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MODELLING REQUIREMENTS FOR ROBUST CONTROL OF DISTILLATION COLUMNS

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Abstract. In order to investigate the level of modelling detail that is required for effective control, five process models of different complexity are studied. It is demonstrated that the high-frequency behavior (initial time response) of the model is much more important for controller design than it's steady-state characteristics. Flow dynamics should be included in a column model which is to be used for controller design.

Keywords. Robust control; Distillation Column; Identification.

1 Introduction

Composition control of distillation columns (Fig. 1) has proved to be difficult to implement in practice. One-point control (one composition under feedback control and the other uncontrolled) is fairly common, while two-point control (feedback control of both compositions) is rarely used. One reason for this is that on-line tuning of two composition loops on a strongly interacting distillation column is very difficult (eg., Wood and Berry, 1973). It is therefore desirable to obtain controller tunings based on some model of the column. The objective of this paper is to investigate what properties such a model should have.

In the paper it is assumed that the product flows D and B are used for level control such that the reflux L and the boilup V are used for composition control. This corresponds to the *LV*-configuration. This may certainly not be the best configuration for two-point control, but it is the most commonly used configuration in industry; probably because it works well for one-point control. Furthermore, we shall assume that single-loop PID controllers are used, that is, top composition y_D is controlled with L, and bottom composition x_B is controlled with V. The choice of simple PID controllers seems to be a good one for the LV-configuration provided they are tuned properly (Skogestad and Lundström, 1990).

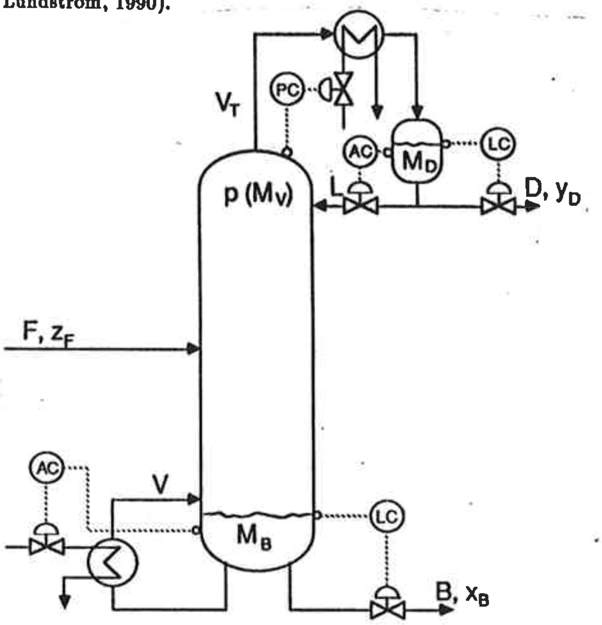

Figure 1. Two-product distillation column with LV-configuration.

The desired linear model of the plant is written

$$\begin{pmatrix} dy_D(s) \\ dx_B(s) \end{pmatrix} = G(s) \begin{pmatrix} dL(s) \\ dV(s) \end{pmatrix} \tag{1}$$

where $G(s)$ is a 2×2 transfer matrix expressing the effect of small changes in the independent flows on the compositions. The traditional approach for distillation columns has been to obtain the four elements of $G(s)$ *independently*, for example, by fitting open loop responses of steps or pulses in reflux and boilup to simple transfer function models (Luyben, 1970, Toijala and Fagervik, 1972, Wood and Berry, 1973). This approach might work for columns that are simple to control. However, as pointed out by Skogestad and Morari (1988b), for columns with large interactions between top and bottom this approach is likely to fail. The main reasons for these problems is that it is very difficult to obtain a good model based on open-loop experiments or simulations unless one explicitly takes into account the expected couplings between the elements when formulating the model. In particular, one is not able to obtain a good model of the low-gain direction of the plant (Skogestad and Morari, 1988b, Andersen et al., 1988).

