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Selecting the best distillation control structure

Sigurd Skogestad Elling W. Jacobsen

Petter Lundström

Chemical Engineering
Norwegian Institute of Technology (NTH)
N-7034 Trondheim, Norway

Abstract

The selection of an appropriate control structure (configuration) is the most important decision when designing distillation control systems. The steady state RGA is commonly used in industry as a tool for selecting the best structure. In this paper it is stressed that decisions regarding controller design should be based on the initial response (high-frequency behavior) rather than the steady state. However, for most distillation configurations the steady state RGA turns out to be a good indicator of expected control quality. One counterexample is the DB-configuration which has infinite steady-state RGA-values. However, the RGA-values at higher frequency are close to one, and good control performance is possible.

Based on a frequency-dependent RGA-analysis and optimal PI controller designs, the (L/D)(V/B)-configuration is found to be the best choice for two-point composition control. The traditional LV-configuration performs much poorer, but is preferable if one-point control is used.

1 Introduction

Consider the simple two-product distillation column in Fig. 1. From a control point it may be viewed as 5×5 system. The optimal controller should, based on all available information (measurements, process model, expected disturbances) manipulate *all* 5 inputs (L,V,V_T,D,B) in order to keep the 5 outputs (levels in top and bottom, pressure, top and bottom composition) as close as possible to their desired values. However, few columns, if any, are controlled using a full 5×5 controller. In order to simplify the design and get a control system which is simpler to understand, to retune, to make failure tolerant and which is insensitive to plant operation, a decentralized control system based on single loops is used in practice. Since the levels and pressure have to be controlled at all times to ensure stable operation, and because these control loops are essentially independent of the composition control, the level and pressure control system is designed first. Engineers often do a good effort in designing this subsysten reliably, but many fail to recognize the profound influence their choice of level controllers has on the remaining composition control problem.

As an example, consider the conventional choice of controlling pressure with cooling (V_T), top level with distillate D, and bottom level with bottoms flow B. This gives rise to the LV-configuration as shown in Fig. 2. It is given this name because the reflux L and boilup V are the remaining independent variables to be used for composition control. The DV-configuration, with completely different characteristics, is obtained if L is used to control top level instead of D. For example, whereas the compositions in the DV- configuration are only weakly sensitive to disturbances in boilup or reflux, the LV-configuration is very sensitive to such changes. In other words, the action of the level loops cause some configurations to have better 'built-in' rejection of disturbances than do others. In particular, the (L/D)(V/B)-configuration is good in this respect.

This paper addresses this important issue of control configuration selection for distillation columns, that is, which two in-

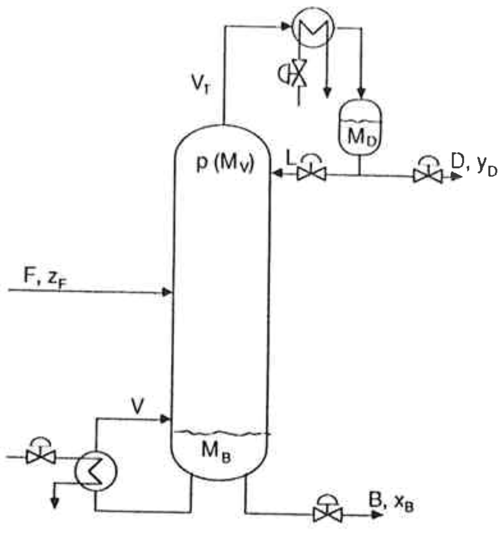

Figure 1. Two product distillation column with single feed and total condenser.

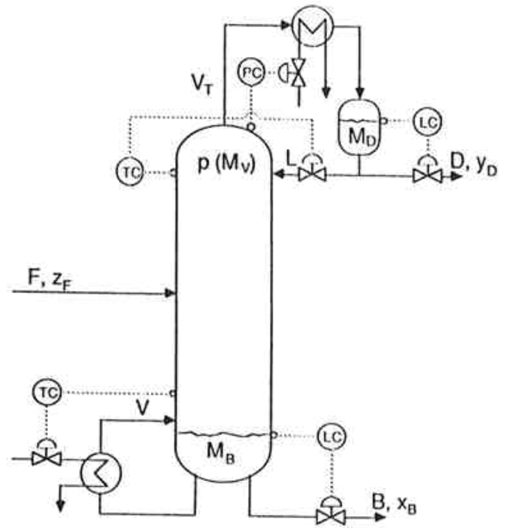

Figure 2. Two product distillation column with LV- configuration.

dependent variables to use for composition control. The issue has been discussed for quite some time by industrial people (eg., Shinskey, 1967, 1984), but has only recently been treated in any detail by academic researchers (eg., Waller, 1986, Skogestad and Morari, 1987a).

Shinskey relies on the steady state value of the RGA for selecting the best control structure. The steady state RGA contains no information about disturbances and dynamic behavior, both of which are crucial for evaluating control performance, so one would not expect that this measure by itself is sufficient to make conclusions with regards to control performance. The fact that this measure turns out to be very useful for distillation columns must therefore be a result of fortunate circumstances. One objective of this paper is to investigate this in more detail.

One counterexample to the usefullness of the steady state RGA is the DB-configuration. It involves using distillate product D and bottom product B for composition control. This control scheme has previously been labeled 'impossible' by most distillation experts (eg., Shinskey, 1984, p. 154) because its RGA is infinity at steady state. Yet, Finco et al. (1989) have recently shown both with simulations and with actual implementation that the scheme is indeed workable. We shall investigate the reasons for this in more detail in this paper.

In the paper three different modes of operation are considered with respect to the difference between the control configurations:

1. Open-loop (manual) operation
2. One-point composition control
3. Two-point composition control.

Although most industrial columns are operated with one-point composition control, the emphasis will be on the last mode. The reason is that this is the most difficult problem and has large potential for economic savings. Although energy savings are often mentioned, the increased yield of valueable product by keeping both compositions at their optimal values (thereby avoiding overpurification at the one end of the column and loss of valuable product at the other end) is probably far more important. The first mode (open-loop) is not too interesting by itself, but it is of interest for the other two modes because low open-loop sensitivity to disturbances (good 'built-in' disturbance rejection) introduces less need for feedback control and makes the remaining control problem simpler.

