MU-OPTIMAL PID SETTINGS FOR DISTILLATION COLUMNS

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AIChE, Wash. DC
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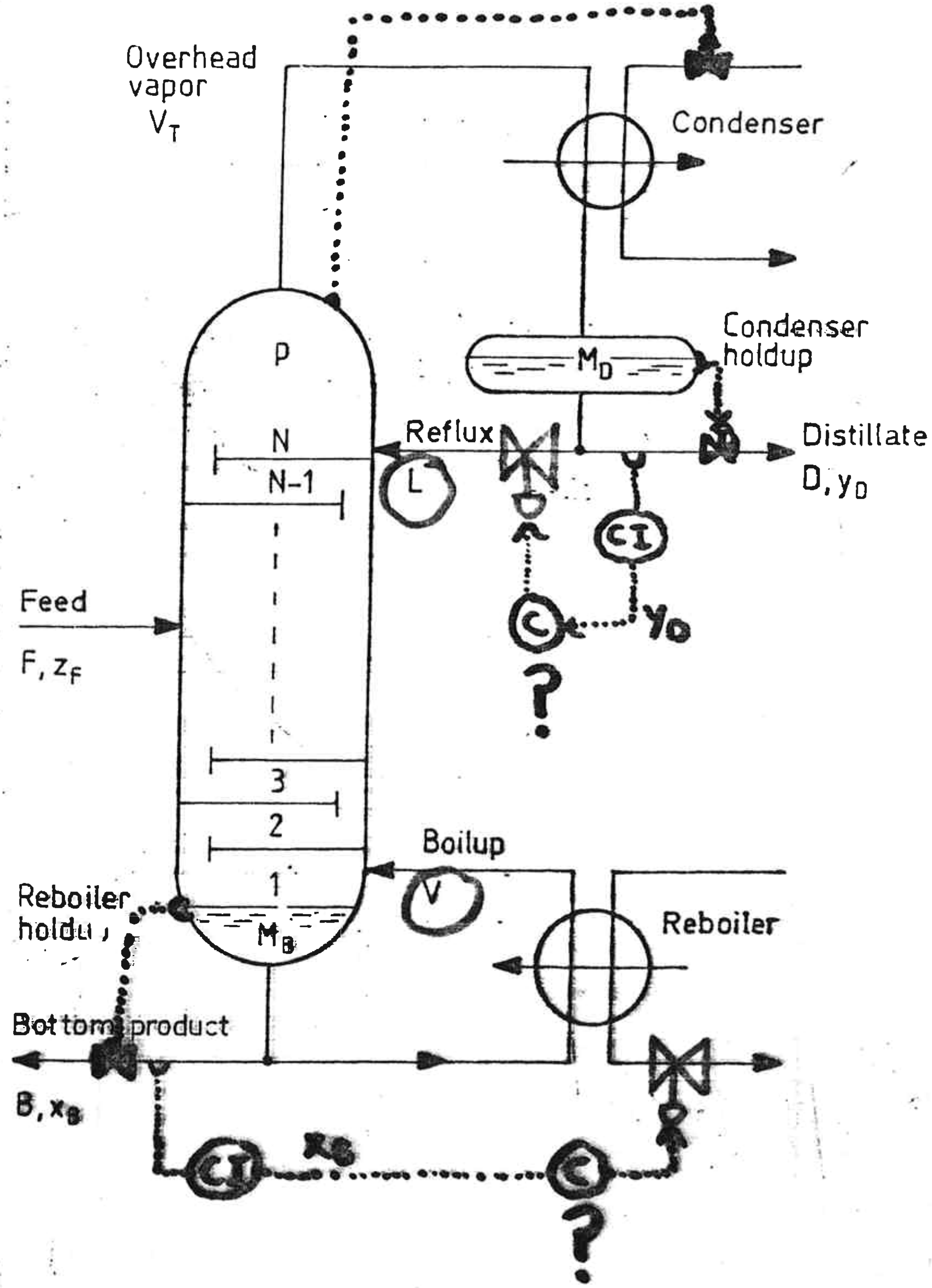
CONSIDER

- TWO-POINT COMPOSITION CONTROL
- CONVENTIONAL CONTROL (LV)
- SINGLE LOOPS



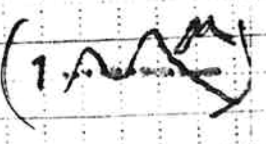
HOW GOOD IS THIS?

- COMPARE CONTROLLERS BASED ON "ROBUST PERFORMANCE"

WORST CASE COMBINATION OF DISTURBANCES and MODEL ERRORS.

DOES "WORST CASE" RESPONSE SATISFY PERFORMANCE REQUIREMENT?

YES $\Leftrightarrow \mu_{RP} < 1$ 

NO $\Leftrightarrow \mu_{RP} > 1$ 

μ: STRUCTURED SINGULAR VALUE (Doyle, 1982)

$\boxed{\Rightarrow \text{MINIMIZE } \mu_{RP}}$.

1. PLANT MODEL

2. UNCERTAINTY:

" - 20 % FOR EACH INPUT
- 0.1 MIN DELAY " } (*)

↓

MAGNITUDE OF INPUT UNC.

UNCERTAINTY WEIGHT

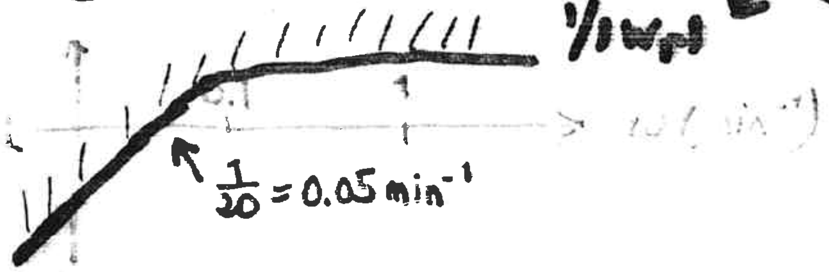
$|W_I|$

3. PERFORMANCE:

" - WANT CLOSED-LOOP TIME CONSTANT LESS THAN 20 MIN. " } (**)

↓

UPPER BOUND ON $\bar{\sigma}(S)$



$\frac{1}{20} = 0.05 \text{ min}^{-1}$

PERFORM. WEIGHT

$1/|W_P|$

μ-ANALYSIS:

$\mu_{NP} < 1$: PERFORMANCE (**) WITH NO UNC.

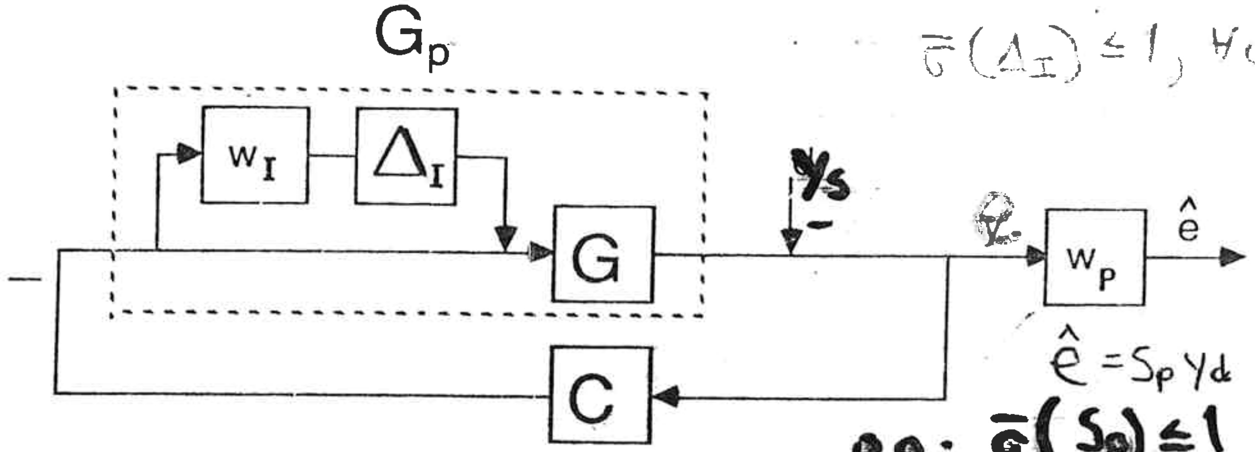
$\mu_{RS} < 1$: STABLE WITH UNCERTAINTY (*)

