

# Control of Unstable Distillation Columns

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## Abstract

The paper addresses control of distillation columns at open-loop unstable operating points. The fact that ideal two-product distillation columns may have right half plane poles has only recently been recognized. The paper provides evidence for the observed instability. It is shown that the columns may be stabilized by use of one-point control, i.e., feedback control of a composition or a temperature inside the column. Finally, the impact of the right half plane pole on achievable control performance is discussed.

## 1 Introduction

Distillation is undoubtedly the most studied unit operation in the process control literature. Two-point (dual composition) control of distillation columns is a difficult task due to several complexities like strong interactions (ill-conditioning), sluggish responses and strong non-linearities. However, all work so far has assumed the columns to be open-loop stable (with level and pressure loops closed). The main reason for this is that most authors have considered dynamic models with inputs (eg., reflux and boilup) given on a molar basis.

For the case of molar inputs there exists several papers on uniqueness and stability of the operating points. Rosenbrock [4], [5] showed that binary distillation columns with constant relative volatility (ideal VLE) and constant molar flows always exhibit unique and asymptotically stable solutions. Doherty and Perkins [1] extended this result to the case with non-ideal VLE, but still assumed binary mixture and constant molar flows (i.e., neglected the energy balance). Sridhar and Lucia [6] include the energy-balance in the model and conclude under certain assumptions that also in this case binary distillation column will exhibit unique and stable solutions. They do however only study a limited set of specifications (eg.  $Q_D Q_B$  and  $LB$ ).

However, Jacobsen and Skogestad [3] argue that real columns only in rare cases have all the manipulated inputs on a molar basis. For instance, fixing the valve position will normally correspond closely to fixing the geometric average of mass and volumetric flow-rate. As they show, the transformation from mass- or volume flows to molar flows is nonlinear due to the composition dependence and may in some cases become singular. A singularity in the transformation will imply that there exists multiple solutions, one of which will be unstable.

The previous paper [3] treats the multiplicity only from a steady-state point of view. In this paper we study the dynamics of columns with multiple solutions, and consider the implications of open-loop instability for feedback control. The last point is important as previous work on distillation control has assumed open-loop stability. The most important question to be answered is whether the achievable closed-loop performance is significantly influenced by the possible right half plane pole.

We start the paper with a brief summary of the previous results on steady-state multiplicity in distillation columns. We then prove the instability observed in [3] and consider the overall dynamics of columns with right half plane (RHP) poles. Finally, we consider whether any fundamentally new control problems are introduced by the multiplicity and instability.

We will limit ourselves to discuss mainly one control configuration (set of specifications), namely the  $L_w V$ -configuration where mass reflux  $L_w$  and molar boilup  $V$  (corresponds closely to heat input  $Q_B$ ) are used as independent variables. This is the most widespread configura-

tion in the industry, and it is the configuration for which multiplicity and instability is most likely [3].

## 2 Results on Steady State Multiplicity in Ideal Distillation

We give here a brief review of the results presented in [3].

Consider the two-product distillation column in Fig.1. If the feed to the column is given there are at least four flows that may be specified: reflux  $L$ , boilup  $V$ , distillate  $D$  and bottoms flow  $B$ . However, for a given column there are only two degrees of freedom at steady-state, that is, only two of these flows may be specified independently. In the following we will denote a specific choice of these two independent variables as a "configuration".

Jacobsen and Skogestad [3] provide a simulation example of steady-state multiplicity in a column separating a mixture of methanol and n-propanol. The column has mass reflux and molar boilup as independent variables, i.e.,  $L_w V$ -configuration. Data for the column are given in Table 1. Note that the energy-balance is excluded, i.e., constant molar flows are assumed. The simulation results are given in Table 2, and the multiplicity is graphically illustrated in Figure 2.

The multiplicity is caused by the transformation between the actual flow-rates (mass) and the molar flow-rates which determines separation. For a binary mixture the transformation between mass reflux,  $L_w$ , and molar reflux,  $L$ , is given by

$$L = L_w/M; M = y_D M_1 + (1 - y_D) M_2 \quad (1)$$

Here  $M_i$  denotes the mole weight of the individual components. One might expect the molar reflux to increase monotonically with the mass reflux, that is,  $(\partial L/\partial L_w)_V > 0$ . However, because  $M$  is a function of composition,  $y_D$ , and thereby of  $L_w$ , this might not be the case. Assuming molar boilup  $V$  fixed and differentiating  $L_w = LM$  on both sides with respect to  $L$  yields

$$\left(\frac{\partial L_w}{\partial L}\right)_V = M + L(M_1 - M_2) \left(\frac{\partial y_D}{\partial L}\right)_V \quad (2)$$

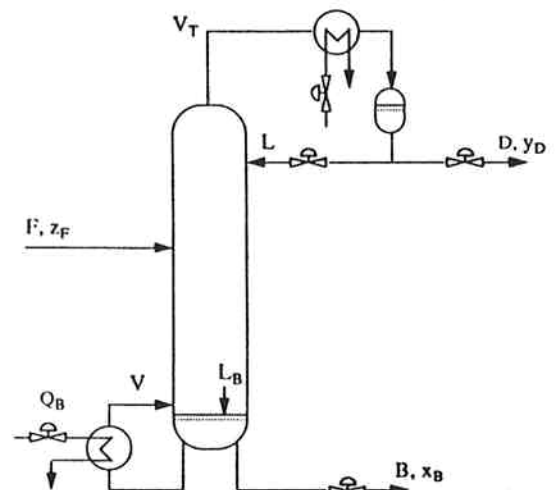


Figure 1. Two product distillation column.