The analysis demonstrates that steady state data may be entirely misleading for the evaluation of control performance. For feedback control, it is the initial response, corresponding to the time constant of the closed-loop system, which is of primary importance.

Example columns. Throghout the paper we shall make use of the seven example columns A-G studied by Skogestad and Morari (1988a). Steady-state data for these columns are given in Table 1. For all examples, we assume constant molar flows, binary mixtures, constant relative volatility and perfect control of pressure and levels. However, in contrast to Skogestad and Morari (1988a) we shall here include the liquid flow dynamics in the column. The steady state holdup on all trays, including reboiler and condenser, is $M_i/F = 0.5$ min.

The steady state gain matrix for the LV-configuration is shown in Table 2 for all columns. Note that scaled (relative, logarithmic) compositions have been used:

$$\Delta y_D^S = \Delta y_D/(1-y_D); \qquad \Delta x_B^S = \Delta x_B/x_B \qquad (1)$$

Shown in Table 2 are also the time constant τ_1,τ_2 and θ_L which are used in the simplified dynamic model discussed below. $\theta_L = N\tau_L$ is overall liquid lag from the top to the bottom of the column. τ_L is computed from linearizing the Francis weir formula. Assuming in addition half of liquid over weir we get

$$\tau_L = \left(\frac{dM_i}{dL_i}\right)_V = \frac{1}{3}\frac{M_i}{L} \qquad (2)$$

The following configurations are considered in the paper: LV (often denoted "energy balance" or "indirect material balance"), DV ("material balance"), DB, (L/D)V ("Ryskamps scheme") and (L/D)(V/B) ("double ratio").

Table 1. Steady-state data for distillation column examples. All columns have liquid feed ($q_F = 1$).

Column	z_F	α	N	N_F	$1-y_D$	x_B	D/F	L/F
A	0.5	1.5	40	21	0.01	0.01	0.500	2.706
B	0.1	1.5	40	21	0.01	0.01	0.092	2.329
C	0.5	1.5	40	21	0.10	0.002	0.555	2.737
D	0.65	1.12	110	39	0.005	0.10	0.614	11.862
E	0.2	5	15	5	0.0001	0.05	0.158	0.226
F	0.5	15	10	5	0.0001	0.0001	0.500	0.227
G	0.5	1.5	80	40	0.0001	0.0001	0.500	2.635

Table 2. Data used in simple model of distillation columns, eq. 8.

Column	$G_{LV}(0)^S$	τ_1	τ_2	θ_L
A	$\begin{pmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{pmatrix}$	194	15	2.46
B	$\begin{pmatrix} 174.79 & -171.7 \\ 90.191 & -90.5 \end{pmatrix}$	250	15	2.86
C	$\begin{pmatrix} 16.023 & -16.0 \\ 9.29 & -10.7 \end{pmatrix}$	24	10	2.44
D	$\begin{pmatrix} 24.585 & -24.2 \\ 21.270 & -21.3 \end{pmatrix}$	154	30	1.54
E	$\begin{pmatrix} 203.4 & -131.5 \\ 22.47 & -22.5 \end{pmatrix}$	82	30	11.06
F	$\begin{pmatrix} 10740 & -10730 \\ 9257 & -9267 \end{pmatrix}$	2996	4	7.34
G	$\begin{pmatrix} 8648.94 & -8646 \\ 11347.06 & -11350 \end{pmatrix}$	20333	30	5.06

2 Modelling.

Because of the large number of trays in some of the columns (110 trays for column D which give a 220th order model if both composition and liquid flow dynamics are included) we choose to use the simple two time- constant dynamic model presented by Skogestad and Morari (1988a). Using a simple model also makes it easier for the reader to interpret and check the results. The two time-constant model is derived assuming the flow and compostion dynamics be decoupled, and then the two separate models for the composition and flow dynamics are simply combined. In reality, the flow dynamics do affect the composition dynamics and the model will be somewhat in error. In particular, the time constant τ_2 may be different (larger) from the one shown in Table 2 which was obtained by Skogestad and Morari (1988a). However, the results in this paper turn out to be only weakly dependent on τ_2. Let $dL_T = dL$ and dV_T represent small changes in the liquid and vapor flows in the top of the column, and let dL_B and $dV_B = dV$ be changes in the bottom. In addition to constant molar flows we make the following assumptions

1. Immediate vapor response (perfect pressure control)

$$dV_T(s) = dV(s) \tag{3}$$

2. Liquid flow from top to bottom as N lags in series and vapor has no effect on holdup

$$dL_B(s) = g_L(s)dL(s) \tag{4}$$

where

$$g_L(s) = 1/(1 + (\theta_L/N)s)^N \tag{5}$$

3. Perfect control of reboiler and condenser level

$$dL_T(s) = dL(s) = dV_T(s) - dD(s) \tag{6}$$

$$dV_B(s) = dV(s) = dL_B(s) - dB(s) \tag{7}$$

To generalize these equations to the case of not constant molar flows and to the case when changes in vapor flow affect holdups (λ in Skogestad and Morari (1988a) is nonzero) additional parameters have to be introduced in equations 3 and 4.

LV-configuration. The simplified model of the LV-configuration then becomes

$$\begin{aligned} dy_D &= \frac{g_{11}^{LV}}{1+\tau_1 s}dL + \left(\frac{g_{11}^{LV}+g_{12}^{LV}}{1+\tau_2 s} - \frac{g_{11}^{LV}}{1+\tau_1 s}\right)dV \\ dx_B &= \frac{g_{21}^{LV}}{1+\tau_1 s}g_L(s)dL + \left(\frac{g_{21}^{LV}+g_{22}^{LV}}{1+\tau_2 s} - \frac{g_{21}^{LV}}{1+\tau_1 s}\right)dV \end{aligned} \tag{8}$$

where g_{ij}^{LV} denote the steady state gains for the LV-configuration. These are given in Table 2 for scaled compositions.

Models for other configurations.