→ $\mu_{RP} < 1$: PERFORMANCE (**) WITH UNC. (*)

RP WITH INPUT UNCERTAINTY (Δ_I)

$\bar{\sigma}(\Delta_I) \le 1, \forall \omega$

RP:

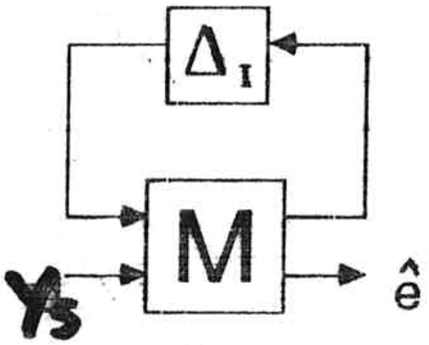


$\hat{e} = S_p y_d$

RP: $\bar{\sigma}(S_p) \le 1$ $\forall \Delta_I$

$\Updownarrow$

RP:

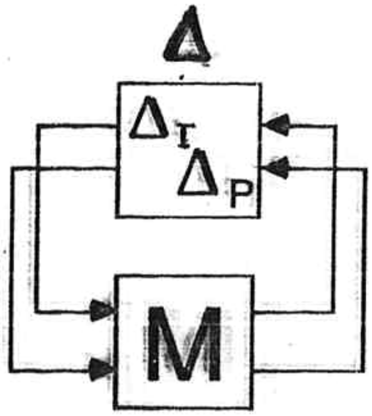


$$M = \begin{bmatrix} w_I CSG & w_I CS \\ w_P SG & w_P S \end{bmatrix}$$

RP: $\hat{e} = S_p d$
$\bar{\sigma}(S_p) \le 1 \quad \forall \Delta_I, \forall \omega$

$\Updownarrow$

RS:



Δ_I : diagonal maxtrix $(n \times n)$

Δ_P : "full" matrix $(n \times n)$

RP $\overset{\text{def.}}{\Leftrightarrow}$ $\bar{\sigma}(w_P(I+G_pC)^{-1}) \le 1, \quad \forall \omega, \forall G_p$

DOYLE (1982) $\rightarrow$ $\Leftrightarrow$ $\mu_\Delta(M) \le 1, \forall \omega$
and NS (M stable)

Given

- TWO-POINT CONTROL
- LV-CONFIGURATION
- SINGLE LOOPS (PI/PID)
- OPTIMIZE ROBUST PERFORMANCE (μ)

QUESTIONS:

1) Single loops close to μ-optimal?
2) Is robust control ($\mu < 1$) possible with LV-configuration?
3) Tuning rules, τ_I = ?
4) Sensitivity of results
   i) Model
   ii) Add disturbance in F and z_F
   iii) Performance and unc. weights
   iv) Operating point
5) One-point control, same tunings work?
6) Other configurations better?

① Single loops close to μ-optimal ?

Consider Column "A":

- $N = 40$, $\alpha = 1.5$, $z_F = 0.5$
  $1 - y_D = x_B = 0.01$
  $q_F = 2.7$
- $M_i/F = 0.5$ min (all trays)

COLUMN A

Approximate two time constant model

$$G_1^S(s) = \begin{array}{cc} \Delta L & \Delta V \\ \left(\begin{array}{cc} \dfrac{87.8}{1+\tau_1 s} & -\dfrac{87.8}{1+\tau_1 s} + \dfrac{1.4}{1+\tau_2 s} \\ \dfrac{108.2}{1+\tau_1 s} & \dfrac{-108.2}{1+\tau_1 s} - \dfrac{1.4}{1+\tau_2 s} \end{array} \right) & \begin{array}{l} \rightarrow \Delta Y_D \\ \rightarrow \Delta X_B \end{array} \end{array}$$

with $\tau_1 = 194$ min and $\tau_2 = 15$ min.

- Uses: gains + τ_1 + τ_2
- Scaled (logarithmic) compositions

  $Y_D = \ln(1-y_D)$

  $X_B = \ln x_B$

- τ_1: External flows

  τ_2: Internal flows

$$\boxed{\min_{C} \mu_{RP}}$$

μ- optimal PI and PID controllers:

$$C_{PI} = K_c \frac{1+\tau_I s}{\tau_I s}$$

$$C_{PID} = K_c \frac{1+\tau_I s}{\tau_I s} \frac{1+\tau_D s}{1+\underbrace{\alpha}_{0.1}\tau_D s}$$

- Optimal K_c, τ_I, τ_D obtained by minimizing μ_{RP} ("Brute force")

ACC 87

Two years ago (~~AIChE 86~~)
I told you PI controller
was "sluggish"

WRONG (if right settings
are used)

In fact

MULTI-VAR. $C_{1\mu}$: $\mu^{*}_{RP} = 0.95$ "old" "μ-optimal" (using H^{∞}-software)

