based on the simplified model performs on the full-order model. This is summarized in Table 3. We see that the controllers based on the models $F1$, $F2$ and H2 peform well, with H2 and $F2$ giving the best design. Model $N1$ gives a poor controller for the full plant; it will in fact give an unstable system. We see that model $F1X$ which is close to the full model only the initial 10 minutes of the open-loop response yields a far better controller than $N1$.

**Table 3.** $\mu_{RP}$-values for PID-controllers of Table 2 when applied to original simplified model and to full linear model.

| Controller (Table 2) | $\mu_{RP}$ Original | $\mu_{RP}$ Full model |
|---|---|---|
| N1 | 1.32 | 2.37 |
| F1 | 0.91 | 1.16 |
| F1X | 1.12 | 1.44 |
| F2 | 0.80 | 0.95 |
| H1 | 0.95 | 1.24 |
| B1 | 1.23 | 1.28 |
| H2 | 0.78 | 0.94 |
| B2 | 0.80 | 0.98 |
| Full | 0.86 | 0.86 |

To compare the four different mathematical methods for model reduction discussed in section 3.2, we also designed controllers for models reduced from 82 to 2 states with each method. The results are given in Table 3 and 4. The results confirm that the methods *not* using the minimal balanced realization step (B2 and H2) yield controllers performing well on the full plant. The two other methods (B1 and H1) yield controllers that perform reasonably well on the full plant, but significantly poorer than B2 and H2. For the methods B1 and H1 one would have to include more states in the reduced models to get satisfactory results.

## 6 Conclusions on modelling requirements

From the analysis presented above we conclude that models $F2$ and H2 seems to be the best models to be used for controller design. Most engineers would have considered mainly the open-loop responses for changes in external flows (see Fig.2), and would then conclude that $N1$ seemed to be a reasonable model. However, as the RGA analysis demonstrated, model $N1$ is incorrect at high-frequencies and therefor useless for controller design. Model $F1X$ which is correct at high frequencies but incorrect at steady-state is in fact a much better model for controller design. These results simply support the well known fact that the high-frequency behavior is much more important than the steady-state properties for control. An important conclusion is that the flow-dynamics should be included in a model to be used for controller design.

# Part II: Model identification from experiments

In this section we discuss how good and simple models may be obtained from open-loop experiments. From part I. we concluded that $F2$ and H2 were the best models. However, estimating $\tau_2$ in model $F2$ will be difficult experimentally. Furthermore, the structure of model H2 is not physically motivated, and will be difficult to fit from experiments. However, model $F1$ seemed to be a reasonably good model structure, and should be well suited to fit from open-loop experiments. We will therefor use an $F1$-type model structure in the model identification.

All experiments are based on open-loop responses from the nonlinear model doing step changes in one input at the time. This may seem a risky way to obtain a model as the plant is ill-conditioned and small errors in the gains may yield an erronous model. On this background Andersen et.al. [1] suggested using multivariable experiments. However, step responses are clearly the easiest way to obtain a model, and as we shall see, they may yield a good model if used together with theoretically established structural properties. The approach we suggest to use when obtaining the model is:

1. Fit only the initial part of the responses. This is motivated by the fact that we are mostly interested in capturing the high-frequency behavior correct. Furthermore, the initial part of the response is less affected by nonlinearities than the reponse towards steady-state [15].

2. Make sure that the flow-dynamics are captured in the model. If there are several other deadtimes/lags in the process, one should make sure that the sum of lags in the off-diagonal elements are larger (actually $\theta_d$ larger) than the sum in the diagonal elements. $\theta_d$ may be obtained individually by measuring the time it takes for a change in liquid flow from the top to reach the bottom of the column.