Models for other configurations are easily obtained from eq. 8 by choosing two independent variables u_1 and u_2 for composition control and expressing dL and dV in terms of these variables using equations 3 to 7. This gives

$$\begin{pmatrix} dL \\ dV \end{pmatrix} = M_{LV}^{u_1u_2}(s)\begin{pmatrix} du_1 \\ du_2 \end{pmatrix} \tag{9}$$

and the linear model for the new configuration

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = G^{u_1u_2}(s)\begin{pmatrix} du_1 \\ du_2 \end{pmatrix} \tag{10}$$

becomes

$$G^{u_1u_2}(s) = G^{LV}(s)M_{LV}^{u_1u_2}(s) \tag{11}$$

If ratios are used as independent variables these must first be expressed as a function of the actual flows, for example with $u_1 = L/D$,

$$d(L/D)(s) = (1/D)dL(s) - (L/D^2)dD(s) \tag{12}$$

DV-configuration. In this case with perfect level control we have $dL = -dD + dV$, that is,

$$M_{LV}^{DV} = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} \tag{13}$$

DB-configuration. In this case $dV = g_L(s)dL - dB$ where $dL = dV - dD$. Eliminating dV yields $dL(1-g_L(s)) = -dD - dB$ and $dV(1 - g_L(s)) = -g_L(s)dD - dB$. We obtain

$$M_{LV}^{DB}(s) = \frac{1}{1-g_L(s)}\begin{pmatrix} -1 & -1 \\ -g_L(s) & -1 \end{pmatrix} \tag{14}$$

Note that

$$\lim_{s\to 0}\frac{1}{1-g_L(s)} = \frac{1}{N\tau_L s} = \frac{1}{\theta_L s} \tag{15}$$

and $\lim_{s\to 0} g_L(s) = 1$. The elements in gain matrix for the DB-configuration will therefore approach infinity at low frequency (steady state) and it will also become singular. The physical interpretation is that a decrease in, for example, D with B constant, will immediately yield a corresponding increase in L. This increase will yield a corresponding increase in V, which subsequently will increase L even more, etc. Consequently, the effect is that the internal flows eventually will approach infinity. Mathematically, there is an integrator at low frequency: As $s \to 0$ we have $dL = dV = -(dD + dB)/\theta_L s$.

Several authors (eg. Häggblom and Waller,1986; Skogestad and Morari, 1987b; Skogestad,1988) discuss how to obtain steady state models for various configurations, but these transformations do not include the flow dynamics which are crucial for the dynamic behavior.

Example columns.

Accuracy of simplified model for Column A. To illustrate the accuracy of the simplified model the singular values and RGA-values as a function of frequency are compared with exact values in Fig. 3. The exact values were obtained by linearizing the full 82th order model which has two states (composition and holdup) for each tray. The hydraulic tray lag in the full- order model is $\tau_L = \theta_L/N = 0.06$min. From Fig. 3 we conclude that the simplified model seems to capture the true behavior quite well.

Different configurations for column A. Fig. 4 show the gain elements g_{ij} as a function of frequency for four configurations. Note in particular how the flow dynamics decouple the dynamics at high frequency.

RGA-values of Columns A-G. Fig. 5 shows the 1,1-element of the RGA as a function of frequency for five different configurations. We shall discuss these values in more detail in Section 5.1. Fig. 5 should be compared with similar figures given in Skogestad and Morari (1988a) for the case with no flow dynamics (that is, $dL_B = dL$). The introduction of flow dynamics are seen to change the results dramatically. Firstly, the response becomes decoupled at high frequency which makes RGA=I. Secondly, the importance of the time constant τ_2 associated with internal flows seems to disappear almost completely.

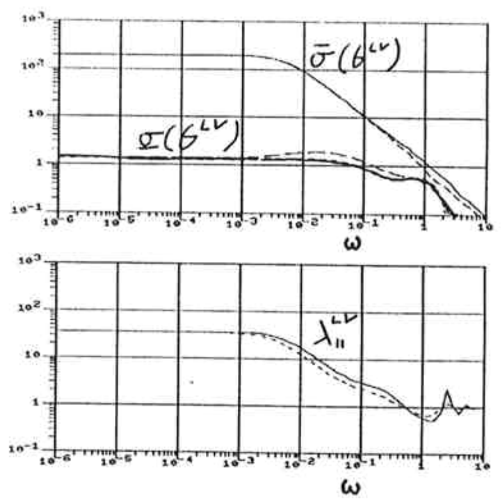

Figure 3. Singular values and λ_{11} as a function of frequency for simplified model and full model. Dotted line: Simplified model.

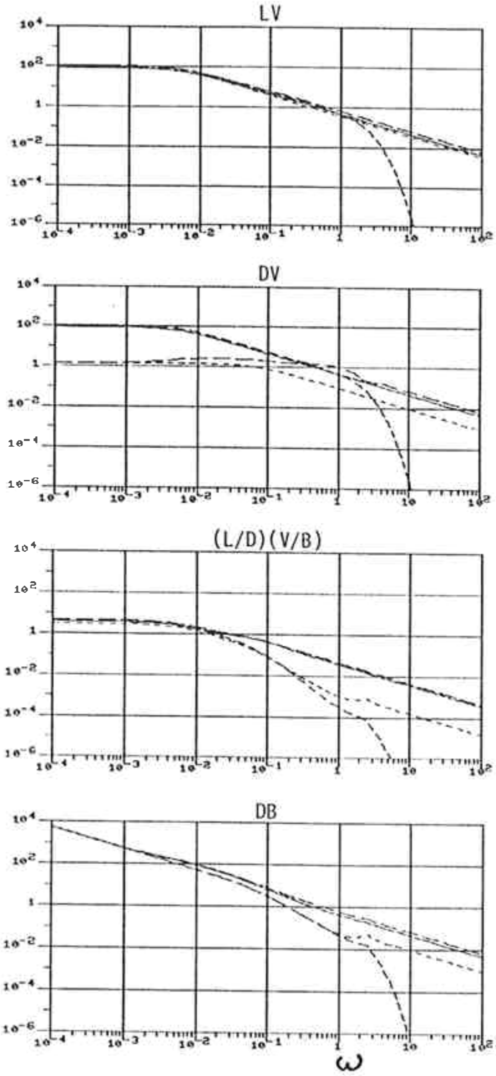

Figure 4. Gain elements as a function of frequency for column A using four different configurations: LV, DV, (L/D)(V/B) and DB.