Table 1. Data for Methanol-Propanol Column.

$z_F$	$F$	$\alpha$	$N$	$N_F$	$M_1$	$M_2$
0.50	1	3.55	8	4	32.0	60.1

Constant molar flows

Feed is saturated liquid

Total condenser with saturated reflux

Liquid holdups are  $M_{Li}/F = 0.5$  min, including reboiler and condenser.

For  $M_1 < M_2$ , which is the most common case (the most volatile component has the smallest molecular weight), the second term on the right hand side of (2) will be negative and the total differential may take either sign. The transformation from  $L_w$  to  $L$  will be singular when  $(\partial L_w/\partial L)_V = 0$ . The singular point corresponds to a pitchfork bifurcation point, i.e., the number of solutions change from one to three. Jacobsen and Skogestad [3] state that solutions with a negative slope between  $L$  and  $L_w$  corresponds to unstable operating points, but they do not prove this rigorously.

### 3 Open-Loop Dynamics

The maximum eigenvalue in different operating points for the methanol-propanol column with constant molar flows and the  $L_wV$ -configuration are shown in Fig.2. From the figure we observe that the eigenvalues at the upper and lower branches are negative, implying stability, while those at the intermediate branch (negative slope) are positive, implying instability of the operating points. The eigenvalues at the singular points are zero as expected since they correspond to bifurcation points.

The purpose of the rest of this section is to prove the observed instability at the intermediate branch and to compare the dynamics of columns with mass or volume inputs with those found for models with molar inputs.

#### 3.1 Stability

Although the dynamic model of a distillation column is of high order, it is well known that the overall composition dynamics in distillation columns may be well approximated by a first order response (eg., [8]). This implies that we may approximate the transfer-function from molar reflux to top composition with (assuming constant molar flows)

$$\left(\frac{\partial y_D}{\partial L}\right)_V(s) = \frac{g_{11}}{1 + \tau_1 s} \quad (3)$$

where  $g_{11} (= (\frac{\partial y_D}{\partial L})_V$  in (2)) is the steady-state gain and  $\tau_1$  is the dominant time-constant. We want to derive the transfer function  $(\partial y_D/\partial L_w)_V(s)$ . We have, with  $V$  constant

$$dy_D(s) = \left(\frac{\partial y_D}{\partial L}\right)_V(s)dL(s) \quad (4)$$

Here  $L = L_w/M$  is a function of both  $L_w$  and  $y_D$  and we get

$$dL(s) = \left(\frac{\partial L}{\partial L_w}\right)_{y_D}(s)dL_w(s) + \left(\frac{\partial L}{\partial y_D}\right)_{L_w} dy_D(s) \quad (5)$$

Table 2. Steady-state solutions for methanol-propanol column with  $V=2.0$  kmol/min and  $L_w$  in the range 48 to 53 kg/min.

	$L$ kmol/min	$D$ kmol/min	$L_w$ kg/min	$y_D$	$x_B$
I	1.064	0.936	48.00	0.534	$3.10e-3$
II	1.143	0.857	50.00	0.584	$3.50e-3$
III	1.463	0.537	50.00	0.9237	$7.80e-3$
IV	1.555	0.445	50.00	0.9969	0.104
V	1.650	0.350	53.00	0.9984	0.233