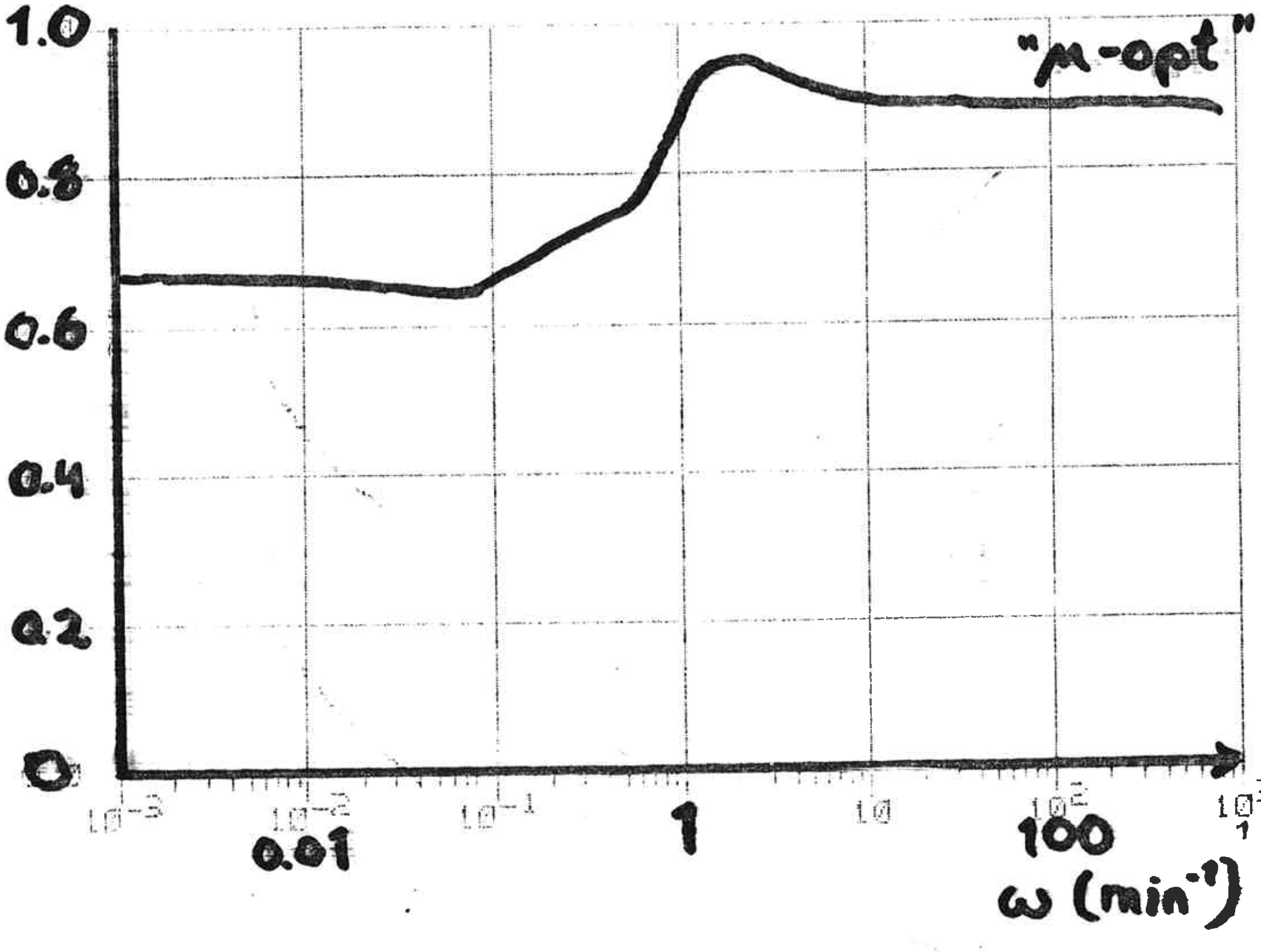
SINGLE LOOPS

PI: $\mu^{*}_{RP} = 0.92$

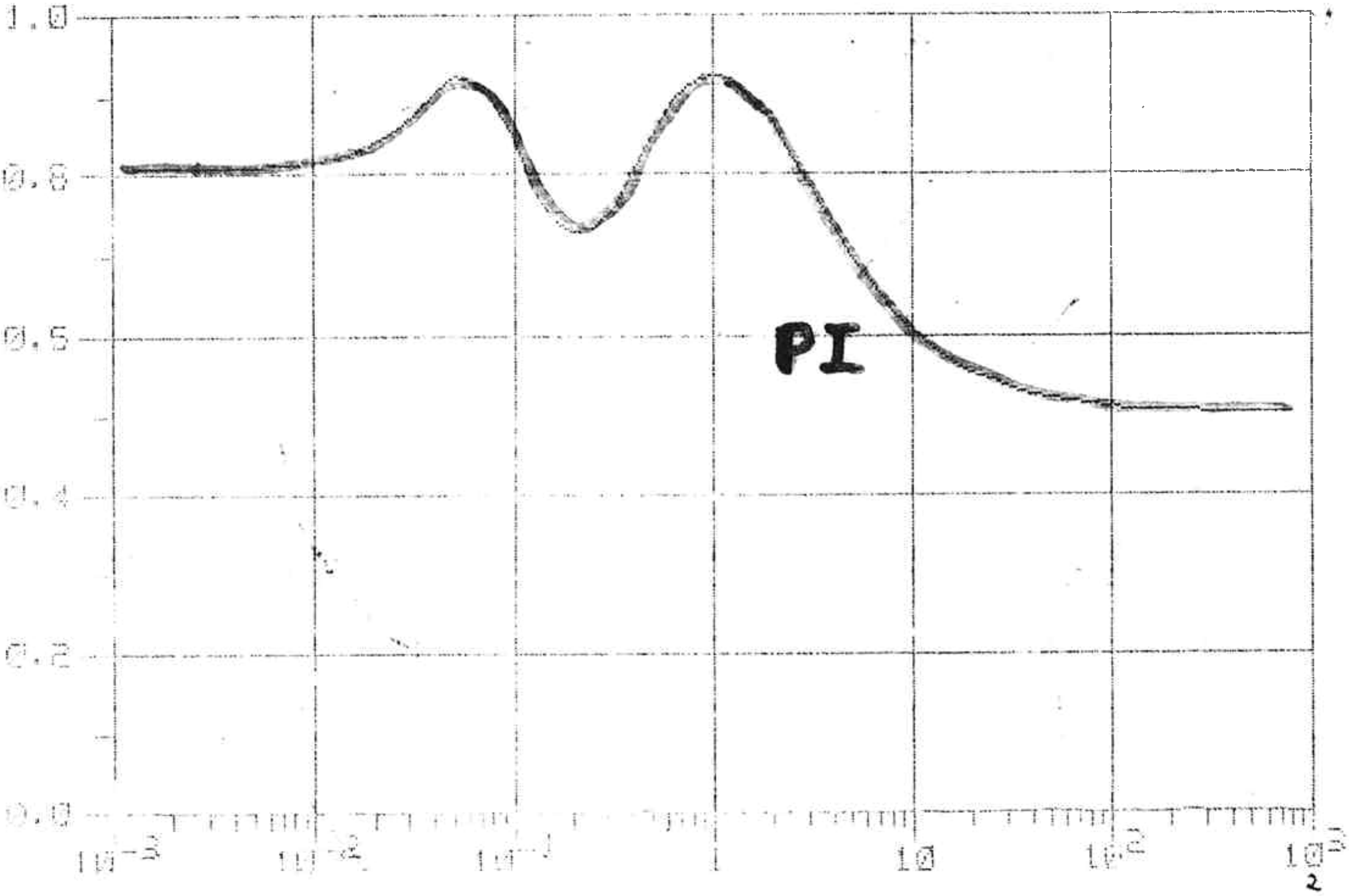