3 Open-loop (manual) operation.

The term "open-loop" should here be put in quotes because we are not talking about an uncontrolled column, but assume the levels and pressure are perfectly controlled and consider the effect of the remaining independent variables on the compositions. For example, the open-loop operation of the LV-configuration assumes levels and pressure to be perfectly controlled with D, B and V_T, and the remaining manipulated variables, L and V, to be constant (in manual). The use of a true open-loop 5×5 model where all five flows L,V,D,B and V_T are independent variables is considered by Skogestad (1989).

Good open-loop performance is then achieved if the effect of disturbances on compositions are small, that is , if the 'built-in' disturbance rejection is good. Disturbances include changes in feed conditions (F, z_F, q_F) and disturbances on all manipulated flows L,V,D,B and V_T.

The steady-state effect of disturbances on compositions may be evaluated quite accurately by considering the steady-state effect of the disturbances on D/B or equivalently D/F. This follows since major effect on product compositions of any change in the column may be obtained by assuming the separation factor S constant (Shinskey, 1984). Differentiating the component material balance with S constant yields

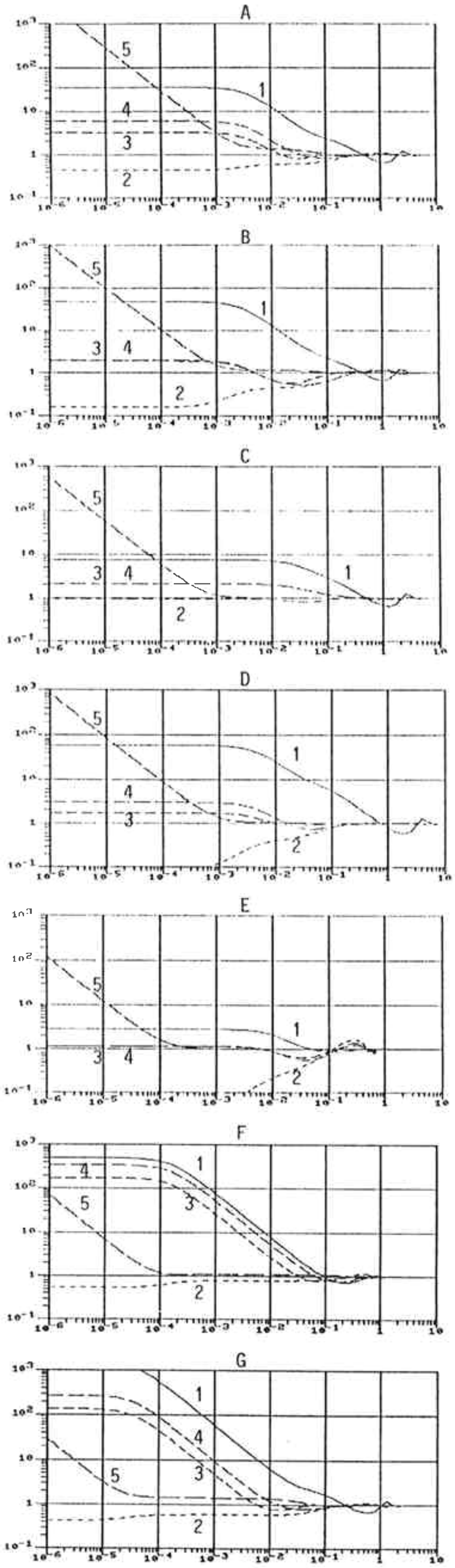

Figure 5. λ_{11}-value as a function of frequency for columns A-G using five different configurations. 1.LV 2.DV 3.(L/D)(V/B) 4.(L/D)V 5.DB

$$\frac{dy_D}{(1-y_D)y_D} = \frac{dx_B}{(1-x_B)x_B} = \frac{1}{I_S}(Fdz_F - (y_D - x_B)Fd(D/F)) \quad (16)$$

where the impurity sum is defined as $I_S = D(1-y_D)y_D + B(1-x_B)x_B$. The effect of various disturbances on $Fd(D/F)$ is given in Table 3. Configurations with small entries in Table 3 are insensitive to disturbances at steady state and should be preferred. We see that the (L/D)(V/B)-configuration is a good choice when the reflux is large. Note that the DB-configuration is not included in Table 3 because D and B are not independent at steady state. In general, the values obtained with the above approximation are quite accurate, in particular, for the gain of the least pure product. For example, the effect of a change in feed rate on y_D with constant L and V is approximately (eq.16)

$$\left(\frac{\partial y_D}{\partial F}\right)_{L,V} = -\frac{(1-y_D)y_D(y_D - x_B)}{D(1-y_D)y_D + B(1-x_B)x_B} F\left(\frac{\partial D/F}{\partial F}\right)_{L,V} \quad (17)$$

where from Table 3, $F\left(\frac{\partial D/F}{\partial F}\right)_{L,V} = 1 - \hat{q}_F - D/F$. For column A the value obtained by this approximation is 0.49 which compares quite well with the actual value of 0.394. For column C, which has an unpure top product, the values are 0.882 and 0.883.

Another interesting point is that the values in Table 3 *happen* to correlate closely with the RGA for most configurations (Skogestad, 1988). This is one of the fortunate circumstances which make the RGA useful for distillation configuration selection.

The differences between the configurations with respect to disturbance sensitivity are smaller at high frequency. Consider Fig. 6 which shows the effect of a disturbance in F on the compositions for column D for three different configurations. Although the gain is entirely different at steady state (infinity for the DB-configuration, zero for the (L/D)(V/B)-configuration) the high frequency gain (initial effect on compositions) is quite similar.

Table 3. $F\left(\frac{\partial D/F}{\partial d}\right)_{u_1,u_2}$ = Linearized effect of flow disturbances on D/F when both composition loops are open.