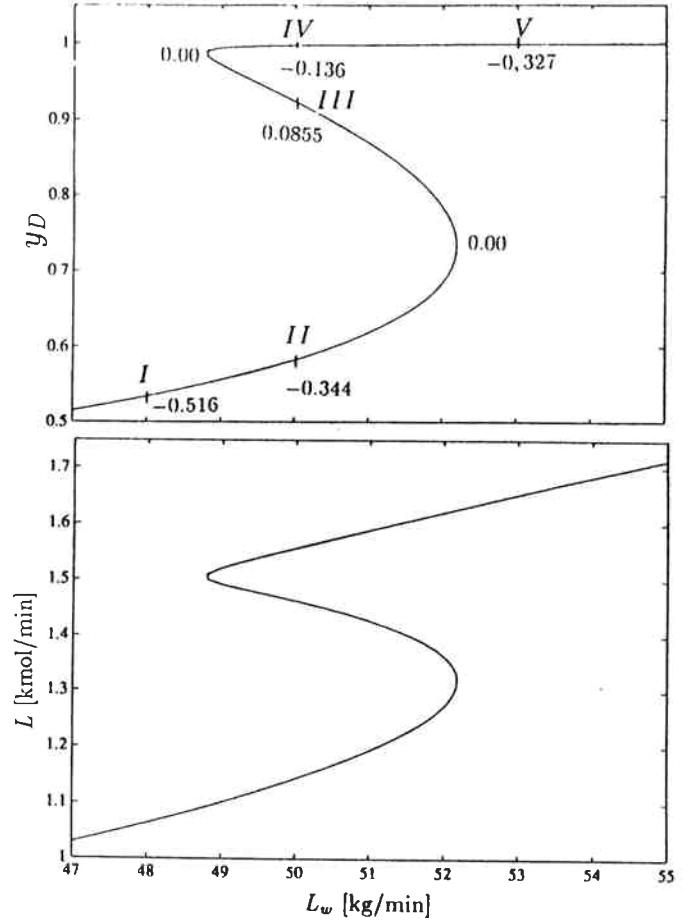


Figure 2. Methanol-propanol column: Multiple steady states for  $L_wV$ -configuration. Mass reflux  $L_w$  is varied while molar boilup  $V$  is fixed at 2.0 kmol/min. On the upper plot the corresponding maximum eigenvalue is shown at some of the steady-state solutions.

Combining (4) and (5) yields:

$$\left(\frac{\partial y_D}{\partial L_w}\right)_V(s) = \frac{\left(\frac{\partial L}{\partial L_w}\right)_{y_D} \left(\frac{\partial y_D}{\partial L}\right)_V(s)}{1 - \left(\frac{\partial L}{\partial y_D}\right)_{L_w} \left(\frac{\partial y_D}{\partial L}\right)_V(s)} \quad (6)$$

The Laplace variables have been deleted for  $(\partial L/\partial L_w)_{y_D}$  and  $(\partial L/\partial y_D)_{L_w}$  since the relationship  $L = L_w/M$  is purely static:

$$\left(\frac{\partial L}{\partial L_w}\right)_{y_D} = y_D^* M_1 + (1 - y_D^*) M_2 \quad (7)$$

$$\left(\frac{\partial L}{\partial y_D}\right)_{L_w} = \frac{L_w(M_2 - M_1)}{(y_D M_1 + (1 - y_D) M_2)^2} \quad (8)$$

From (6) we now find that the dominant pole is given by

$$\lambda_{max} = -\frac{1}{\tau_1} \left(1 - \frac{g_{11} L (M_2 - M_1)}{y_D M_1 + (1 - y_D) M_2}\right) \quad (9)$$

The pole will be in the right half plane when

$$\frac{g_{11} L (M_2 - M_1)}{y_D M_1 + (1 - y_D) M_2} > 1 \quad (10)$$

This is exactly the same criterion as Jacobsen and Skogestad [3] found for a negative slope between mass and molar reflux. Thus, a necessary and sufficient condition for instability for the  $L_wV$ -configuration is

$$\left(\frac{\partial L}{\partial L_w}\right)_V < 0 \quad (11)$$

In other words, solution branches with a negative slope between  $L$  and  $L_w$  represents unstable solutions, provided the column is stable on a molar basis. This result is in accordance with numerical results and also with what one would expect for a pitchfork bifurcation.

Equation (9) gives an approximate way of calculating the dominating pole for the  $L_wV$ -configuration from data computed for molar inputs. The expression gives a correct value of zero  $\lambda_{max}$  at the singular points. From (9) we see that large internal flows  $L$  and  $V$ , (to make  $L$  large we must increase both  $L$  and  $V$  since  $L \leq V$ ) will most probably yield instability. This is discussed in more detail in [3].

In the general case with more complex dynamics, (3) may be replaced by

$$\left(\frac{\partial y_D}{\partial L}\right)_V(s) = \frac{g_{11}(1 + b_1s + b_2s^2 + \dots + b_{n-1}s^{n-1})}{1 + a_1s + a_2s^2 + \dots + a_ns^n} \quad (12)$$

This follows since distillation column dynamics generally have a pole excess of 1 [8]. We may now use the Routh-Hurwitz stability criterion (all coefficients in the pole polynomial should have the same sign) to conclude that in the general case (10) and (11) are sufficient conditions for instability.

### 3.2 Overall Dynamics

The analysis above showed that the dominant pole, and thereby the low-frequency dynamics are strongly influenced by the transformation between mass and molar reflux. The multiplicity and instability is caused by this pole crossing the imaginary axis. However, the effect on the high frequency dynamics is unclear. Figure 3 shows the magnitude and phase of the transfer-functions from molar and mass reflux respectively on top composition  $y_D$  (keeping boilup  $V$  fixed) in operating point III of the methanol-propanol column. From the figure we see that the phase of the two transfer-functions differs with 180 degrees at low frequencies as expected since the transfer function from mass-reflux is non-minimum phase due to the RHP pole. However, the phases approach each other at intermediate frequencies and becomes identical at high frequencies. The magnitudes of the two systems are almost identical at all frequencies. The fact that the magnitudes are similar also at low frequencies is more a coincidence for this operating point; the dominant poles are equal in magnitude (-0.0078 and 0.0086). However, the dynamics of the two systems are different in the region where the phases differ.