Better than the "optimal"!

PID: $\mu^{*}_{RP} = 0.85$

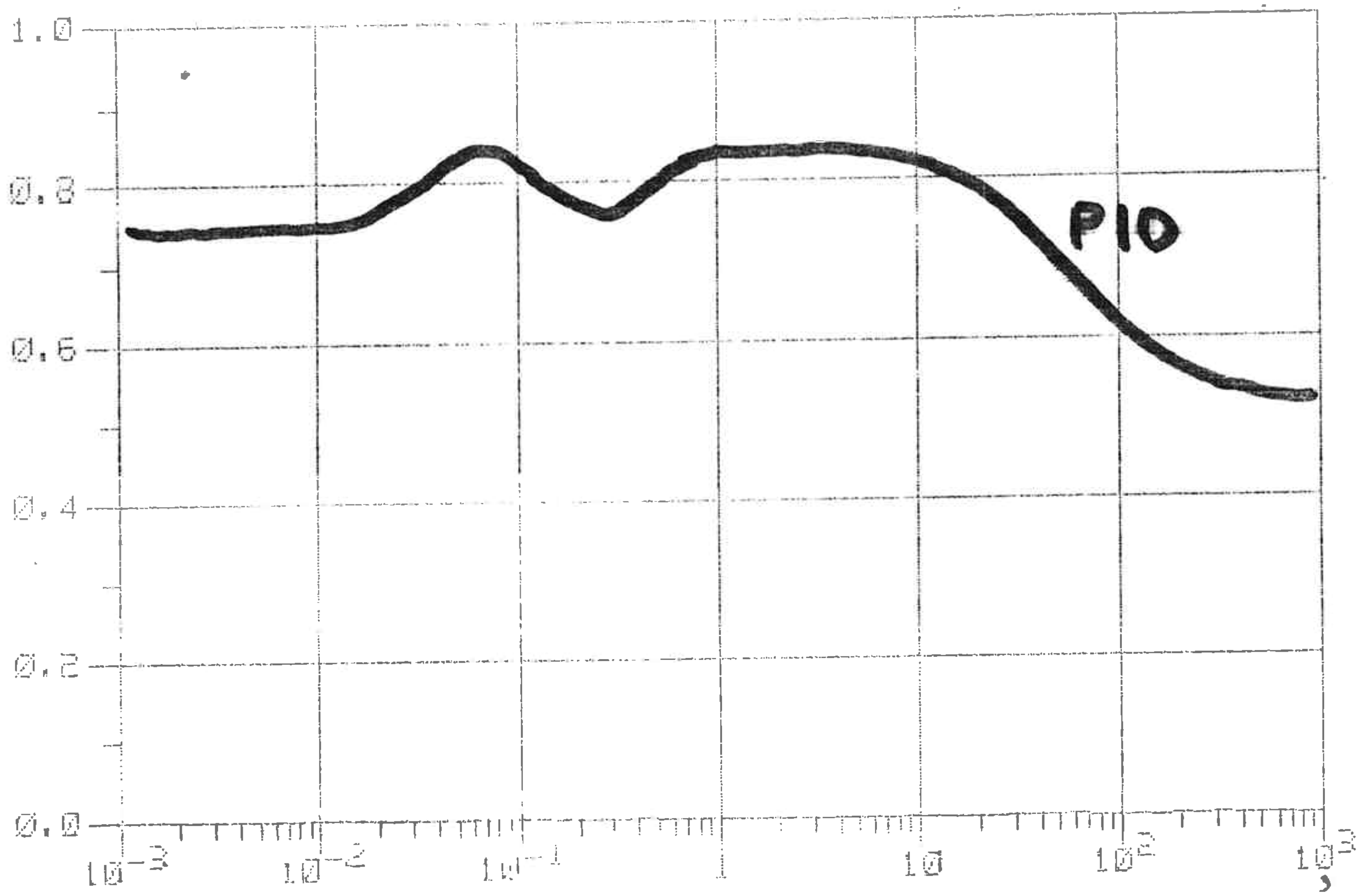
Even better!!



