AIChE, Miami 1986
SS

The first 8 pages were presented by Pierre Grosdidier

LARGE RGA-ELEMENTS ARE BAD NEWS

(Everybody knows that. But WHY?)

1) Intuitively "obvious" from the definition

$$\lambda_{ij} = \frac{(\partial y_i / \partial u_j)_{u_k}}{(\partial y_i / \partial u_j)_{y_k}} = \frac{g_{OL}}{g_{CL}}$$

(Applies to Decentralized control ONLY)

2) HOWEVER, large RGA-elements are also claimed to imply that the plant is FUNDAMENTALLY difficult to control (irrespective of controller)

⇓

We want to look at

RGA as an indicator of SENSITIVITY to MODEL UNCERTAINTY ("changes in process parameters")

TWO "JUSTIFICATIONS" FOR WHY LARGE RGA-ELEMENTS ARE ALWAYS BAD:

If the RGA-elements are large then small changes in the elements in G imply large changes in the elements of G^{-1}

a) From RGA-definition:

$$G = \{g_{ij}\}, \quad G^{-1} = \{\hat{g}_{ij}\}$$

This is the relative change in ij-th element of

$$\boxed{\frac{d\hat{g}_{ji}}{\hat{g}_{ji}} = -\lambda_{ij}\frac{dg_{ij}}{g_{ij}}}$$ (Grosdidier et.al., 1985)

and you get the relative change in the inverse-element.

multiply it by the corresponding RGA element

Sensitivity to error

CLOSE

b) From relationship to condition number, $\gamma^*(G)$:

Bounded below

Bounded above

2x2: $$\boxed{\|RGA\|_1 - \frac{1}{\gamma^*(G)} \leq \gamma^*(G) \leq \|RGA\|_1}$$

(Grosdidier et.al (1985), Nett & Manousiouthakis (1986))

1) Afine $$\|RGA\|_1 = \sum |\lambda_{ij}|$$

2) Define $$\gamma^*(G) = \min_{D_1, D_2} \gamma(D_1 G D_2)$$

Almost identical for 2x2 plants

AND ill-conditioned plants ($\gamma(G) >> 1$) are BELIEVED to be difficult to control.

⇒ Forget computing $\gamma^*(G)$. Use $\|RGA\|_1$

From to imply that large RGA elements are bad but does not prove anything

RGA AND MODEL UNCERTAINTY

1) UNCORRELATED ELEMENT UNCERTAINTY

Each element : $\left|\dfrac{\Delta g_{ij}}{g_{ij}}\right| < r$

For this particular case $\|RGA\|_1$ turns out to be an excellent sens. indicator

For this process there [illegible] a very [illegible] relat[illegible]

2x2 : Theorem. (Skogestad and Morari, 1985)

INTEGRAL ACTION on all outputs possible

$$\begin{cases} \text{IF} \quad r < \dfrac{1}{\|RGA\|_1} & (\text{"TIGHT"}) \\ \text{ONLY IF} \quad r < \dfrac{1}{\|RGA\|_1 - 1} & \end{cases}$$

λ_{11} about 5

Example : $\|RGA\|_1 = 20$

We may design a control system

$r < 0.0\cancel{0}50 \Rightarrow$ STABLE

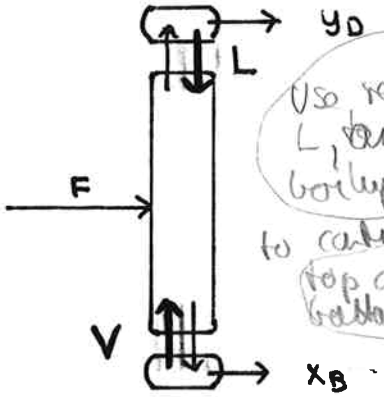
$r > 0.0\cancel{0}53 \Rightarrow$ UNSTABLE

nxn : Similar result

Nice result, but of limited usefulness

"PROBLEM" : Elements usually CORRELATED

Example: HIGH-PURITY DISTILLATION COLUMN



Use reflux L, and boilup V to control top and bottom comp

$y_D = 0.99$
$x_B = 0.01$
$N = 40$
$\alpha = 1.5$
$L/D = 5.4$
$z_F = 0.5$

$$\begin{pmatrix} \Delta y_D \\ \Delta x_B \end{pmatrix} = \overset{G}{\begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix}} \begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix}$$

If the relative err. in the elements is larger than 0.7% the plant may become singular

$$\lambda_{11} = 35.1 \qquad 1/\|RGA\|_1 = 0.007 \quad (0.7\%)$$

perturbed plant

$$G_p = \begin{pmatrix} 0.878 - 0.006 & -0.864 - 0.006 \\ 1.082 + 0.007 & -1.096 + 0.007 \end{pmatrix} \Rightarrow \text{UNSTABLE} \ (G_p \text{ singular})$$

If this uncert. was true this plant would be ~~top~~ impossible to control.