Configuration u_1u_2	Disturbance d: dF	dv_d	dD_d	dB_d
$LV, \frac{L}{V}V, \frac{L}{V}L$	$1-\hat{q}_F-D/F$	1	0	0
$DX, \frac{D}{X}X$	$-D/F$	0	1	0
$BX, \frac{B}{X}X$	B/F	0	0	-1
$\frac{D}{B}X$	0	0	B/F	$-D/F$
$\frac{L}{D}V, \frac{D}{L+D}V$	$-\frac{V'/F}{1+L'/D}$	$\frac{1}{1+L'/D}$	$\frac{L'/D}{1+L'/D}$	0
$L\frac{V}{B}, L\frac{B}{V+B}$	$\frac{L'/F}{1+V'/B}$	$\frac{1}{1+V'/B}$	0	$-\frac{V'/B}{1+V'/B}$
$\frac{L}{D}\frac{V}{B}, \frac{D}{L+D}\frac{V}{B}$	0	$\frac{1}{r'}$	$\frac{L'/D}{r'}$	$\frac{-V'/B}{r'}$

Applies to steady state
$dv_d = (1-\epsilon_V)dV_d - (1-\epsilon_L)dL_d$
ϵ_V and ϵ_L represent deviations from constant molar flows
$\hat{q}_F$ - fraction of liquid in feed
X - any other manipulated input u except D, B and $\frac{D}{B}$.
subscript d denotes an additive disturbance on this flow.
$V' = (1-\epsilon_V)V; \quad L' = (1-\epsilon_L)L; \quad r' = 1 + L'/D + V'/B$

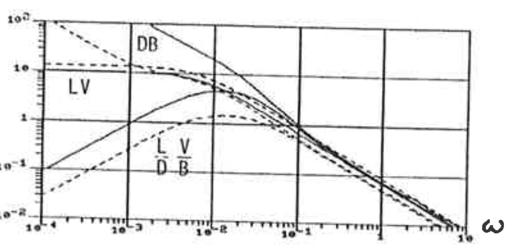

Figure 6. Effect of disturbance in F on the compositions for column D using three different configurations; LV, (L/D)(V/B), DB. Solid line: $(\delta y_D^S/\delta F)$ Dotted line: $(\delta x_B^S/\delta F)$.

4 One-point composition control

For one-point control we shall introduce the following convention: The u_1u_2-configuration means that u_1 is used for composition control and u_2 is constant (in manual). Most industrial columns use one-point control, usually of top composition, that is y_D is controlled by u_1. The composition in the other end of the column is then left uncontrolled or is possibly slowly adjusted by changing u_2 manually. The automatic control of the one composition is quite simple, and the choice of u_1 is usually not crucial, although reflux L or boilup V is usually used because they directly affect compositions. The more important choice is which manipulated variable u_2 is left in manual because this affects the behavior of the uncontrolled composition.

One should never choose u_2 to be D or B because this locks the material balance and makes it impossible to adjust D/F as desired in the case of disturbances in feed rate and feed composition (recall eq. 16). Acceptable choices for u_2 are for example L,V,L/D and V/B, and because of the strong interaction between the top and the bottom of the column one generally finds that controlling the one composition also gives reasonably good control of the 'uncontrolled' composition. Unfortunately, there is no simple method (like Table 3 for open-loop) to find the best choice for u_2. However, based on a large number of numerical evaluations of $(\partial y_D/\partial d)_{x_B,u_2}$ and $(\partial x_B/\partial d)_{y_D,u_2}$ for various disturbances d we found the best choice for u_2 to be L or V if disturbances in z_F, L and V are considered. Using a ratio, for example, u_2=L/D, is of course best for disturbances in F. However, if the feed rate is measured then using feedforward action and choosing u_2 as L/F or V/F may be equally good.

In many cases the reason for using one-point control is that the column is operating at maximum capacity. In this case u_2 is not free to choose and will be equal to the flow which is limited (for example, V if boilup or vapor flows are limiting, V_T if cooling is limiting). Again, one should not operate the column such that D or B are at their maximum values.

The conclusion is that the LV- or VL-configuration (possibly with a feedforward action from F to u_2) is the best choice for one-point *composition control*. This is probably the reason for why this choice is the preferred configuration in industry (however, as we shall see below the LV- configuration is not the best for two-point control). If reflux or boilup is large, then *level control* with D or B may be difficult. In this case the DV- or BL-configurations may be preferred for one-point control.

The (L/D)(V/B)-configuration is almost as good as the LV-configuration with respect to composition control, gives better level control if the reflux is large, and is easier transferred to two-point control. If one disregarded the problems of implementation (all flows L,V,D and B must be measured or estimated) it would have been the best overall choice.

5 Two-point control.

5.1 Two-point control and the Relative Gain Array

The 1,1-element of the RGA (denoted λ_{11} in the following) is shown as a function of frequency in Fig. 5. Traditionally, only the steady state value of the RGA has been considered (eg., McAvoy, 1983). However, newer results demonstrate that the RGA as a function of frequency yields even more information: Skogestad and Morari (1987a) showed that large RGA-values indicate a plant which is very sensitive to element-by- element uncertainty, and even more importantly, to input uncertainty, and that simple control structures (decentralized controllers) should be used for such plants. Nett (1987) have demonstrated that RGA-values close to 1 at frequencies corresponding to the closed-loop bandwidth means that single-loops controllers may be designed independently. Therefore, a column will be easy to control with single loops for a given configuration if the RGA is close to one at frequencies corresponding to the desired closed-loop bandwidth which in this case is about 0.1 min^{-1}. This statement will be the basis for the discussion that follows. Because of the liquid flow dynamics the plant will be triangular at high frequency and the RGA will aproach one for all configurations. As one measure of how easy a configuration is to control we shall consider the frequency, ω_1, where the RGA approaches one. We shall see that this frequency turns out to be closely related to the steady state RGA for most configurations (except for the DB-scheme). Consequently the steady state behavior reflects what happens at high frequency. This provides added justification for using the steady state RGA.