3. Check the sign of the model at steady-state. It is important that the sign is correct also at steady-state. This may be checked by computing $\lambda_{11}(0)$ which should be greater than 1 for almost any column with the $LV$-configuration. Due to the ill-conditioning at steady-state, only small errors in the individual steady-state gains may yield an incorrect sign in the gain of the weak direction (corresponding to internal flows). If the steady-state sign of the model is incorrect ($\lambda_{11}(0) < 1$), change one of the individual steady-state gains so that the sign becomes correct. Correct also the corresponding time-constant so that the high-frequency dynamics are unchanged. Due to the flow-dynamics, the column will not be ill-conditioned at high frequencies, and it is therefor likely that the high-frequency directions will be correctly captured.

We will demonstrate this approach by obtaining step responses from the nonlinear model.

**Experiment 1.** In this experiment we use the $LV$-configuration and do step responses in reflux and boilup separately. We do a pertubation of 0.1 % in each input. This is of course an unrealistic small step change from a practical point of view, but as we shall see, even in this ideal case one may easily end up with an erronous model. As we are mostly interested in the high-frequency behavior of the column we fit only the initial 60 minutes of the responses. The model thereby obtained is

$$\hat{G}(s) = \begin{pmatrix} \frac{0.728}{1+164s} & \frac{-0.903}{1+224s} \\ \frac{0.949}{1+154s} g_L(s) & \frac{-0.933}{1+144s} \end{pmatrix}, \theta_L = 2.5min. \quad (9)$$

Note that we have used four different time-constants. This was done to get the best fit of the responses. Figure 5 shows the responses of the nonlinear model together with the responses of the fitted model. Note that the step changes where made so that the compositions increased in both experiments, ie. we increased $L$ and decreased $V$. From the figure we see that the initial 60 minutes are well fitted by the model, while there are quite large deviations at steady-state. The fitted model predicts a larger steady-state gain for top composition $y_D$ than found for the nonlinear response. This may be explained by the fact that the gain in the process decreases with increased purity ($y_D$) (e.g., [15]), and this is reflected in the nonlinear response. The steady-state gain from $L$ to $y_D$ found in the nonlinear simulation is approximately 0.682, the fitted gain is 0.728 and the true linear gain is 0.87. For responses in bottom composition we see the same effect, but in this case the purity decreases ($x_B$ increases) and the gain increases. This shows that even for these very small step changes, the reponses becomes significantly affected by the nonlinearities. The change in gain due to change in purity is the main nonlinearity in distillation [15].

From (9) we easily see that the sign of the gain for the weak direction of the plant is wrong. The steady-state gains tells us that

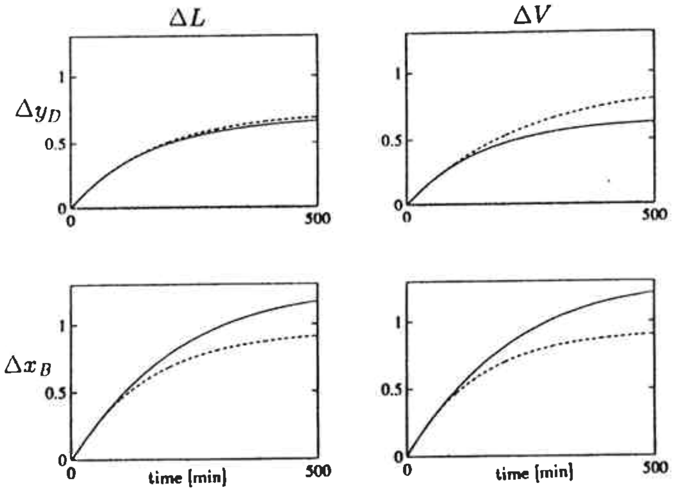


**Figure 5.** Open-loop responses for 0.1 % increase in reflux $L$ and 0.1% decrease in boilup $V$. Responses are scaled so that it corresponds to $\Delta L = -\Delta V = 1$. Solid line: nonlinear response. Dashed line: fitted response.

an increase in internal flows ($dL = dV$) will decrease purity in both ends, while we from theory know that this gain always will be positive in distillation, i.e., internal flows increases the separation in the column. This is also seen when computing $\lambda_{11}(0)$ which yields a value of -3.8 which we know should be larger than 1. Computing the transmission zeros we also find that there is a multivariable RHP zero at $0.0072 min^{-1}$ which is closely linked to the negative $\lambda_{11}(0)$ [7].