The most important conclusion to draw from Fig.3 is that it is mainly the dominant pole that is influenced by the transformation from mass to molar flows. The initial response (high-frequency) is unaffected. Similar results are obtained for the three other transfer-functions of the 2x2 system.

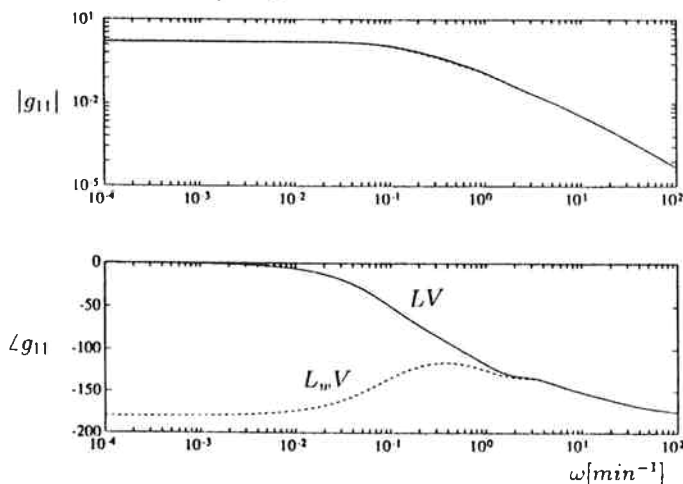


Figure 3. Frequency response for transfer-function from reflux to top composition  $y_D$  for operating point III of methanol-propanol column. The magnitude for mass-reflux  $L_w$  is scaled by  $M$ .

## 4 Feedback Control

### 4.1 Limitations imposed by RHP poles and zeros

As we have seen, columns operating with mass or volume inputs may be open-loop unstable, and will require feedback control (in addition to level and pressure control) for stabilization. From control theory it is well known that unstable poles by themselves do not represent any bandwidth limitations; on the contrary they put a lower limit on allowable bandwidth of the closed-loop system. Problems will therefore only arise if there are bandwidth limitations like right half plane zeros at frequencies comparable to the right half plane pole ("The system goes unstable before we are able to observe what is happening") or if there are constraints ("we can not counteract the instability").

Freudenberg and Looze [2] have extended the Bode Integral Theorem valid for minimum phase systems to systems containing RHP zeros and RHP poles. The integrals are for scalar systems, but similar relations are obtained if one considers the maximum singular value of the sensitivity function for multivariable systems.

For a system with a real RHP zero at  $z$  and a RHP pole  $p$  the following restriction applies to the sensitivity function,  $S$ , of the closed-loop system

$$\int_0^{\infty} \log|S(j\omega)|W(z,\omega)d\omega = \pi \log\left|\frac{p+z}{p-z}\right| \quad (13)$$

(With no RHP pole ( $p = 0$ ) the integral equals zero). Here

$$W(z,\omega) = \frac{2z}{z^2 + \omega^2} \quad (14)$$

We see that the weighted area of  $\log|S|$  for  $|S| > 1$  must be larger than the equivalent area for  $|S| < 1$ . The form of the weight  $W$  (equals  $2/z$  at low frequency and falls off with a  $-2$  slope from  $\omega = z$ ) implies that essentially all the positive area has to be at frequencies lower than  $z$ , and there have to be a peak  $|S| > 1$ . The peak will become increasingly large as the crossover frequency approaches  $z$ . We see from (13) that as the RHP zero approaches the RHP pole, the peak goes to infinity. This implies that we in general must require

$$p < z \quad (15)$$

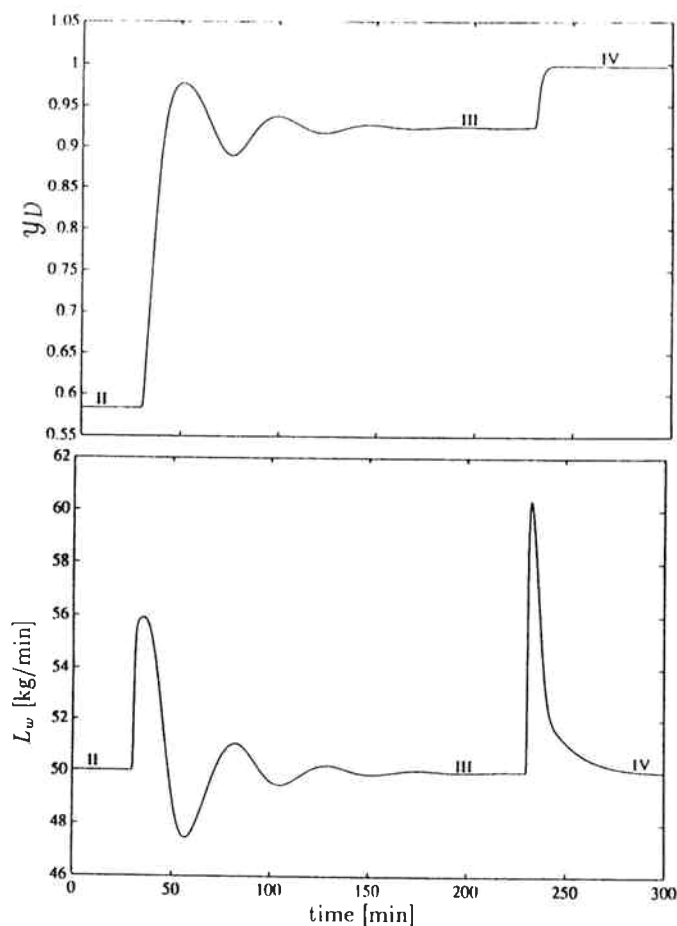
For the distillation column  $p = \lambda_{max}$ , and RHP zeros are most likely caused by dead-times,  $\theta_d$ , in measurements and actuators. Using a Padé approximation for  $\theta_d$  results in a RHP zero at  $z = 2/\theta_d$ .