"μ-opt" : μ_{∞} = 0.95

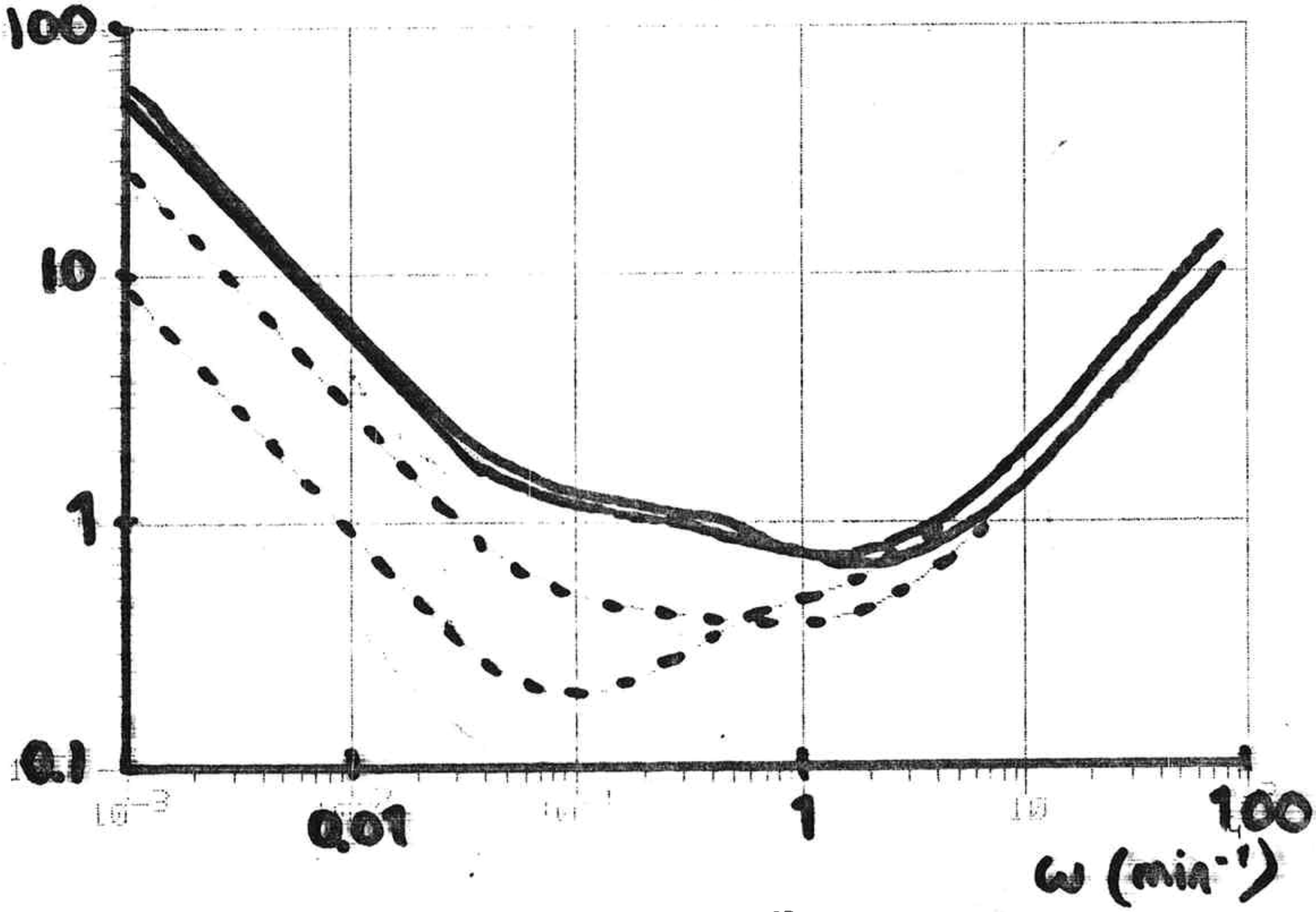


PI : μ_{RP} = 0.92

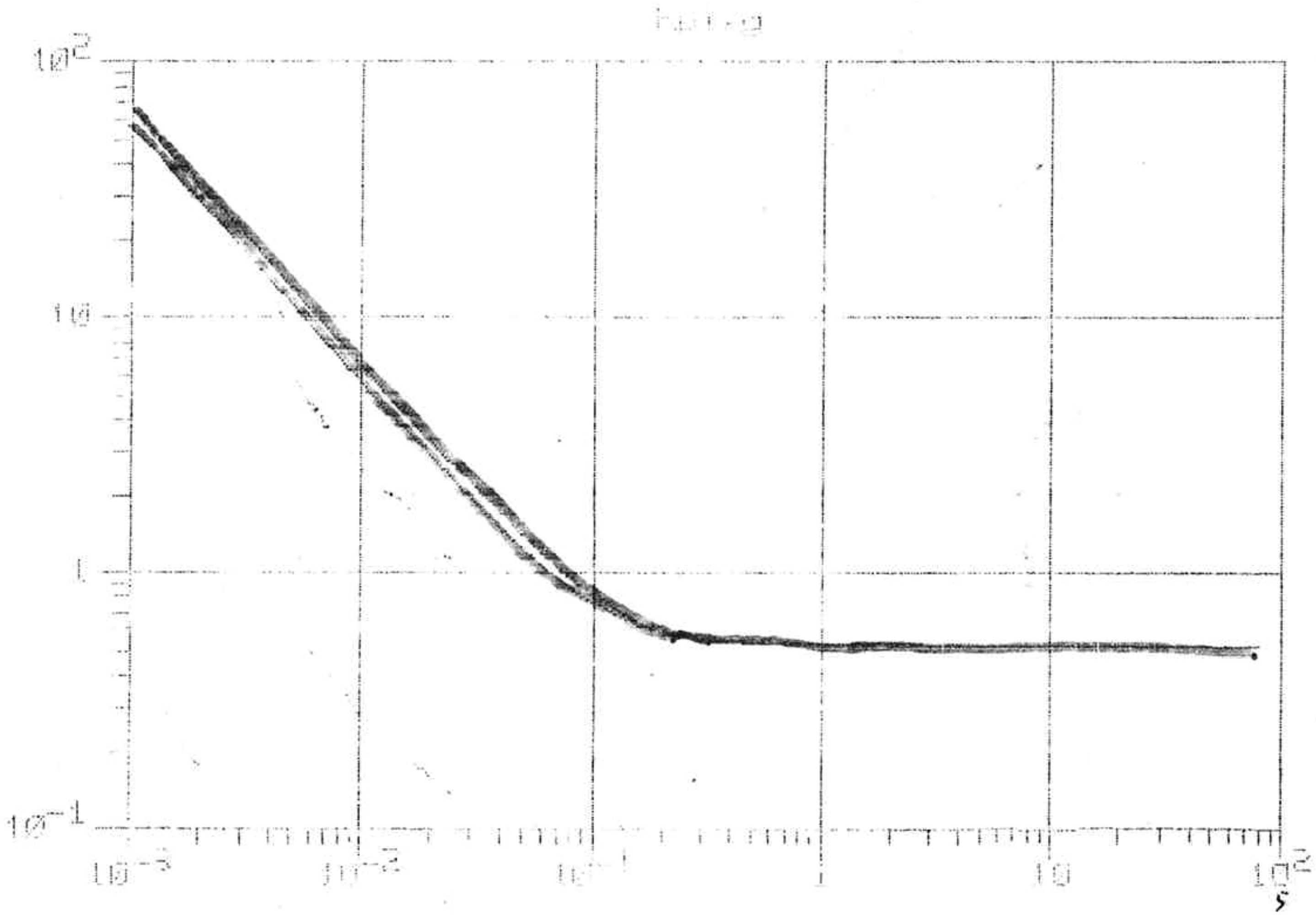


PID : μ_{RP} = 0.85

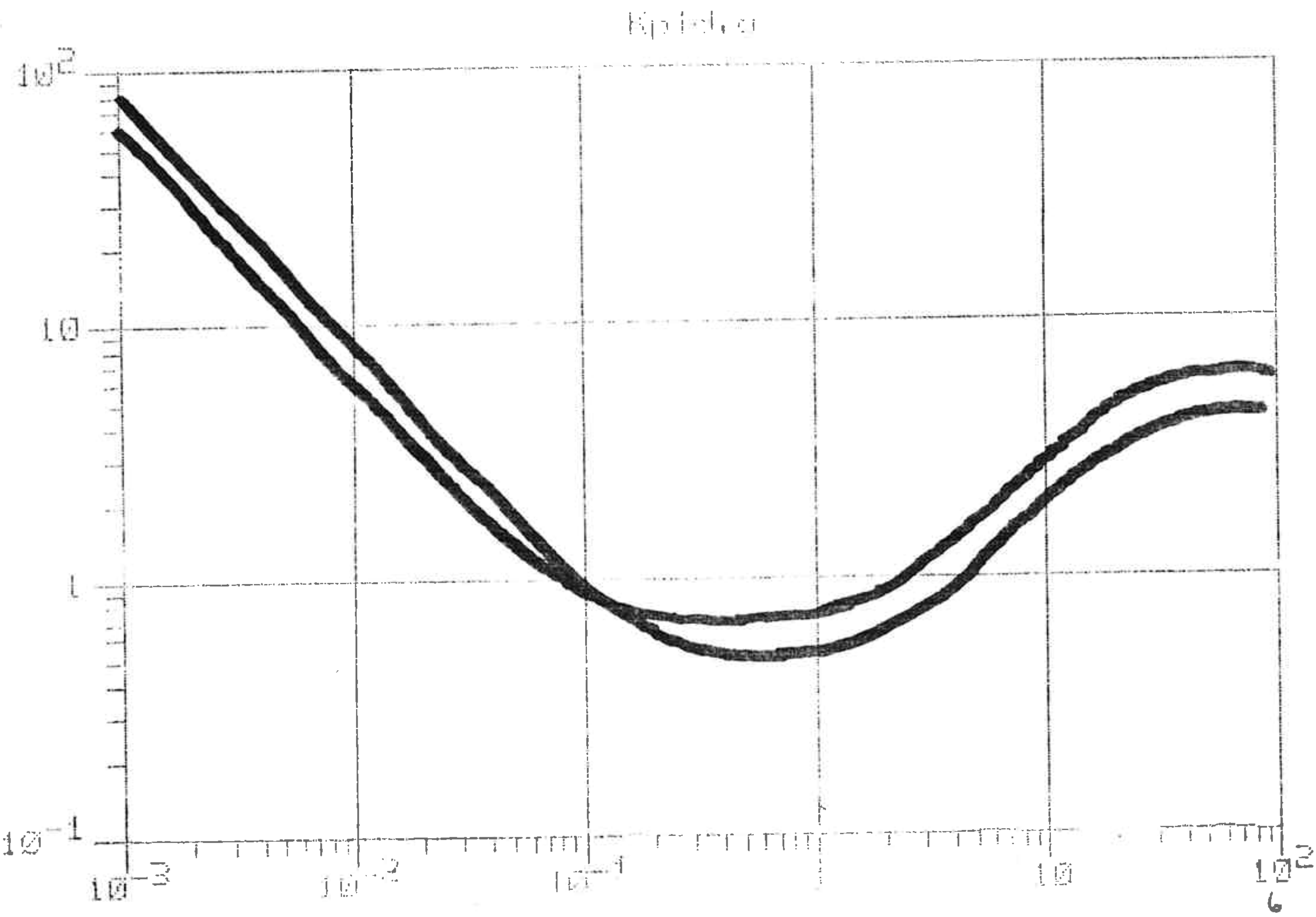
CONTROLLER GAINS



— "μ-optimal"