The following five control configurations are included in Fig. 5: $LV, DV, \frac{L}{D}\frac{V}{B}, \frac{L}{D}V, DB$. The results are discussed for each configuration:

LV-configuration: λ_{11} starts out at its steady state value until it reaches the first corner frequency where it starts falling off with a slope -1 on the logarithmic plot. Taking a close look one can for column A,B,D and G observe a small intermediate region where the slope is somewhat less than -1. This is the effect of the time constant τ_2 as shown below. λ_{11} then continues falling off with a -1 slope down to unity. For all the columns considered the frequency at which this happens is at

$$\omega_1^{LV} \approx 1/\theta_L \tag{18}$$

which is the frequency at which the response becomes decoupled. The shape of the λ_{11}-curve can be divided into three cases for the LV-structure:

(1) $\theta_L \leq \frac{\tau_1}{\lambda_{11}(0)}$. The curve breaks off at $1/\tau_1$ and there is then a flat region which starts at the frequency $1/\tau_2$.

(2) $\theta_L \approx \frac{\tau_1}{\lambda_{11}(0)}$. In this case the curve again breaks off at about $1/\tau_1$, but now the flat region has disappeared.

(3) $\theta_L \geq \frac{\tau_1}{\lambda_{11}(0)}$. The break-off frequency is no longer at $1/\tau_1$, but rather at $\omega_1^{LV}/\lambda_{11}(0)$.

The three situations are illustrated in Figure 7 for column A at four different values of θ_L. Physically, different values of θ_L may be obtained by changing the plate geometry such that the holdup is more or less affected by the liquid flows. Note that increasing the influence of the flow dynamics, that is increasing τ_L, makes the decoupling take place at lower frequency and is thereby beneficial for control purposes. The reader should however notice that there usually is a close correlation between τ_1 and τ_L, as they both depend on the amount of holdup in the column (Skogestad and Morari, 1988a). In fact, it may be shown that for well-designed columns we have $\tau_1 \approx \lambda_{11}^{LV}(0)\theta_L$ and this is indeed seen to be the case with the example columns.

$\frac{L}{D}V$-configuration: From Figure 5 we observe that the shape of the RGA-curve for this configuration follows that of the LV-configuration and that the frequency for crossing with one is reduced by a factor corresponding to the ratio between their steady-state RGA-values, that is

$$\omega_1^{(L/D)V} \approx \omega_1^{LV}\frac{\lambda_{11}^{(L/D)V}(0)}{\lambda_{11}^{LV}(0)} \tag{19}$$

where $\lambda_{11}(0)$ is the steady-state RGA-value.

$\frac{L}{D}\frac{V}{B}$-configuration: The same arguments as above apply to the this configuration and we have

$$\omega_1^{(L/D)(V/B)} \approx \omega_1^{LV}\frac{\lambda_{11}^{(L/D)(V/B)}(0)}{\lambda_{11}^{LV}(0)} \tag{20}$$

The fact that the shape of the λ_{11}-curve is similar for the $LV, \frac{L}{D}V$ and $\frac{L}{D}\frac{V}{B}$ configurations assures one that selecting the one with lowest λ_{11} value at steady state guarantees the same to hold under dynamic conditions.

DV-configuration: The λ_{11} for the DV- configuration shows a different behavior than for the three discussed above. The main difference is of course that this configuration always has $\lambda_{11}(0)$ less than 1. The steady state value of the 1,1-element in the RGA for this configuration is approximately $1/(1 + Bx_B/D(1 - y_D))$, that is , it is close to one for columns with a pure bottom product (for columns with a pure top product the LB-configuration has RGA close to one). The RGA-value becomes one at the same frequency as for the LV-configuration, that is,

$$\omega_1^{DV} \approx \omega_1^{LV} \tag{21}$$

However, for columns with a pure top product the RGA is close to one at all frequencies and this value is not too meaningful. One disadvantage with the DV-configuration for two-point control, even for columns with a pure bottom product, is that the RGA-values and thereby control behavior may depend strongly on the operating conditions.

DB-configuration: The RGA for the DB-configuration is infinity at steady state. It falls off with a -1 slope and the low-frequency asymptote crosses one at (Skogestad et al., 1989)

$$\omega_1^{DB} \approx \omega_1^{LV}\frac{1}{1/(Bx_B) + 1/(D(1-y_D))}\frac{\ln S}{L_B} \tag{22}$$

The term multiplying ω_1^{LV} is much less than one for high-purity columns and for columns with large reflux. Thus, although the RGA for the DB- configuration is much worse (higher) than for the LV-configuration at low frequency, it is significantly better (closer to one) in the frequency range important for feedback control, that is, in the frequency range from about 0.01 to 1 min^{-1}. The observation of Finco et al. (1989) that the DB-configuration gived better control performance than the LV-configuration for a propane- propylene column (similar to column D) is therefore not surprising from the RGA-values. In fact, from the RGA-values all the columns seem to be quite simple to control using the DB-configuration.

Conclusion: The discussion above shows that there is a close correlation between the RGA at steady state and its value at all frequencies. This partly explains why the use of steady state gain information for selecting the best control configuration for two point control of distillation columns may be used with success, even if it from a control point of view is the frequency (dynamic) behaviour that is important. One interesting,but not too surprising, result is that due to flow dynamics the value of λ_{11} becomes unity for moderate to high frequencies for all configurations, which implies a decoupling of the loops at these frequencies. This decoupling is beneficial for control purposes.

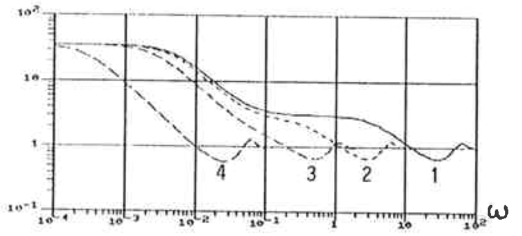

Figure 7. λ_{11} as a function of frequency for column A with four different values of θ_L. **1**.0.1 min **2**.1 min **3**.5.6 min **4**.100 min

5.2 Comparison of control performance

In this section we compare configurations for columns A and D based on their achievable control performance using single-loop PI controllers for composition control. Single-loop controllers are chosen because this is the preferred controller structure in industry. The controllers were tuned to optimize robust performance using the same weights for performance and uncertainty as used by Skogestad and Morari (1988b). This corresponds to allowing 20% error on each manipulated input and a variable time delay of up to 1 min, and the performance objective is that the worst case response should have a closed-loop time constant better than 20 min. Furthermore, the worst- case amplification of high-frequency disturbances should be less than two. Note that the controllers were optimized with respect to set-point changes, and the analysis does not take into account that some configurations are less sensitive to disturbances than others (recall Fig. 6). Mathematically, these optimal tunings were obtained by minimizing the structured singular value μ as discussed in more detail by

Skogestad and Morari (1988b). Crudely, one could say that the value of μ at a given frequency expresses the worst case weighted error. This error should be less than one at any frequency to satisfy our performance objective. Since single-loop controllers generally are insensitive to model errors we would not expect very different PI tunings if some other performance objective than μ had been used.