It is obvious that the fitted model, as is, is useless for designing controllers to be used on the plant. As we should always pair on positive RGA-elements, the model suggest the reverse pairing as compared to the correct model, i.e., $L$ with $x_B$ and $V$ with $y_D$. Designing a controller with integral action for this pairing would yield an unstable system when applied to the true plant [6]. If we paired the usual way, ie. $L$ with $y_D$ and $V$ with $x_B$, we would not obtain a stabilizing controller with the model. However, we have fitted only the initial part of the responses and know that these are less affected by nonlinearities than the steady-state responses. In addition the process is not ill-conditioned at high frequencies ($|\lambda_{11}(j\infty)| = 1$). We may therefor assume that the initial responses of the model are reasonably correct. Furthermore, we know that except for getting the correct sign of the plant at steady-state, the low-frequency behavior of the plant (and model) is of little importance for the control properties. We therefor propose to simply fix the sign of the model at low-frequency. This may be done by correcting one of the individual steady-state gains so that $\lambda_{11}(0)$ is greater than one. To get a reasonably correct value of $\lambda_{11}(0)$ we use the simple expression [12]:

$$\lambda_{11}(0) = \frac{1}{Bx_B + D(1-y_D)} L \frac{2}{N} \frac{L+F}{F} \quad (10)$$

Inserting the nominal values in (10) we find $\lambda_{11}(0)$=50 (the correct value is 35). We choose to correct the off-diagonal element 2,1 to obtain $\lambda_{11}(0)$=50, and the new steady-state gain becomes $k_{21}$=0.737. Note that we scale the corresponding time constant accordingly so that the initial response is unchanged. This yields $g_{21}(s) = 0.737/(1+120s)$. The corrected model has no RHP zeros. Figure 6 shows the RGA plotted as a function of frequency for the corrected model, F1fit1. We see that we have a resonably good fit of the RGA for the full linear model at high frequencies.

To check the quality of the obtained model, we use it for controller design as described in section 5, and apply the controller to the full plant. The results are given in Table 5. We see that the model yields a controller that performs reasonably well on the full plant. Comparing with the results in Table 4, we see that we obtain a controller with a performance similar to what was obtained with the "correct" F1-

model. This is of course the best we could expect since we used a $F1$-type model structure.

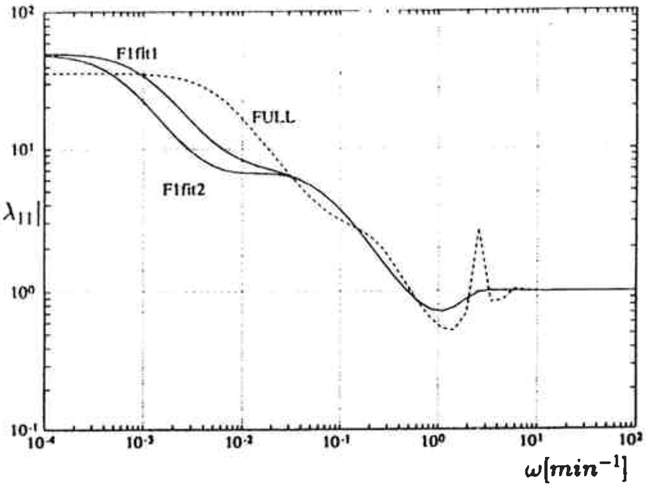


**Figure 6.** $\lambda_{11}$ as a function of frequency for the fitted models F1fit1 and F1fit2.

**Table 4.** $\mu_{RP}$-optimal PID-tunings for fitted models F1fit1 and F1fit2 and $\mu_{RP}$-values when applied to fitted model and to full linear model.