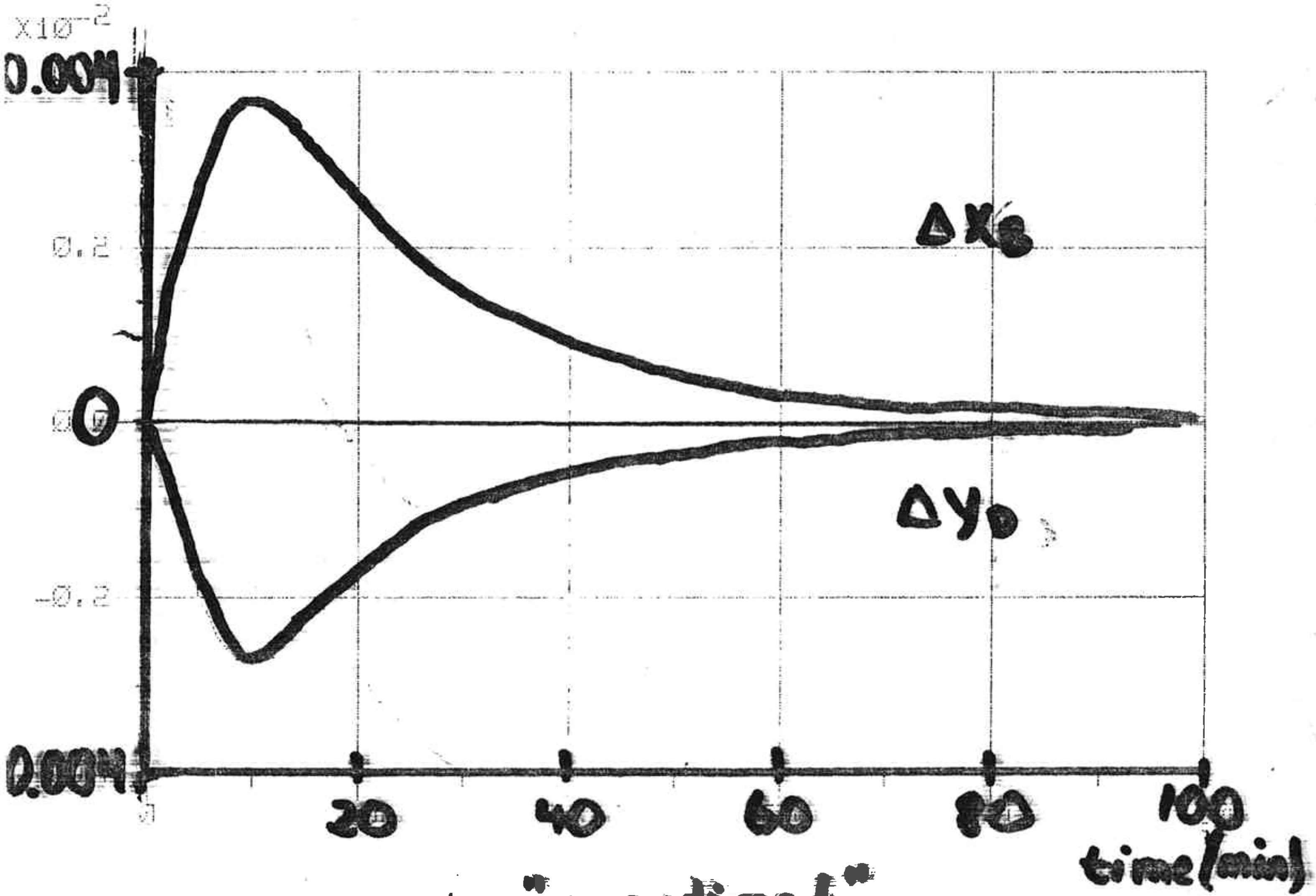
— PE



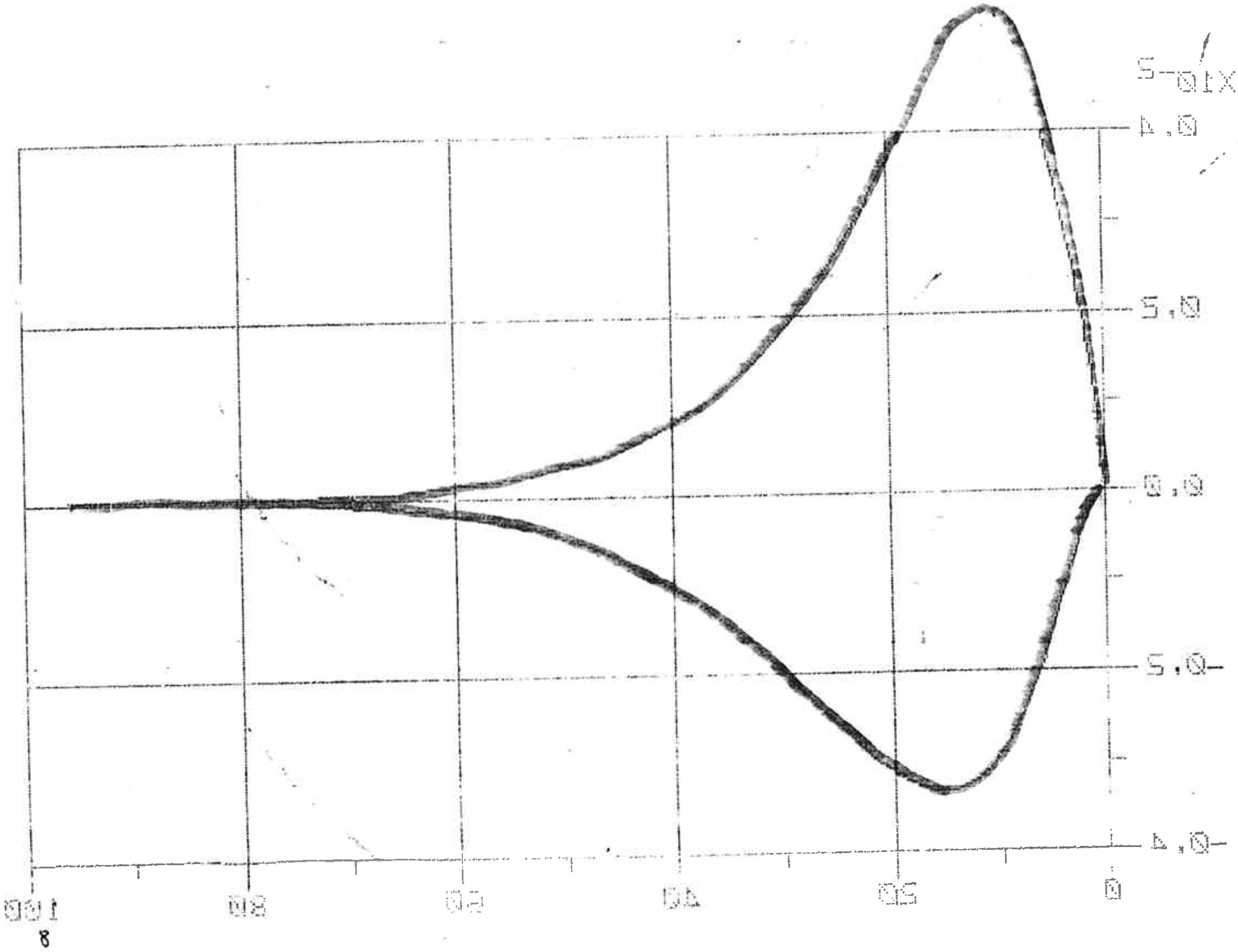
— PID

~~F: + 20%~~

z_F: + 20%



— "μ-optimal"



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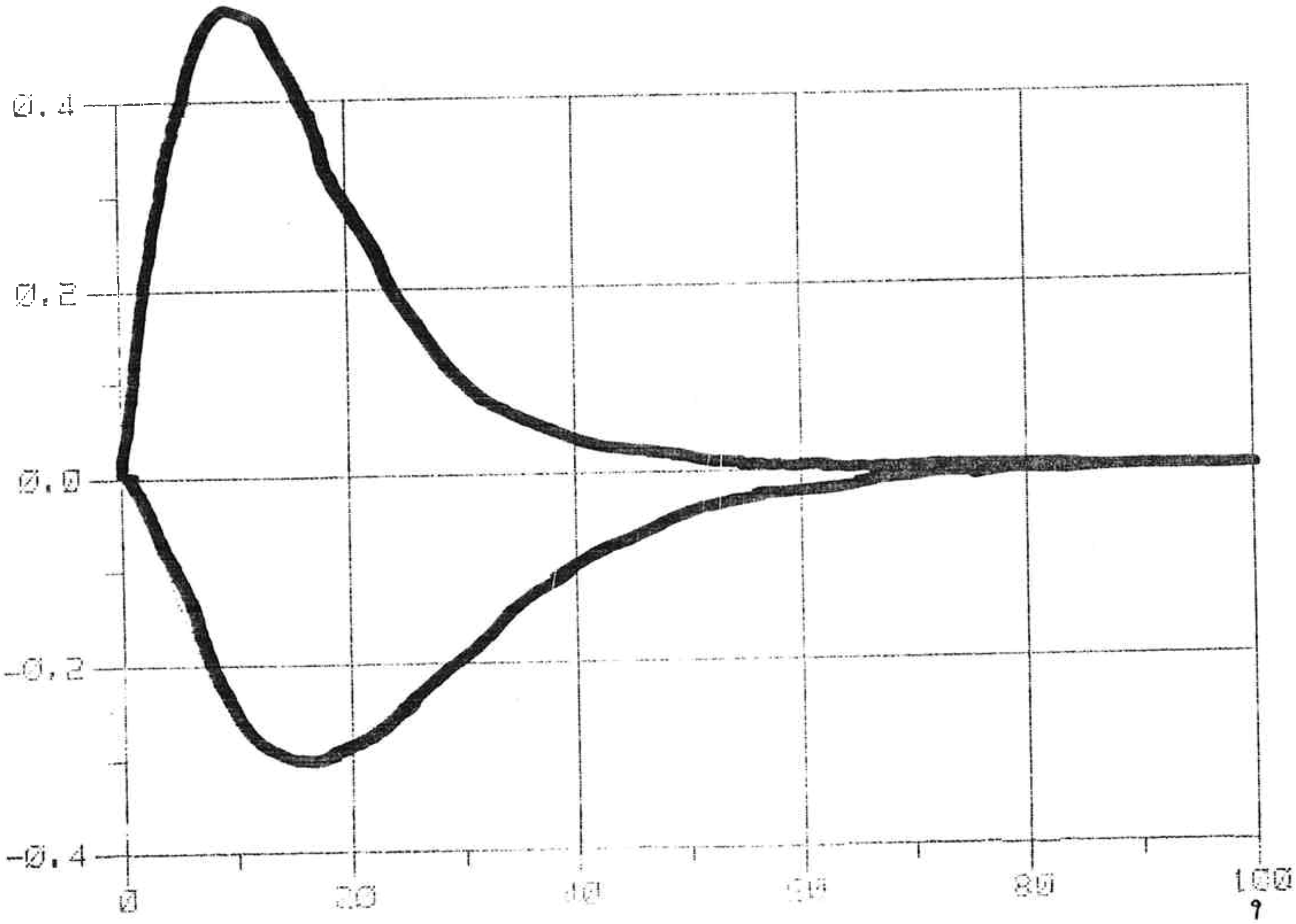
DISTURBANCE RESPONSE