For two-point composition control using single loops the convention is that u_1 is used to control y_D and u_2 for x_B. Four different configurations ($u_1 u_2$) are considered: LV, DV, DB and (L/D)(V/B). The controllers parameters and the minimized peak μ-values for the 8 cases are summarized in Table 4. The μ-values are also shown as a function of frequency in Fig. 8.

The results in Table 4 correlate very well with what we expect from the RGA-values at high frequency. The μ-analysis shows that the best configuration, with two PI controllers, is the (L/D)(V/B)-configuration. For this configuration the peak μ-value is about 0.7 for both columns. This is significantly below one, and indicates that the performance requirement may be tightened, that is, we may require a faster closed-loop time constant than 20 min. In fact, the value of 0.7 is getting close to the theoretical minimum of 0.5 (the 0.5 comes from the requirement of disturbance amplification less than 2 at high frequency). The μ-value for the DB-configuration is slightly worse than the (L/D)(V/B)-configuration for column A and almost as good for column D. This is also what one would expect from the RGA-plots. The LV- and DV- configurations give considerably higher μ-values. Again, this is in accordance with the RGA-plots.

If μ is very different from one (as for the (L/D)(V/B)- configuration) then from an engineering point of view it may be better to *not* fix the performance weight, and instead vary some parameter in this weight (eg., some measure for the bandwidth) to make μ for robust performance equal to one. Different designs can then be compared based on their maximum achievable bandwidth. Two disadvantages with this approach are: 1) If the system is not even robustly stable, then it is impossible to achieve μ for robust performance equal to one by adjusting the performance weight. 2) It introduces an outer loop in the μ-calculations.

Table 4. μ-optimal PI-settings for column A and D; $C(s) = k\frac{1+\tau_I s}{\tau_I s}$. Gains k_y and k_x are for scaled compositions (eq.1).

Column	Configuration	μ	τ_{Iy} min	τ_{Ix} min	k_y	k_x
A	L, V	0.991	11.21	7.27	0.49	0.34
	$\frac{L}{D}, \frac{V}{B}$	0.676	28.37	23.70	4.25	3.98
	D, V	1.083	14.13	1.10	0.43	0.43
	D, B	0.829	31.78	30.02	0.41	0.65
D	L, V	1.090	15.07	1.46	3.96	0.47
	$\frac{L}{D}, \frac{V}{B}$	0.723	20.65	11.27	44.79	138.3
	D, V	1.053	7.93	0.51	4.08	0.32
	D, B	0.744	39.95	24.48	1.61	1.26

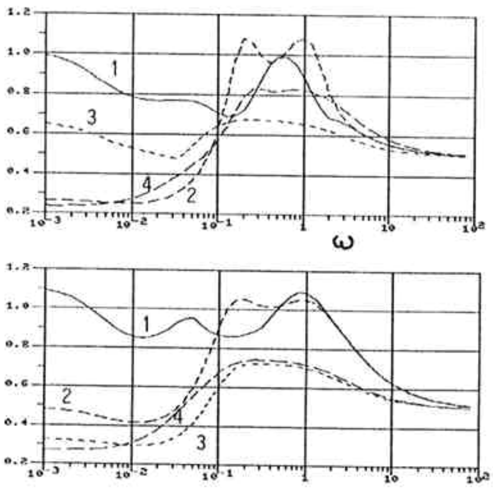

Figure 8. Structured Singular Value, μ, as a function of frequency for column A and D using four different configurations. 1.LV 2.DV 3.(L/D)(V/B) 4.DB

5.3 Simulations

Simulations of the LV-, DV-, (L/D)(V/B)- and DB-configurations for column D (a propane-propylene splitter) with a full-order nonlinear model and the PI controllers given in Table 4 are shown in Fig. 9. The correlation between these simulations and the μ-values is quite good, and the simulations support the conclusions made above.

However, one should note that the simulations shown here are for a specific choice of setpoints and model error (setpoint change in y_D and no error on the inputs and no time delay). One should not necessarily expect a correlation between a single simulation, and the worst case response (worst case combination of setpoints and model errors) for which μ is a measure. For example, although the μ-values are almost identical, the simulations in Fig. 9 indicate that the DB-configuration is better than the (L/D)(V/B)-configuration. However, with some other choice of setpoint change and model error the result may be different. This illustrates one of the main advantages of μ-analysis as opposed to simulations: one does not have to search for the worst case - μ finds it automatically.

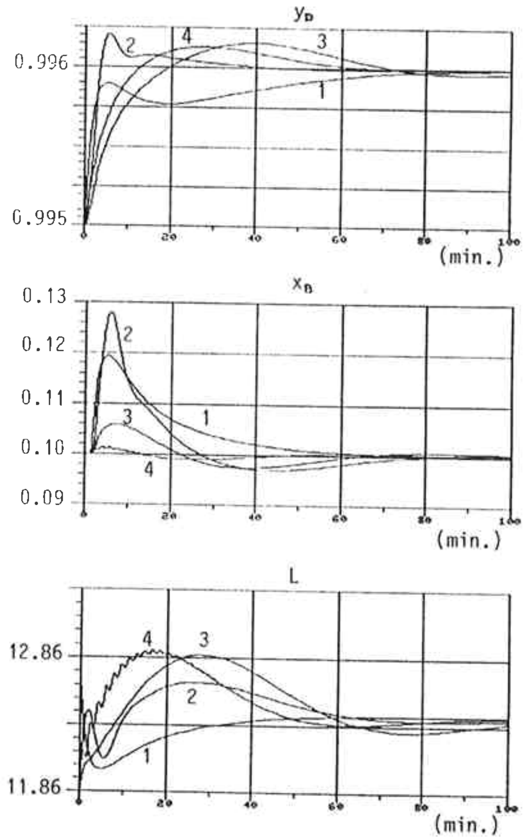

Figure 9. Time reponses for y_D, x_B and L for a setpoint-change in y_D, column D, using four different configurations: 1.LV, 2.DV, 3.(L/D)(V/B), 4.DB.