| Model | $k_y$ | $k_x$ | $\tau_{Iy}$ | $\tau_{Ix}$ | $\tau_{Dy}$ | $\tau_{Dx}$ | $\mu_{RP}$ – Original | $\mu_{RP}$ – Full |
|---|---|---|---|---|---|---|---|---|
| F1fit1 | 37 | 44 | 3.53 | 4.00 | 1.54 | 0.94 | 0.87 | 1.05 |
| F1fit2 | 53 | 41 | 4.00 | 3.11 | 1.31 | 0.89 | 0.91 | 1.11 |

**Experiment 2.** In experiment 1 we saw that even a 0.1 % pertubation in the inputs gave nonlinear effects in the responses. In practice one will of course have to make significantly larger changes in the inputs, and nonlinearities will affect the responses even more. One should be careful about trying to fit the nonlinearities into a linear model. To demonstrate this consider Fig.7 which shows the response in bottom composition $x_B$ to a 1 % change in reflux $L$. The response now seems to be 2.order. However, as discussed above the gain will increase as $x_B$ increases, and this explains the seemingly 2. order of the response. We know the linear responses in distillation columns should be almost pure 1.order. Trying to fit the response to a second order model would yield a poor model. However, as Skogestad and Morari

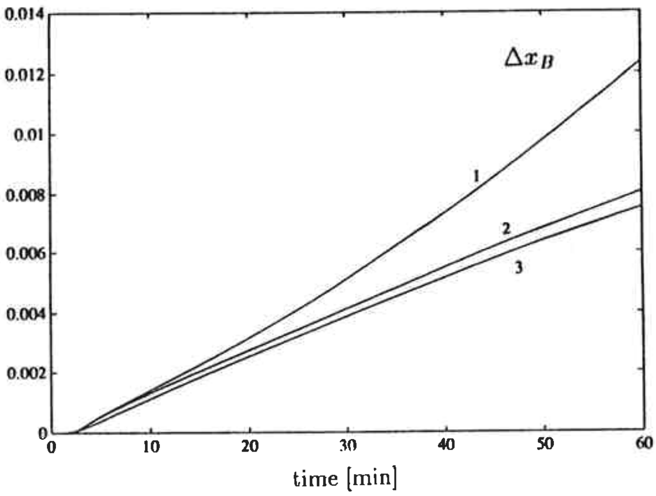


**Figure 7.** Open-loop response in $x_B$ to a 1 % increase in reflux. 1) Nonlinear response. 2) Nonlinear logarithmic response ($\Delta log(x_B)/100$). 3) Linear response.

[15] show, the nonlinearities may be partly counteracted by using the logarithm of the compositions as measurements, i.e. $log(1 - y_D)$ and $log(x_B)$. As they show, especially the initial responses will be linearized by the logarithm. This is illustrated in Fig.7 which shows the nonlinear response in $x_B$ with and without logarithmic measurements together with the linear response. The figure clearly demonstrates that the logarithm linearizes the initial response of the column.

In this case we therefor fit the logarithmic responses, and again we fit only the initial 60 minutes. This yields the model

$$\hat{G}(s) = \begin{pmatrix} \frac{0.456}{1+98s} & \frac{-0.423}{1+99s} \\ \frac{1.02}{1+168s} g_L(s) & \frac{-0.903}{1+141s} \end{pmatrix}, \theta_L = 2.5min. \tag{11}$$

For this model we find $\lambda_{11}(0) = -21$ which is clearly incorrect. Using the same procedure as above (change $k_{21}$ to yield $\lambda_{11}(0)$=50) we find the new $g_{21} = 0.954/(1 + 157s)$. The RGA of the corrected model, F1fit2, is shown in Fig.5, and we see that we get a similar fit as obtained in Experiment 1.

The results obtained when using this model for controller design is given in Table 5. Again we see that we get a model which yields a controller with a relatively good performance, and comparable to what was obtained with the $F1$-model.