(not designed for)



— PID

(1)

$\boxed{\text{CONCLUSION: YES, SINGLE-LOOP CONTROL IS CLOSE TO } \mu\text{-OPTIMAL}}$

LV-configuration for "column A"

(2) Is robust control ($\mu < 1$) possible with LV-configuration?

- Consider single-loop (PI, PID) control of 7 example columns

- Same weights, same kind of model as above

- Compute optimal μ_{RP} (minimize wrt. K_c, τ_I, τ_D)

EXAMPLE COLUMNS

Column	z_F	α	N	N_F	$1-y_D$	x_B	D/F	L/F	N_{min}	$(L/F)_{min}$
A	0.5	1.5	40	21	0.01	0.01	0.500	2.706	22.7	1.95
B	0.1	1.5	40	21	0.01	0.01	0.092	2.329	22.7	1.82
C	0.5	1.5	40	21	0.10	0.002	0.555	2.737	20.7	1.66
C_3-split D	0.65	1.12	110	39	0.005	0.10	0.614	11.862 !	66.1	7.76
E	0.2	5	15	5	0.0001	0.05	0.158	0.226	7.55	0.197
F	0.5	15	10	5	0.0001	0.0001	0.500	0.227	6.80	0.071
G	0.5	1.5	80	40	0.0001	0.0001	0.500	2.635	45.4	2.00

Table I. Steady-state data for distillation column examples. All columns have liquid feed ($q_F = 1$).

Dynamic:
- Immediate flow response
- M_i/F = 0.5 min, all trays

GAINS FOR LV

Column	$G_{LV}(0)^S$	$\gamma(G_{LV}(0)^S)$	RGA $\lambda_{11}(G_{LV}(0))$
A	$\begin{pmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{pmatrix}$	141.7	35.1
B	$\begin{pmatrix} 174.79 & -171.7 \\ 90.191 & -90.5 \end{pmatrix}$	229.2	47.5
C	$\begin{pmatrix} 16.023 & -16.0 \\ 9.29 & -10.7 \end{pmatrix}$	31.3	7.53
D	$\begin{pmatrix} 24.585 & -24.2 \\ 21.270 & -21.3 \end{pmatrix}$	234.9	58.7
E	$\begin{pmatrix} 203.4 & -131.5 \\ 22.47 & -22.5 \end{pmatrix}$	36.7	2.82
F	$\begin{pmatrix} 10740 & -10730 \\ 9257 & -9267 \end{pmatrix}$	2014	499
G	$\begin{pmatrix} 8648.94 & -8646 \\ 11347.06 & -11350 \end{pmatrix}$	6939	1673

DIFFICULT COLUMNS!

TIME CONSTANTS (min)

Column	$\tau_1 = \tau_{1e}$	τ_2
A	194	15
B	250	15
C	24	10
D	154	30
E	82	30
F	2996	4
G	20333	30
	EXTERNAL FLOWS	INTERNAL FLOWS

MINIMIZE μ_{RP} wrt. K_c, τ_I, τ_D

	PI	PID	
A	0.92	0.85	
B	0.95	0.85	
C	0.94	0.87	
D	1.23	1.14	(C_3-splitter)
E	0.86	0.75	
F	1.00	0.83	
G	0.97	0.90	

Table. Optimal μ_{RP}-values

2 **CONCLUSION: Yes, robust control ($\mu \le 1$) is possible**

3 TUNING RULES

	$\tau_{I1} = \tau_{I2} = \frac{\tau_2}{2}$ (min)	τ_{D1} (min)	τ_{D2} (min)	K_{C1}	K_{C2}	% higher μ_{RP}
A	7.5	0.5	0.4	0.6	0.6	1%
B	7.5	0.5	0.4	0.6	0.6	2%
C	5	0.5	0.4	0.6	0.5	<1%
D	15	0.3	0.3	5.6	5.9	<1%
E	15	0.4	0.5	0.1	1.0	<1%
F	2	0.5	0.2	0.7	0.7	6%
G	15	0.5	0.8	0.02	0.09	1%

Table. μ-optimal tuning parameters for PID controller. τ_I fixed $= \tau_2/2$.
1: Top loop ; 2: Bottom loop

CONCLUSION

EQUAL FOR BOTH LOOPS!
$$\left\{\begin{aligned} \tau_I^* &\approx \tau_2/2 \\ \tau_D^* &\approx \theta_d/2 \end{aligned}\right.$$

TIME CONSTANT FOR <u>INTERNAL</u> FLOWS

DEAD TIME (1 min in examples)

④ SENSITIVITY OF RESULTS (μ^*_{RP})

i) **MODEL** (Use N+1'th order instead of simplified 2nd order)

Yes, quite sensitive for some columns (Col. D, E)

New $\mu^*_{RP} > 2$

ii) **ADD DISTURBANCE** (F, z_F)

No, insensitive (setpoints "worst case") for two-point control.

iii) **PERFORMANCE AND UNCERTAINTY WEIGHT**

W_P W_I

M-value quite sensitive, but tuning <u>rules</u> are <u>not</u>.

iv) **Operating point** (AIChE 86; CES, 1, 1988)

Quite insensitive

(Gains change; counteracted by using <u>logarithmic</u> compositions.)

(5)

ONE POINT CONTROL, same tunings work?

