LETTERS TO THE EDITOR

To the Editor:

In their recent paper (32(8), p. 1312, 1986), Li and Toor discuss in detail simultaneous mixing and reaction in a turbulent, tubular reactor. Conclusions from their experim re:

a) The eff ss δ of a
diffusion-reac sely pro-
portional to t the reac-
tant diffusivi Reynolds
number ud/ι imeter d
are held con action is
changed. Th ed from
very rapid r id liquid
phases, was g

(1)

and was als ing slab
model. They ab thick-
ness δ, origin ixed only
by hydrodyn on diffu-
sivity."

b) From E "there is
no predicted y on mix-
ing or reacti mixing is
not controlle usion."

Both conc ical, and
(b) especiall ost previ-
ous thinking rets them
in terms of a romixing
model (Bald 84).

Slab thick *ty*

According e rate of
micromixing ipation of
concentratio cular dif-
fusion) is hi helor mi-
croscale λ_B,

(2)

(3)

Using the r pressure
drop, the ra tion ϵ can
be shown to

$\epsilon \sim u$ (4)

Substituting Eq. 4 into the definition of the Kolmogorov scale, Eq. 3 and combining with Eq. 2 gives:

1/2 (5)

With cons id Re, Eq. 5
simplifies the experi-
mental res δ should be
interpreted icromixing,
not the thic it region.

Diffusion *rmining* *step*

The expe Li and Toor
is that the profiles for
rapid secon using sepa-
rate feed s luenced by
diffusivity, e stoichio-
metric rati nstant. If
neither diff al kinetics
were rate- is? Our
model inco diffusion
within stret ating vor-
tices, where enerations
are needed t ixing. The
average life rtex τ_w is
given by:

(6)

The half- ion in the
absence of r

$t_{DS} = 2(\nu$ $5\,Sc)$ (7)

The condi o be com-
pleted withi eration to
make the ra f vortices
rate-determi

$t_{DS} \ll$)0 (8)

The Schmidt r's exper-
iments satisf explains
why he deduc ffect.

For suffici . 8 is no
longer satisfi ixing in,
for instance, a can be-
come diffusi e is also
needed with s. Their
time scales may be independent of D, but not the product-determining spatial concentration distributions.

Literature cited

Baldyga, J., and J. R. Bourne, "A Fluid Mechanical Approach to Turbulent Mixing and Chemical Reaction," *Chem. Eng. Comm.*, **28**, pp. 243 and 259 (1984).

Li, K. T., and H. L. Toor, "Turbulent Reactive Mixing with a Series-Parallel Reaction—Effect of Mixing on Yield," *AIChE J.*, **32**, 1312 (1986).

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To the Editor:

We would like to note some interesting connections between the results of Mijares, Cole, Naugle, Preisig and Holland (32(9), p. 1439, 1986) concerning criteria for selecting pairings for decentralized control and the work of Grosdidier and Morari (1985, 1986).

Let G be the plant transfer function and let $\tilde{G}$ be a simplified model of this plant containing the diagonal elements of G only.

$$\tilde{G} = \text{diag}\,\{g_{11}, g_{22}, \ldots, g_{nn}\}$$

G and $\tilde{G}$ are assumed stable. A diagonal controller $C = \text{diag}\,\{c_i\}$ with integral action is to be used to control the plant. Let $\tilde{H} = \text{diag}\,\{\tilde{h}_i\} = \tilde{G}C(I + \tilde{G}C)^{-1}$ denote the closed-loop transfer matrix of the diagonal system

$$\tilde{h}_i = g_{ii}c_i/(1 + g_{ii}c_i)$$

Based on stability arguments for inverting a matrix, Mijares et al. propose to use the "Jacobi Eigenvalue Criterion" for selecting the best pairing of controlled and manipulated variables: The pairing which minimizes $\rho(E)$ is selected, where $E = (G - \tilde{G})\tilde{G}^{-1} = G\tilde{G}^{-1} - I$, and E is evaluated at steady state ($\omega = 0$). $\rho(E)$ is the spectral radius that is defined as the magnitude of the largest eigenvalue of E. [Mijares et al. consider the matrix $A = I - \tilde{G}^{-1}G$, but this does not change the condition since $\lambda_i(E) = -\lambda_i(A)$ (λ_i denotes the eigenvalue)].