## 7 Discussion

*Other control configurations* We have in this paper only dicussed the $LV$-configuration, i.e., $L$ and $V$ as independent variables. However, other configurations are possible, and the model determined for one configuration may easily be transformed to another configuration by using simple algebraic relations (e.g., [17]). For the $DV$-configuration the input $D$ will correspond to external flows and input $V$ to internal flows. One might think that it should be relatively easy to obtain the weak direction of the plant from a step change in $V$. This is however not necessarily true: Because of the flow-dynamics there will be a temporary change in external flows when changing $V$. This results in a large overshoot in bottom composition, and the transfer-function should contain a minimum phase zero close to the imaginary axis in addition to two poles. Furthermore, the $DV$-configuration depends on the tuning of the level controllers, and one will observe that this affects the responses. There will for instance be a small inverse response in $y_D$ for a change in $V$ because of nonperfect level control. Due to these complications we found it difficult to obtain a good model using the $DV$-configuration. Using the $LV$-configuration (which is independent of the level controllers) and correcting the model as discussed above yielded far better models.

*Effect of deadtimes.* We have assumed an input deadtime of 1 minute when designing controllers. This implies that we are able to have a relatively small closed loop time-constant (20 min.). However, if the deadtime in the system is significantly larger, the closed-loop constant will have to be reduced accordingly. In this case it will be necessary to have a good fit of the model also at lower frequencies, and the models we have obtained from open-loop experiments may not be sufficient. In cases with large deadtimes it will be difficult to obtain a good model from single step response experiments.

*Measurement noise.* We have not included any measurement noise in our simulations. However, when fitting the reponses to 1.order transfer-functions with only two parameters, it is unlikely that randomly distributed noise will affect the results significantly.

*Multivariable controllers.* One might argue that when true multivariable controllers are to be applied to the plant one needs a better fit of the individual elements in the model. This may certainly be true. However, it is also clear that due to uncertainty in the inputs, a multivariable controller that tends to decouple the process will perform poorly on an ill-conditioned plant [13].

## 8 Conclusions

1. The most important model characteristic for controller tuning is the high-frequency dynamics (initial response) corresponding to

the time-constant of the closed-loop system.

2. Because of the importance of the high-frequency behavior, the flow-dynamics should be included in the model.

3. An accurate model of the steady-state behavior is not very important for controller design. However, it is important to know the sign of the plant, that is, one must know the sign of the steady-state RGA. For almost any column with the $LV$-configuration $\lambda_{11}(0)$ should be larger than 1.

4. Models obtained simply from fitting individual transfer-functions from open-loop experiments may prove entirely useless for controller design. However, if the obtained model is corrected according to established structural properties, the resulting model will most likely be good.

5. One may easily enter the nonlinear region when doing open-loop experiments. The nonlinearities may effectively be counteracted by using logarithmic measurements.

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**NOMENCLATURE** (also see Fig. 1) .

$G(s)$ - linear model of column
$K = G(0)$ - steady-state gain matrix
$k_{ij}$ - steady state gains for column
$RGA$ - Relative Gain Array, elements are $\lambda_{ij}$
$x_B$ - mole fraction of light component in bottom product
$y_D$ - mole fraction of light component in distillate (top product)
$z_F$ - mole fraction of light component in feed

*Greek symbols*

$\alpha = \frac{y_i/x_i}{(1-y_i)/(1-x_i)}$ - relative volatility
$\lambda_{11}(j\omega) = (1 - \frac{g_{12}(j\omega)g_{21}(j\omega)}{g_{11}(j\omega)g_{22}(j\omega)})^{-1}$ - 1,1-element in RGA.
$\omega$ - frequency ($\min^{-1}$)
$\tau_1$ - dominant time constant for external flows (min)
$\tau_2$ - time constant for internal flows (min)
$\tau_L = (\partial M_i/\partial L)_V$ - hydraulic time constant (min)
$\theta_L = (N-1)\tau_L$ - overall lag for liquid response (min)

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