In the paper we discuss these issues and investigate what modelling detail is needed for $G(s)$ to design controllers which also perform well on the real plant. As an example, we shall consider column A (Table 1) where the open-loop responses are very well described by a simple first-order response with the same time constant τ_1 in all elements of $G(s)$ (Skogestad and Morari, 1988b). This model is compared to the full linear model and to simple models where a different value τ_2 is used for the time constant of the low-gain direction (internal flows) and to models where the liquid flow dynamics are included. Even though the open-loop responses of the models are very similar, there are significant "hidden" differences. To illustrate this we plot as a function of frequency the Relative Gain Array (RGA) (Bristol, 1966) of the models. The RGA shows the degree of interactions in the models. We also perform simulations with a given controller and show that the models behave quite differently under closed-loop. Furthermore, we design controllers based on the simplified models and test how they perform on the full-order model.

This paper is a continuation of the work of Skogestad and Morari (1988a), Skogestad and Morari (1988b) and Skogestad and Lundström (1990). Skogestad and Morari (1988b) found that the initial column response in terms of logarithmic compositions should be nearly linear, and proposed the simple two time-constant model based on the fundamental difference between external and internal flows which is used in this paper. Skogestad and Morari (1988a) used this model to study LV-control of column A under various operating conditions. The structured singular value, μ, was used as a performance indicator. They confirmed that the effect of nonlinearity may be effectively counteracted by using logarithmic compositions. They also found that

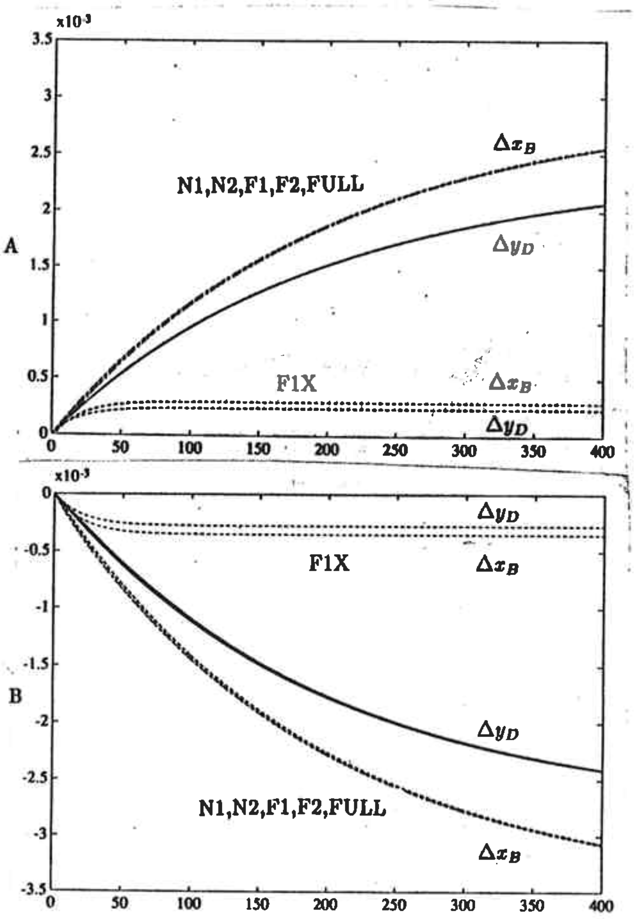

Figure 2. Open-loop linear response of various models to a 0.1% increase in boilup (A) and reflux (B).