### 4.2 One-point Control

Good control of distillation columns requires two-point control, i.e., feedback control of both product compositions. However, in order to stabilize an open-loop unstable column one-point control will suffice. This is also the way most industrial columns are operated. An unstable column operating with the  $L_wV$ -configuration may be stabilized by controlling either top or bottom composition, or any other variable related to composition, eg. a temperature inside the column. All transfer-functions involving composition dependent variables will contain the RHP pole. If the composition measurements involves large deadtimes that complicates stabilization, faster temperature measurements may be used instead.

With a specified structure of the controller, eg. a PI-controller, we must require a larger distance between the RHP pole and the RHP zero than given by (15) in order to stabilize the column. For operating point III of the methanol-propanol column the RHP-pole is at  $p = 0.086 \text{ min}^{-1}$  and we get problems stabilizing the column with a PI-controller when the deadtime exceeds 11 min. ( $z = 2/\theta_d = 0.182 \text{ min}^{-1}$ ). Typical composition measurement delays (GC-analysis) in industrial columns may be up to 30 min. Note also that as the limit (15) is approached, the closed-loop performance becomes highly deteriorated with a high peak on the sensitivity function, according to (13).

**Nonlinear Simulations.** Figure 4 shows nonlinear simulations



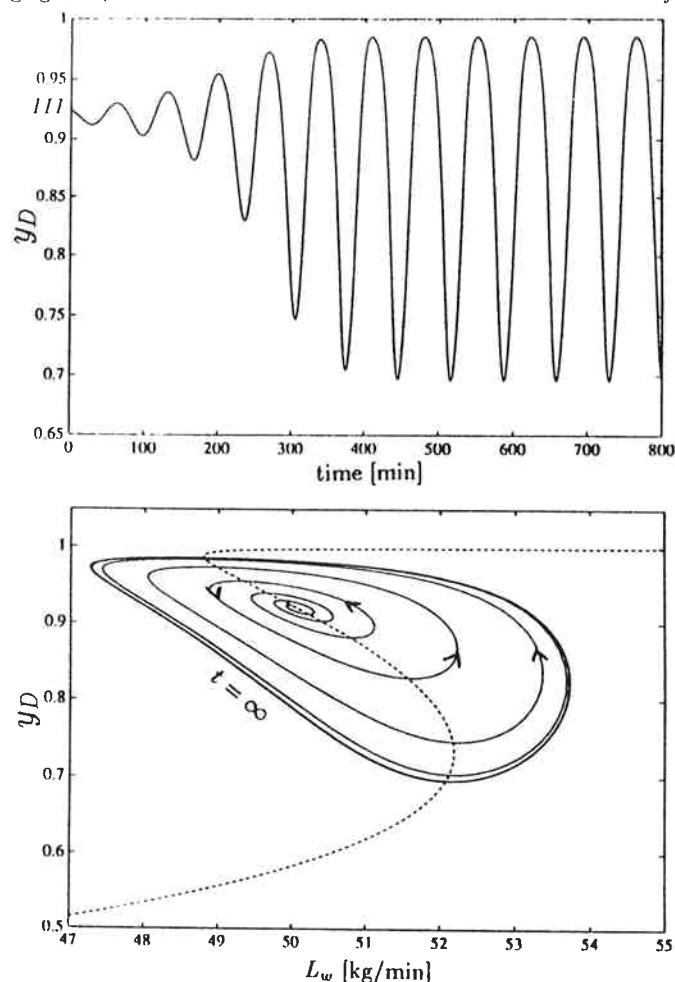
**Figure 4.** Nonlinear simulation of methanol-propanol column with one-point control of top-composition  $y_D$  using mass reflux  $L_w$ . Setpoint changes from operating point II to III and from III to IV. Boilup  $V = 2.0$  kmol/min. Controller parameters:  $k = 3.0$  and  $\tau_I = 11.0$  min. Gain is for logarithmic composition, i.e.,  $\log(1 - y_D)$ .

of the methanol-propanol column using a single-loop PI-controller<sup>1</sup> between top composition  $y_D$  and mass-reflux  $L_w$ . Molar boilup  $V$  is kept constant at 2.0 kmol/min. The figure shows the responses to setpoint changes in  $y_D$  from operating point II (open-loop stable) to operating point III (open-loop unstable) and then further on to operating point IV (open-loop stable) (see Table 2). Logarithmic measurement  $Y_D = \ln(1 - y_D)$  was used in the controller as this reduces the non-linearity of the initial response between different operating points [8]. From the figure we see that the controller is able to stabilize the open-loop unstable operating point III with a RHP pole at  $0.086 \text{ min}^{-1}$ . The simulations also show that the same controller may be used in these three widely differing operating points. The reason why the same controller may be used in all three operating points is simply that the initial response (high-frequency dynamics) is similar in all operating points. From the plot of mass-reflux  $L_w$  against time we see that the steady-state change in the input is zero, showing that the three operating points are multiple solutions.