This criterion may also be derived from Corollary 2.1 in Grosdidier and Morari

(1986) that states:

"Assume that all individual loops are stable (i.e., $\tilde{h}_i$ stable) and have been chosen to have identical transfer functions, i.e., $\tilde{H} = \tilde{h}I$ [for example, $c_i(s) = k(s)/g_{ii}(s)$]. Then the overall system with all loops closed is stable if

$$|\tilde{h}(j\omega)| < \rho^{-1}(E(j\omega)) \quad \forall\omega\text{"} \quad (1)$$

In particular, this condition shows that decentralized control with integral action ($\tilde{h}(0) = 1$) is always possible if $\rho(E(0)) < 1$, and a reasonable criterion for selecting pairings is to choose the one with the smallest $\rho(E(0))$. However, if the process dynamics were known, this information should also be used and $\rho(E(j\omega))$ should be kept small as seen from Eq. 1. Thus, Condition 1 also extends the "Jacobi Eigenvalue Criterion" to nonzero frequencies. Condition 1 is derived by Grosdidier and Morari (1986) using the Nyquist criterion which leads to the stability condition $\rho(\tilde{H}E) < 1$. The approach taken by Mijares et al. is less general, but may be helpful, for example, for persons with a background in process design rather than in control.

Condition 1 is only sufficient, and a decentralized controller with integral action may be possible even if $\rho(E(0)) > 1$. To illustrate this, consider the controller $C(s) = k/s\hat{C}(s)$ where $\hat{C}(s)$ is diagonal and satisfies $\hat{C}(0) = \tilde{G}^{-1}(0)$. According to Theorem 7 in Grosdidier et al. (1985), there exists a k^* such that this particular controller results in a stable closed-loop system for any $k\epsilon(0,k^*]$ (integral controllability), if and only if Re $\{\lambda_i(G\tilde{G}^{-1}(0))\} > 0$, $\forall i$. From the identity $\lambda(G\tilde{G}^{-1}) = \lambda(E + I) = \lambda(E) + 1$, we see that this is equivalent to requiring $Re\ \{\lambda_i(E(0))\} > -1$, $\forall i$. This interesting condition is given by Mijares et al. and is proved here to complement their derivation. Consequently, decentralized control with integral action is possible also with $\rho(E(0)) > 1$ when the real parts of the eigenvalues of $E(0)$ are all larger than -1.

One restriction of Eq. 1 is the assumption of identical loop responses. While this is always satisfied at steady state, where $\tilde{H}(0) = I$, this is not likely to be satisfied at nonzero frequencies. Starting from the stability condition $\rho(\tilde{H}E) \leq 1$, Grosdidier and Morari (1986) derive a generalized version of Eq. 1, which also applies when the responses $\tilde{h}_i$ for each loop are *not* identical;

$$|\tilde{h}_i(j\omega)| < \mu^{-1}(E(j\omega)) \quad \forall\omega, \quad \forall i \quad (2)$$

μ is the Structured Singular Value and is computed with respect to a diagonal structure. Note that $\mu(E) \geq \rho(E)$, and therefore Eq. 1 always gives the least restrictive bound on $|\tilde{h}_i|$. By replacing $|\tilde{h}_i|$ by $\bar{\sigma}(\tilde{H})$, condition 2 may easily be extended to cases where C is block-diagonal.

Literature cited

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Grosdidier, P., and M. Morari, "Interaction Measures for Systems Under Decentralized Control," *Automatica*, **22**(3), 309 (1986).

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