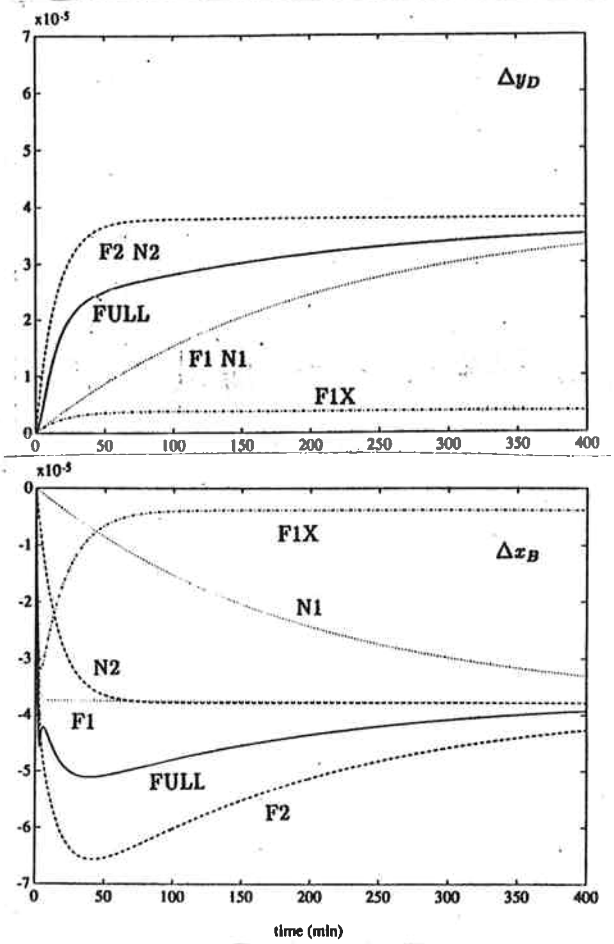

Figure 4. Open-loop linear response of various models to a 0.1% increase in internal flows ($\Delta L/F = \Delta V/F = 0.0027$).

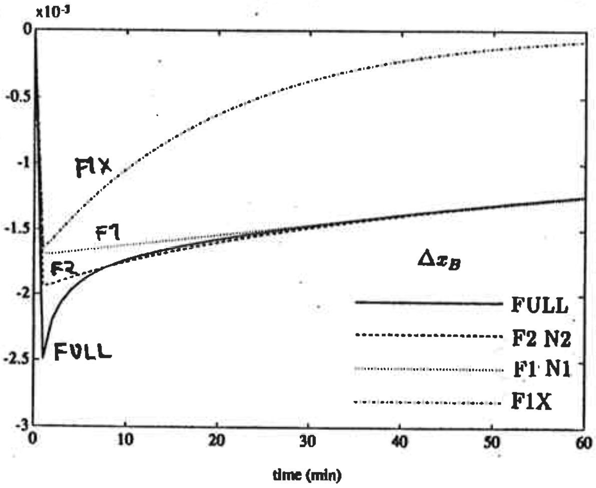

Figure 3. Open-loop linear response in x_B of various models to a 1 minute pulse in boilup.

Note that the steady-state gain for the changes in internal flows is about one hundreth of that for external flows. This implies that in a real plant in open-loop the external flow responses will dominate and it would not be possible to observe effects, such as those in Fig.4., of changes in the internal flows alone. However, under feedback control effects of internal flows may be observed also in practice. This reason is that the controller in order to keep compositions constant may have to make large changes in the internal flows and thus amplify the importance of the low-gain direction. This is clearly shown in the simulations in Section 3.3.

3.1.3 Initial response

For a model

$$dy(s) = \frac{k}{1+\tau s} du(s)$$

the slope of the initial response ($\lim_{t\to 0} dy(t)/dt$) to a step in u of magnitude Δu is

$$\frac{k}{\tau}\Delta u \qquad (7)$$

Skogestad and Morari (1988b) have shown that for distillation columns the initial response in terms of logarithmic compositions is only weakly affected by nonlinearity caused, for example, by changes in the operating point. Logarithmic compositions means using the logarithm of the amount of impurity of key component as outputs, that is choose the outputs y to be $ln x_B$ and $ln(1-y_D)$ for a binary separation. Consequently, if we measure the outputs in terms of a logarithmic compositions then the ratio k^S/τ may be obtained quite reliably from experiments or simulations, and it is not expected to change significantly with operating conditions (superscript S denotes that compositions are measured in terms of logarithmic (scaled) compositions). The unscaled gains are related to the scaled gains in the bottom by $k = x_B k^S$ and in the top by $k = (1-y_D)k^S$. For the simulations in Fig.2 we see that slope of the inital response for x_B is approximately 0.000015 which of course agrees with Eq.7 ($k/\tau\Delta u \approx 1.1/194 \cdot 0.0027 = 0.000015$). In terms of logarithmic compositions the initial slope is 0.0015.