One should be careful about detuning a controller in an open-loop unstable process as the bandwidth may become lower than the minimum allowable and the closed-loop system unstable. This is illustrated in Fig.5. where the controller gain has been reduced by a factor of two compared to the simulations in Fig.4. Operating point III is now closed-loop unstable, and a small setpoint change makes the system start drifting away. However, this does not imply that the column goes globally unstable in the sense that physical constraints are violated.

<sup>1</sup>Tuned to yield reasonably fast response. Note that Ziegler-Nichols tuning rules resulted in a closed-loop unstable system.

Since there exists steady-state solutions above and below the unstable solution the column goes into a stable limit cycle. If the controller gain is reduced further the limit cycles will continue, but now with a longer period of each cycle and with higher peaks in composition. There will exist cases where there are no solutions either above or below the unstable solution (in fact, the solutions would then correspond to one of the product flows being negative). In this case the column is likely to go globally unstable as either the condenser or reboiler would run dry.



**Figure 5.** Nonlinear simulation of methanol-propanol column with one-point control of top-composition  $y_D$  using mass reflux  $L_w$ . Controller gain reduced by a factor of 2 compared to Fig.4. Upper plot: Time as independent variable. Lower plot: Phase-plane plot. Dashed line shows steady-state solutions.

### 4.3 Two-Point Control

As pointed out above one-point control is sufficient to stabilize an unstable operating point, but high performance control requires control of both product compositions. There exist a large amount of literature on two-point control of distillation columns, but all is based on open-loop stable models. We want to investigate whether the potential instability for mass-flows will affect the achievable closed-loop performance of the column significantly.

In order to compare achievable performance for the stable model with molar reflux and the unstable process with mass reflux we design controllers with optimized performance for both cases. As a design objective we use the structured singular value,  $\mu$  (see e.g., [7]). This implies that we may include model uncertainty in the design, which is of utmost importance when designing controllers for ill-conditioned plants [7]. In the design we use an uncertainty on each channel given by the weight

$$w_I(s) = 0.20 \frac{5\theta_d s + 1}{0.5\theta_d s + 1} \quad (16)$$

This means that we allow for a deadtime  $\theta_d$  in addition to 20 % uncertainty in each input. The performance weight used is given by

$$w_P(s) = \frac{1}{P} \frac{\tau_{cl}s + P}{\tau_{cl}s} \quad (17)$$

This implies that the worst case peak of  $\bar{\sigma}(S)$  should be less than  $P$ , and that the closed-loop time-constant should be less than  $\tau_{cl}$ . We will design controllers for different values of  $\theta_d$ , and for each design we adjust the performance weight until a  $\mu$ -value of 1 is achieved. This is done by first increasing  $\tau_{cl}$  and then increasing  $P$  if necessary. A  $\mu$ -value of 1 implies that we can guarantee the specified performance for all plants within the model uncertainty.

We will again consider operating point *III* of the methanol-propanol column with a RHP pole at  $0.086 \text{ min}^{-1}$ . We use two single-loop PID-controllers as this is the preferred controller structure in the industry, and design controllers for deadtimes between 1 and 5 min. The results are given in Table 3.

**Table 3.** Robust performance parameters (see Eq.17) obtained for stable *LV*-configuration and unstable *L<sub>w</sub>V*-configuration in operating point *III* of the methanol-propanol column. All parameters for minimized  $\mu_{RP}=1.00$  using two single-loop PID controllers.

$\theta_d[\text{min}]$	<i>LV</i>		<i>L<sub>w</sub>V</i>	
	$P$	$\tau_{cl}(\text{min})$	$P$	$\tau_{cl}(\text{min})$
1.0	2.5	25	2.5	32
2.0	2.5	59	2.5	140
3.0	2.5	100	3.0	227
4.0	2.5	161	5.0	345
5.0	3.0	175	6.0	455