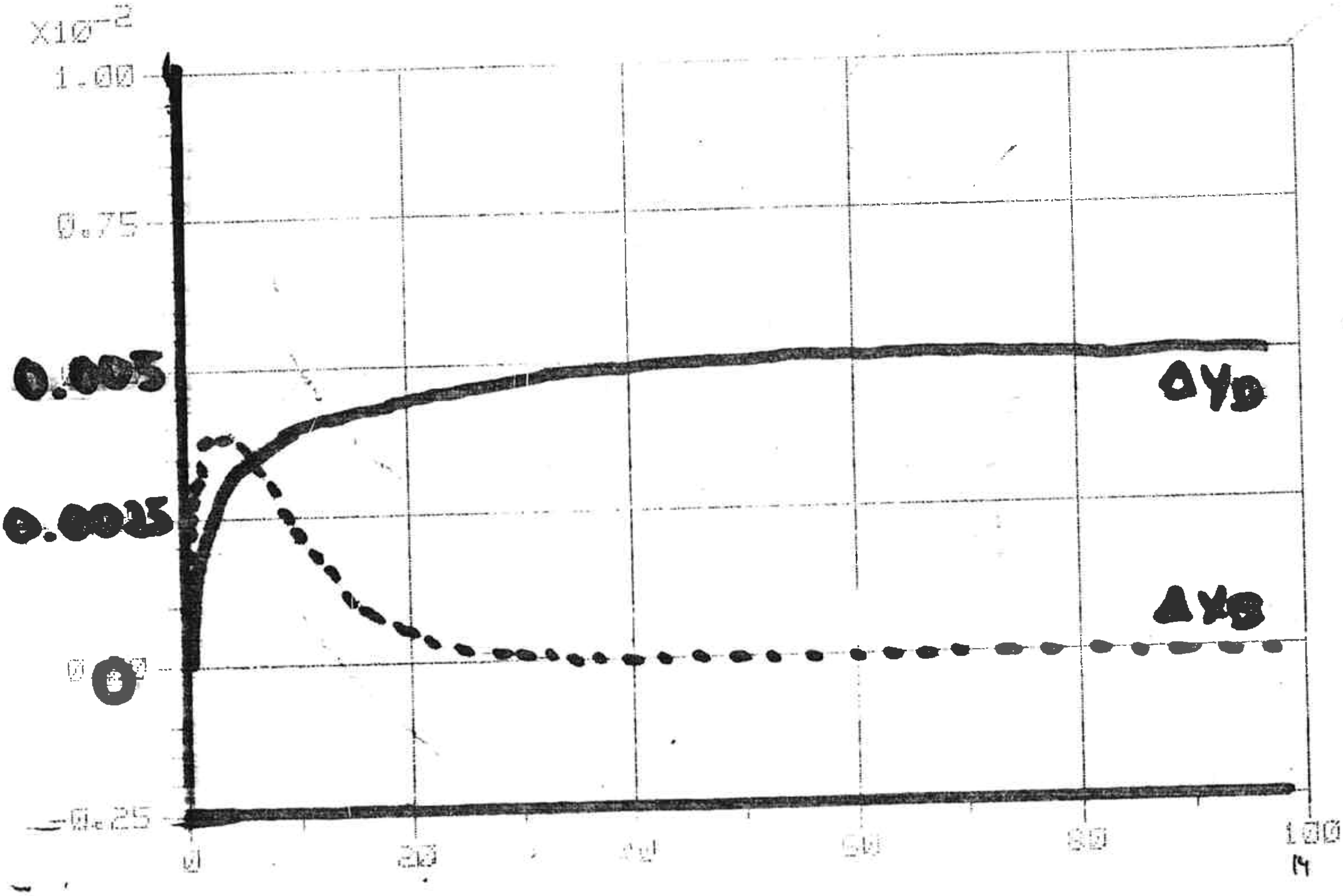
<u>Column A</u>: <u>Yes</u> (Surprising

since $\underset{y_D\text{-loop}}{\lambda_{11}} = \frac{g_{OL}}{g_L} = 35$ $(\omega = 0)$.)

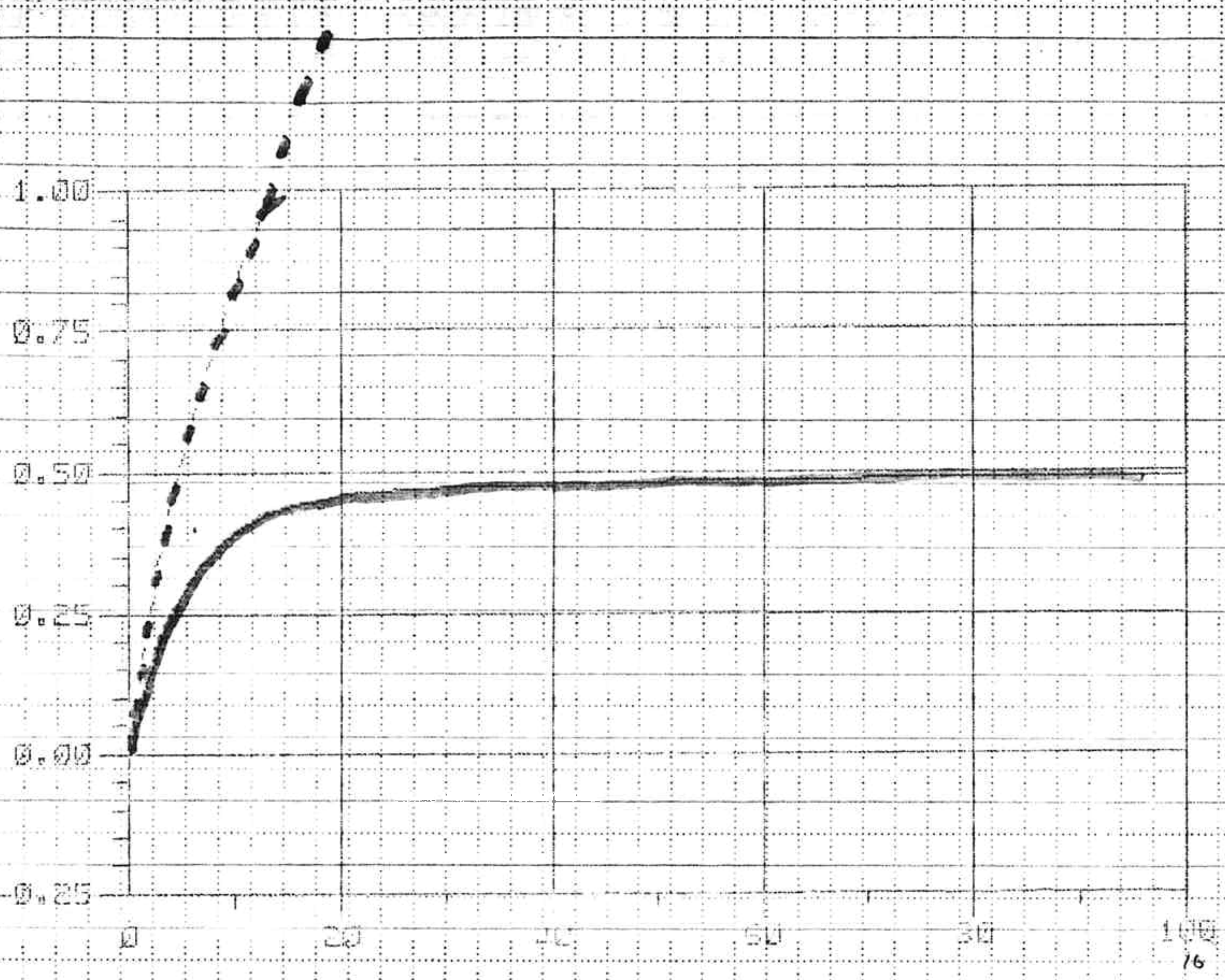
AT STEADY STATE LOOP GAIN IS 35 TIMES HIGHER WITH LOOP 2 (X_B) IN MANUAL.

<u>Other columns</u>: <u>Not tested</u>.

- COLUMN A, PID
- SET-POINT CHANGE IN Y_D



— BOTH LOOPS CLOSED



— LOOP 2 (X_B) IN MANUAL;
SAME TUNINGS FOR LOOP 1 (Y_D).

6

OTHER CONFIGURATIONS BETTER ?

So far: No extensive results with μ

Expect:

- LV quite good for reasonably easy columns (L/F not too high), $\frac{L}{D}\frac{V}{B}$ better for difficult (Col. D)
- OPTIMAL CONTROLLER STRUCTURE:

	Configuration	Inverse-based vs. Diagonal
THIS PAPER →	LV	Diagonal (PID)
	DV	Inverse-based (Decoupler)
	$\frac{L}{D}\frac{V}{B}$	Diagonal or Inverse simpler

Yes

Yes, most cases

$\tau_2/2$

(Yes)

No

No

No

Yes?

$\frac{L}{\delta}\frac{v}{\delta}$ col D