3.1.4 Effect of nonlinearity

The step size for L and V used in the simulations in Fig. 2 is only 0.1%. In Fig.5 the response of the non-linear model to a 10% step increase in L is shown. Such a large increase might

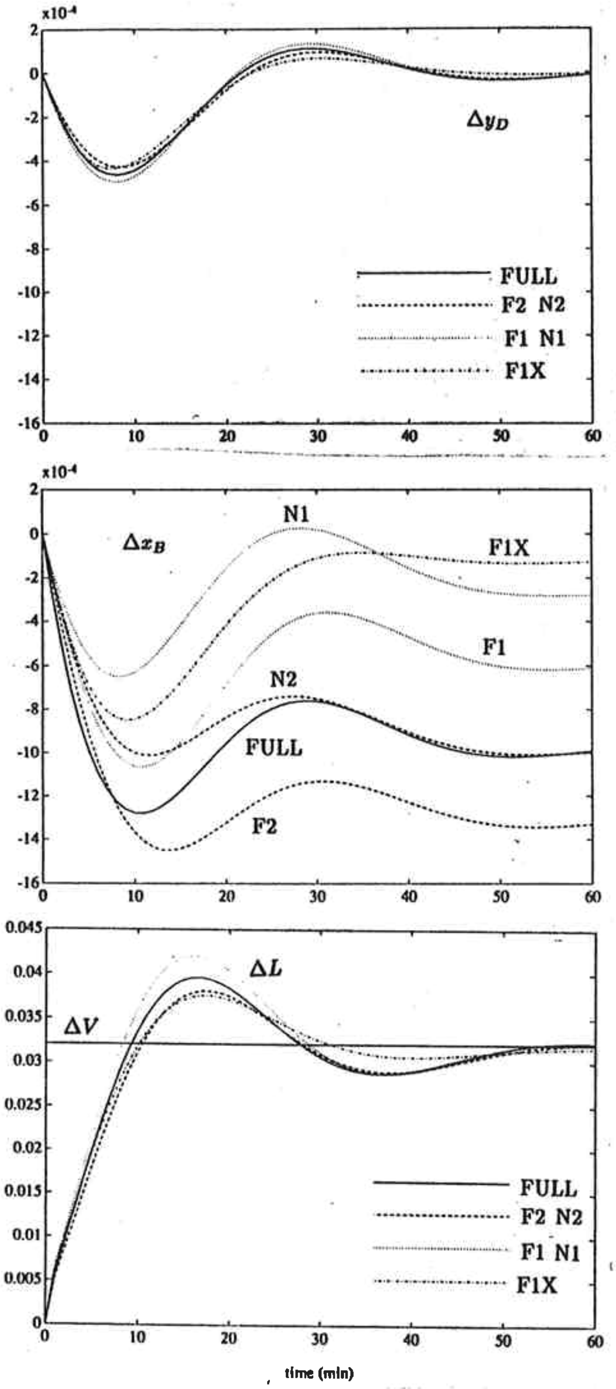

Figure 8. One-point control of y_D. Linear response of various models to a 1% increase in boilup. PID-settings $k_y, \tau_{Iy}, \tau_{Dy}$ from entry 'Full' in Table 2.

of the controllers. The structured singular value, μ, is then a reasonable performance index. Robust performance (that is, the worst-case response is acceptable) is satisfied if μ_{RP} is less than 1, and designs with low μ_{RP}-values are preferable. Uncertainty and performance are defined as in Skogestad and Morari (1988a). Single-loop PID-controllers were tuned for the various models by minimizing μ_{RP}. The results are summarized in Table 2 (Skogestad and Lundström, 1990). We see that the optimal μ_{RP}-value becomes lower as flow dynamics (θ_L) and internal flows (τ_2) are included. This may be explained in terms of lower RGA-values at high frequency (Fig.7) which makes control easier.

However, it is of course of no practical significance how well the *models* may be controlled; the interesting point is how well the controllers tuned based on the simplified models perform on the full-order model. This is summarized in Table 3. We see that

Table 2. μ_{RP}-optimal PID-tunings for various column models. $C_{PID}(s) = k\frac{1+\tau_I s}{\tau_I s}\frac{1+\tau_D s}{1+0.1\tau_D s}$.