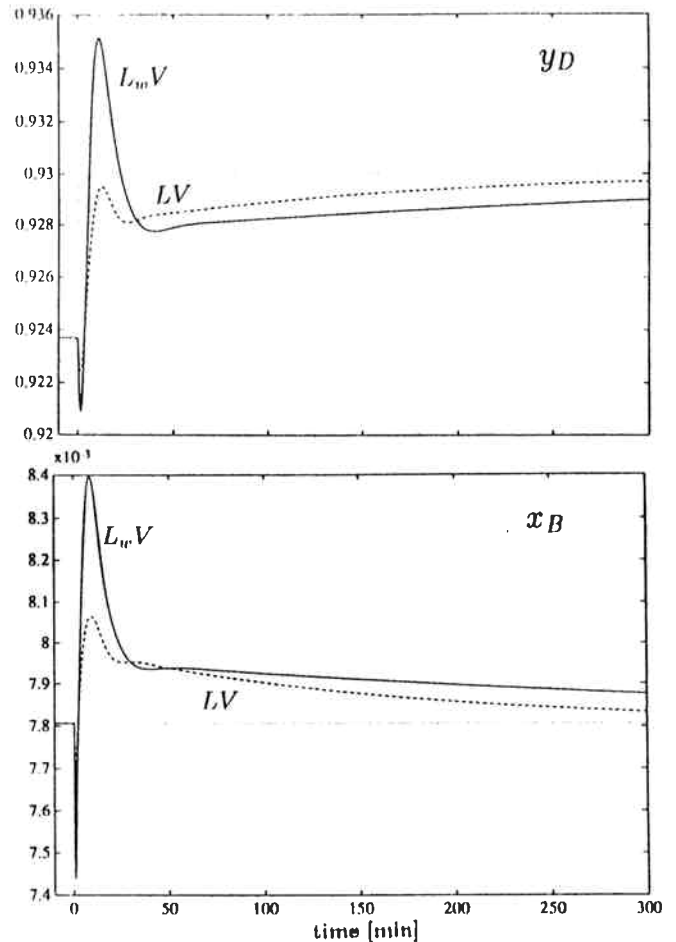
For a deadtime of 1 min. we see from Table 3 that there is only a small difference between the achieved robust performance of the two systems. However, as the deadtime is increased we must allow for higher peaks in the sensitivity function for the open-loop unstable model than for the open-loop stable model. This is also as expected from (13). With a deadtime of 5 minutes (RHP zero at  $0.4 \text{ min}^{-1}$ ) the response for the *LV*-configuration is poor ( $\tau_{cl} = 175 \text{ min}$ . with a maximum peak  $P = 3.0$ ), while the response for the *L<sub>w</sub>V*-configuration is unacceptable ( $\tau_{cl} = 455 \text{ min}$ . and  $P = 6.0$ ). This implies that when the operating point is open-loop unstable (with the *L<sub>w</sub>V*-configuration) and the system in addition has significant deadtime one should consider using a different configuration (see Discussion).

**Nonlinear Simulations.** Figure 6 shows responses to setpoint changes in top composition  $y_D$  using two single loop PID-controllers for the *LV*-configuration and the *L<sub>w</sub>V*-configuration. The simulations include 5 minutes deadtime (using a Padé approximation) and 20 % input uncertainty. The controller parameters were obtained from the  $\mu$ -optimal design above, and are given in Table 4. The simulations demonstrate the fact that the performance for the case with mass-reflux is clearly worse than for the case with molar reflux. The *L<sub>w</sub>V*-configuration has a much larger overshoot as well as a longer settling period.

**Table 4.** Controller parameters for closed-loop simulations in Fig. 6. (Correspond to last entry in Table 3.)

	$k_y$	$\tau_{Iy}$	$\tau_{Dy}$	$k_x$	$\tau_{Ix}$	$\tau_{Dx}$
<i>LV</i> - configuration	0.0687	6.55	2.33	0.0680	5.29	0.180
<i>L<sub>w</sub>V</i> - configuration	4.064	32.27	2.91	0.0280	4.79	0.209

Gains are for logarithmic compositions, i.e.  $\log(1 - y_D)$  and  $\log(x_B)$ .



**Figure 6.** Nonlinear simulation of methanol-propanol column at operating point *III* with two-point control using *LV*- and *L<sub>w</sub>V*-configuration. Response to a setpoint change in  $y_D$  using two single-loop PID controllers. Deadtime  $\theta_d = 5 \text{ min}$ . PID-settings from Table 4.

## 5 Discussion

**Other Configurations.** We have in this paper only considered the *L<sub>w</sub>V*-configuration. This is also the most widespread configuration in industry. However, there are many different configurations that may be used, and using reflux and boilup for composition control is not necessarily the best choice (see e.g., [9]).

Jacobsen and Skogestad [3] showed that instability for the *D<sub>w</sub>V*-configuration is very unlikely. Thus, if instability is observed for the *L<sub>w</sub>V*-configuration one may change to the *D<sub>w</sub>V*-configuration by changing condenser level control from distillate to reflux. However, the *D<sub>w</sub>V*-configuration may for some columns be a bad choice [9]. Another configuration which may be a good choice for many columns is the ratio configuration  $(L/D)(V/B)$  which is unlikely to be unstable when all liquid flows are measured in the same units.

**Effect of column design.** The methanol-propanol column that we have studied in this paper is not optimally designed for the product compositions of operating point *III*. In fact it is doubtful that the column would be unstable if it was optimally designed as the internal flows then would be significantly smaller. This is probably true for most separations, i.e., an optimally designed column will be open-loop stable. However, few industrial columns are operated close to an optimal operating point. The main reason is of course that optimal product compositions will change with prices. In addition many columns operate with high internal flows (over-fractionating) in order to assure that specifications are kept when disturbances enter the column. It is

therefor likely that many industrial columns may have problems with open-loop right half plane poles. The fact that this have not been reported previously is probably due to the fact that open-loop instability has been believed to be impossible, and problems have therefor been explained by other means.