6 Conclusion

The results in this paper are summarized below. The arguments mainly refer to composition control, although comments on level control are included.

LV-configuration. A good choice for one-point control, but is not recommended for two-point control because of sensitivity to disturbances (Table 3) and relatively poor control performance due to interactions between control loops.

DV-configuration. One-point control: D must *always* be used for automatic control (never in manual). May be better than LV for columns with large reflux because top level control is simpler. Two-point control: Works poorly when bottom product is not purer than top; possibly a good choice for columns with a pure bottom product (has to be investigated further). Disadvantage: Very poor performance if failure leeds to D constant (for example measurement in top fails).

DB-configuration. Unacceptable performance if used for one-point control. Two-point control: Good control quality, in particular for column with high purity and/or large reflux. Level control also favors this configuration for columns with large reflux. The main disadvantage is that it lacks integrety; performance is very poor if failure gives D or B constant. In particular, one can not put one of the loops in manual.

(L/D)(V/B)-configuration. Overall this is the best best choice for all modes of operation. The main disadvantage is the need for measurements of all flows L,D,B and V which makes it more failure sensitive and more difficult to implement.

(L/D)V-configuration. Behaves somewhere between LV and (L/D)(V/B).

One important lesson to learn from the results on the DB-configuration is that one should be careful about making conclusions with regards to control quality based on steady state arguments. It is a fact that integral control is impossible (leads to instability) if the gain matrix may change sign (determinant changes sign). However, for the DB-configuration the steady-state gain matrix is "simply" infinite and singular, and there is no possiblity for change in sign.

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NOMENCLATURE (also see Fig. 1) .

g_{ij} - steady state gains for column
$I_s = Dy_D(1-y_D) + Bx_B(1-x_B)$ - "impurity sum"
$L = L_T$ - reflux flow rate (kmol/min)
L_B - liquid flow rate into reboiler (kmol/min)
M_i - liquid holdup on theoretical tray no. i (kmol)
q_F - fraction liquid inn feed
RGA - Relative Gain Array, elements are λ_{ij}
$S = \frac{y_D(1-x_B)}{(1-y_D)x_B}$ - separation factor
$V = V_B$ - boilup from reboiler (kmol/min)
V_T - vapor flow rate on top tray(kmol/min)
x_B - mole fraction of light component in bottom product
y_D - mole fraction of light component in distillate (top product)
z_F - mole fraction of light component in feed

Greek symbols

$\alpha = \frac{y_i/x_i}{(1-y_i)/(1-x_i)}$ - relative volatility
$\lambda_{11}(s) = (1 - \frac{g_{12}(s)g_{21}(s)}{g_{11}(s)g_{22}(s)})^{-1}$ - 1,1-element in RGA.
ω - frequency ($\min^{-1}$)
$\bar{\sigma}(G), \underline{\sigma}(G)$ - maximum and minimum singular values
τ_1 - dominant time constant for external flows (min)
τ_2 - time consant for internal flows (min)
$\tau_L = (\partial M_i/\partial L)_V$ - hydraulic time constant (min)
$\theta_L = N\tau_L$ - overall lag for liquid response (min)

REFERENCES

Finco, M. V. , W. L. Luyben and R. E. Pollek, 1989, "Control of Distillation Columns with Low Relative Volatility", *Ind.Eng. Chem. Res.*, **28**, 76-83.

Häggblom, K. E. and K. V. Waller, 1986,"Relations between steady-state properties of distillation control system structures", *Preprints DYCORD 86*, 243-248, Bournemouth, UK, Dec. 1986.

McAvoy, T.J., 1983, *Interaction Analysis*, Instrument Society of America, Triangle Park, NC, USA.

Nett, C. N., 1987, Presented at 1987 American Control Conference, Minneapolis, MN., June 1987.

Shinskey, F. G., 1967, *Process Control Systems*, McGraw- Hill, New York. Shinskey, F. G., 1984, *Distillation Control*, 2nd Edition, McGraw-Hill, New York

Skogestad, S. and M. Morari, 1987a, "Control Configuration Selection for Distilation Columns", *AIChE Journal*, **33**, **10**, 1620-1635.

Skogestad, S. and M. Morari, 1987b, "A Systematic Approach to Distillation Column Control", Distillation 87, Brighton, Published in *I.Chem.E. Symposium Series*, **104**, A71- 86.

Skogestad, S. and M. Morari, 1987c "Implication of Large RGA-Elements on Control Performance", *Ind. & Eng. Chem. Res.*, **26**, 11, 2121-2330.

Skogestad, S., 1988, "Disturbance Rejection in Distillation Columns", *Proc. CHEMDATA 88*, 365-371, 19th European Symp. on Computer Applications in the Chemical Industry, Göteborg, Sweden, June 1988.

Skogestad, S. and M. Morari, 1988a, "Understanding the Dynamic Behavior of Distillation Columns", *Ind.Eng.Chem.Res*, **27**, 10, 1848-1862.

Skogestad, S. and M. Morari ,1988b, "LV-control of a High-Purity Distillation Column", *Chem. Eng. Sci.*, **43**, 1, 33-48.

Skogestad, S., 1989, "Modelling and Control of Distillation Columns as a 5x5 system", Proc. CACHI 89, 20th European Symp. on Computer Applications in the Chemical Industry, Erlangen, BRD, April 1989.

Skogestad, S., E.W.Jacobsen and M. Morari, 1989, "DB-control of distillation columns", Work in progress.

Waller, K, 1986, "Distillation Control System Structures", *Preprints DYCORD 86*, 1-10, Bournemouth, UK, Dec. 1986.