Model	μ_{RP}	k_y	k_x	τ_{Iy} min	τ_{Ix} min	τ_{Dy} min	τ_{Dx} min
N1	1.32	438	130	178.86	1.87	0.32	0.23
N2	0.84	65	45	12.19	4.31	0.51	0.47
F1	0.91	85	38	7.77	3.61	0.81	1.11
F2	0.80	38	36	6.49	5.80	1.13	0.91
Full	0.86	22	32	3.51	4.71	1.22	0.61

Table 3. μ_{RP}-values for PID-controllers of Table 2 when applied to original simplified model and to full linear model.

Controller (Table 2)	μ_{RP} Original	μ_{RP} Full model
N1	1.32	2.37
N2	0.84	1.11
F1	0.91	1.16
F2	0.80	0.95
Full	0.86	0.86

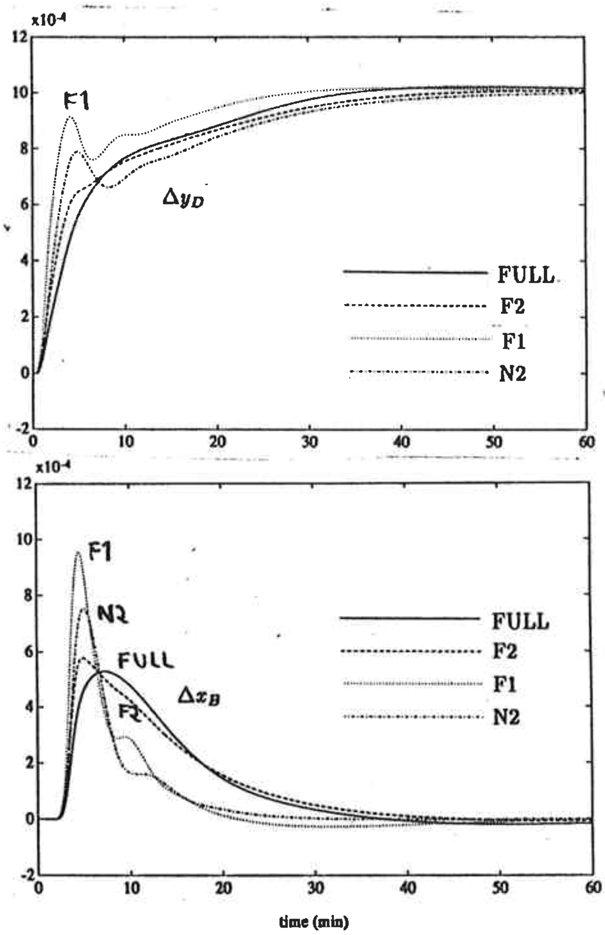

Figure 9. Two-point PID control. Nonlinear response to a set-point change in y_D. PID-settings for various models from Table 2. Model uncertainty: 1 min time delay on each input. Response for N1 is unstable and is not shown.

the controllers designed with the largest model accuracy perform best on the full-order model. As expected, model N1 gives a controller with poor performance when applied to the full model (μ_{RP} is 2.37). The three models N2, F1 and F2 all yield controllers which perform well, with F2 giving the best design; the

5 Conclusions

1. Models obtained by simply fitting open-loop responses may prove entirely useless for evaluating control behavior.

2. In step response experiments one easily gets into the nonlinear region and the obtained model is not valid at the operating point of interest. For example, the observed second-order responses for distillation columns are often an artifact caused by nonlinearity. The effect of nonlinearity may be counteracted using logarithmic compositions.

3. The low-gain directions (corresponding to internal flow changes in distillation), which are often strongly amplified under feedback control, are very difficult to observe from open-loop responses.

4. The most important model characteristic for controller tuning is the high-frequency dynamics (initial response) corresponding to the time-constant of the *closed*-loop system. Recall, for example, the famous Ziegler Nichols tuning rules which are based only on information about the plant at the frequency where the phase lag is -180° and which uses no information about the steady-state.

5. Because of the general importance of the initial response, flow dynamics should be included in the model. Modelling the effect of changes in internal flows (τ_2) is less important, but does also yield some improvement.

6. An accurate model of the steady-state is not very important for controller design. However, in order to use integral action, one must know the sign of the plant, that is, one must know the sign of the steady-state RGA (Grosdidier et al., 1985). We have shown rigorously (Appendix) that for almost any column λ_{11} for the LV-configuration must be larger than 1.