**Multiplicity and instability for molar inputs.** We have in this paper only considered models with constant molar flows, i.e., without the energy balances. However, Jacobsen and Skogestad [3] show that when the energy balances are included in the model, even molar inputs (e.g., molar reflux and molar boilup) may yield multiple solutions and unstable operating points. This type of instability may also be experienced in industrial columns as it is the size of the molar flows that determines separation in distillation. The control problems associated with the RHIP pole in this case are similar to the ones discussed for mass or volume inputs.

## 6 Conclusions

1. Two-product distillation columns operating with mass- or volume inputs may be open-loop unstable. The probability of instability is increased with increased internal flows.
2. An unstable operating may be stabilized by use of one-point control provided the bandwidth of the controller is sufficiently high. If the system becomes closed-loop unstable due to a too low bandwidth, the column may go into a stable limit cycle provided there exists stable solutions above and below the unstable solution.
3. The RHIP pole of an unstable operating point will degrade the performance of the closed-loop system. This will become more marked as the deadtime in the system is increased.

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## NOMENCLATURE (see also Fig.1)

$A$  - State matrix of distillation column.  
 $B$  - bottoms flow (kmol/min)  
 $D$  - distillate flow (kmol/min)  
 $F$  - feed rate (kmol/min)  
 $L$  - reflux flow rate (kmol/min)  
 $M$  - mole weight, usually of top product (kg/kmol)  
 $M_1$  - pure component mole weight of most volatile component (kg/kmol)  
 $M_2$  - pure component mole weight of least volatile component (kg/kmol)  
 $N$  - no. of theoretical stages in column  
 $N_F$  - feed stage location (1-reboiler)  
 $P$  - maximum allowed peak on sensitivity function.  
 $p$  - right half plane pole. ( $\text{min}^{-1}$ )  
 $Q_B$  - heat input to reboiler.  
 $Q_D$  - heat removal in condenser.  
 $S$  - sensitivity function.  
 $V$  - boilup from reboiler (kmol/min) (determined indirectly by heating  $Q$ )  
 $x_B$  - mole fraction of most volatile component in bottom product  
 $y_D$  - mole fraction of most volatile component in distillate (top product)  
 $z$  - right half plane zero ( $\text{min}^{-1}$ ).  
 $z_F$  - mole fraction of most volatile component in feed

### Greek symbols

$\alpha = \frac{y_1/x_1}{(1-y_1)/(1-x_1)}$  - relative volatility (binary mixture)  
 $\lambda_i(A)$  -  $i$ 'th eigenvalue of  $A$ .  
 $\lambda_{max} = \max_i |\lambda_i(A)|$  - maximum eigenvalue = dominant pole  
 $\mu$  - structured singular value.  
 $\bar{\sigma}$  - maximum singular value

$\tau_{cl}$  - required closed-loop time constant (min).

$\theta_d$  - deadtime (min)

$\omega$  - frequency ( $\text{min}^{-1}$ )

### Subscripts

$w$  - flow rate in kg/min

## References

- [1] Doherty, M.F. and J.D. Perkins, 1982, "On the Dynamics of Distillation Processes-IV. Uniqueness and Stability of the Steady State in Homogenous Continuous Distillation", *Chem. Eng. Sci.*, **37**, 3, 381-392
- [2] Freudenberg, J.S. and D.P. Looze, 1988, "Frequency Domain Properties of Scalar and Multivariable Feedback Systems", Springer-Verlag
- [3] Jacobsen, E.W. and S. Skogestad, 1991, "Multiple Steady States in Ideal Two-product Distillation", *Accepted for publication in AIChE J.*
- [4] Rosenbrock, H.H., 1960, "A Theorem of "Dynamic Conservation" for Distillation", *Trans. Instn. Chem. Engrs.*, **38**, 20, 279-287.
- [5] Rosenbrock, H.H., 1962, "A Lyapunov Function with Applications to Some Nonlinear Physical Problems", *Automatica*, **1**, 31-53.
- [6] Sridhar, L.N. and A. Lucia, 1989, "Analysis and Algorithms for Multistage Separation Processes", *I & EC Res.*, **28**, 793-803.
- [7] Skogestad, S., M. Morari and J.C. Doyle, 1988, "Robust Control of Ill-Conditioned Plants: High-Purity Distillation", *IEEE Trans. Autom. Control*, **33**, 12, 1092-1105
- [8] Skogestad, S. and M. Morari, 1988, "Understanding the Dynamic Behavior of Distillation Columns", *Ind. & Eng. Chem. Research*, **27**, 10, 1848-1862.
- [9] Skogestad, S., P. Lundström, E.W. Jacobsen, 1990, "Selecting the Best Distillation Control Structure", *AIChE J.*, **36**, 5, 753-764.