NOMENCLATURE (also see Fig. 1) .

$C(s)$ - transfer function of controller
$G(s)$ - linear model of column
$K = G(0)$ - steady-state gain matrix
k_{ij} - steady state gains for column
RGA - Relative Gain Array, elements are λ_{ij}
x_B - mole fraction of light component in bottom product
y_D - mole fraction of light component in distillate (top product)
z_F - mole fraction of light component in feed

Greek symbols

$\alpha = \frac{y_i/x_i}{(1-y_i)/(1-x_i)}$ - relative volatility
$\lambda_{11}(j\omega) = (1 - \frac{g_{12}(j\omega)g_{21}(j\omega)}{g_{11}(j\omega)g_{22}(j\omega)})^{-1}$ - 1,1-element in RGA.
ω - frequency (min^{-1})
τ_1 - dominant time constant for external flows (min)
τ_2 - time constant for internal flows (min)
$\tau_L = (\partial M_i/\partial L)_V$ - hydraulic time constant (min)
$\theta_L = (N-1)\tau_L$ - overall lag for liquid response (min)

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APPENDIX

Consider a binary separation. Define the separation factor

$$S = \frac{y_D/(1-y_D)}{x_B/(1-x_B)} \tag{9}$$

and assume that the following component and overall material balances apply

$$Fz_F = Dy_D + Bx_B \tag{10}$$

$$F = D + B \tag{11}$$

F and z_F are assumed constant in the following (disturbances not considered). Differentiating these equations yields

$$Ddy_D + Bdx_B = -(y_D - x_B)dD \tag{12}$$

$$\frac{1}{y_D(1-y_D)}dy_D - \frac{1}{x_B(1-x_B)}dx_B = d\ln S \tag{13}$$

and the steady-state gains may be evaluated by eliminating dy_D or dx_B from these equations. The 1,1-element in the RGA is given by

$$\lambda_{11} = k_{11}k_{22}/\det K \tag{14}$$

where by definition for the LV-configuration

$$k_{11} = \left(\frac{\partial y_D}{\partial L}\right)_V; \quad k_{22} = \left(\frac{\partial x_B}{\partial V}\right)_L \tag{15}$$

and by manipulating Eq.12-13 we derive for the LV-configuration

$$\det K = \frac{(y_D - x_B)}{\beta(\partial D/\partial L)_V}\left(\frac{\partial \ln S}{\partial L}\right)_D \tag{16}$$

where β is a positive constant defined as

$$\beta = \frac{D}{x_B(1-x_B)} + \frac{B}{y_D(1-y_D)} \tag{17}$$

Make the following physical assumptions

A1. $k_{11} > 0, k_{22} < 0$. This must apply for any column provided the stage efficiency is not reduced considerably as the flows increase (see Häggblom, 1988).

A2. $\left(\frac{\partial \ln S}{\partial L}\right)_D > 0$. This is equivalent to requiring $\left(\frac{\partial y_D}{\partial L}\right)_D = -\frac{B}{D}\left(\frac{\partial x_B}{\partial L}\right)_D > 0$. This means that an increase in internal flows (D constant) should improve separation.

A3. $\left(\frac{\partial D}{\partial L}\right)_V < 0$. This must hold for any column provided flooding does not occur as L is increased.

With these assumptions, which should apply to any column not operating close to flooding, we get for the LV-configuration $\det K < 0$ and $\lambda_{11} > 0$. If we in addition make the assumption

A4. $k_{21} = \left(\frac{\partial x_B}{\partial L}\right)_V > 0; \quad k_{12} = \left(\frac{\partial y_D}{\partial V}\right)_L < 0$

which is closely related to Assumption A1, then $\frac{k_{12}k_{21}}{k_{11}k_{22}}$ is positive and λ_{11} cannot be between 0 and 1 (Häggblom, 1988). We conclude that for columns where Equations 10 and 11 and Assumptions A1-A4 hold, the 1,1-element in the steady-state RGA of the LV-configuration is always larger than 1. The results may be extended to multicomponent mixtures if flows and compositions are evaluated on a pseudo-binary basis. It should also be noted that if both A1 and A4 hold, then A3 is not needed since it follows as a special case from Eq.12.