IN PRACTICE: Correlations between the elements

$$G_p = G + \begin{pmatrix} -d & d \\ d & -d \end{pmatrix} \Rightarrow \text{NO STABILITY PROBLEMS}$$

This result does not explain why ~~RGA~~ large RGA-elements are bad

Lets look at another unc. descr. for which the RGA is useful

2) UNCERTAINTY ON THE MANIPULATED INPUTS

~~which also turns out~~

Example: Want to increase reflux (L) from

100 to 110 kmol/min ($\Delta L = 10$)

but actual change is

100 to 111 kmol/min ($\Delta L = 11$)

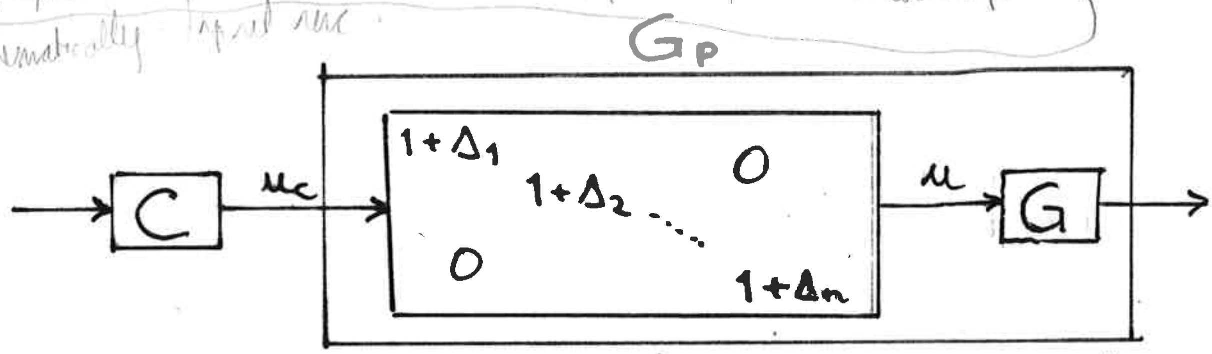
i.e. relative INPUT UNCERTAINTY

is 10 %

THIS SOURCE OF UNCERTAINTY IS ALWAYS PRESENT

→ so if you cannot handle this source of unc. you are in bad shape

Mathematically input unc.



Perturbed → $G_p = G(I + \Delta_I)$, $\Delta_I = \begin{pmatrix} \Delta_1 & & 0 \\ & \Delta_2 \ddots & \\ 0 & & \Delta_n \end{pmatrix}$

Δ_i = relative uncertainty on input i

Now. How do we relate this to the RGA?

Look at how closed-loop transf. funct. is affected

(14)

want small

Loop transfer matrix

$$G_p C = G(I+\Delta_I)C = \underset{\text{nominal}}{GC}\,(I + \underbrace{C^{-1}\Delta_I C}_{\text{error matrix}})$$

For tight control

use INVERSE-BASED controller $C = k(s)\, G^{-1}$

$\Rightarrow$ "Error" matrix $C^{-1}\Delta_I C = G\Delta_I G^{-1}$

Has G and G^{-1}. Not too surprising that it is closely related to the RGA.

$$2\times 2: \quad = \begin{pmatrix} \lambda_{11}\Delta_1 + \lambda_{12}\Delta_2 & -\lambda_{11}\frac{g_{12}}{g_{22}}(\Delta_1 - \Delta_2) \\ \lambda_{11}\frac{g_{21}}{g_{11}}(\Delta_1 - \Delta_2) & \lambda_{21}\Delta_1 + \lambda_{22}\Delta_2 \end{pmatrix}$$

$n \times n$: similar

LARGE RGA-ELEMENTS

$\Rightarrow$ LARGE ELEMENTS ON DIAGONAL OF ERROR MATRIX

$\Rightarrow$ POOR RESPONSE (OR EVEN INSTABILITY) FOR $\Delta_I \neq 0$

Example: HIGH-PURITY DISTILLATION COLUMN

$$C = \frac{0.7}{s} G^{-1} = \frac{0.7}{s} \begin{pmatrix} 39.94 & -31.49 \\ 39.43 & -32.00 \end{pmatrix}$$

Nominal: $GC = \frac{0.7}{s} I$ (1st order with time constant 1.4 min)

Error matrix:

$$G \Delta_I G^{-1} = \begin{pmatrix} 35.1\,\Delta_1 - 34.1\,\Delta_2 & -27.7\,(\Delta_1 - \Delta_2) \\ 43.2\,(\Delta_1 - \Delta_2) & -34.1\,\Delta_1 + 35.1\,\Delta_2 \end{pmatrix}$$

"WORST CASE" for $\Delta_1 = -\Delta_2$.

$$\begin{matrix} \Delta_1 = 0.2 \\ \Delta_2 = -0.2 \end{matrix} \Rightarrow \quad G \Delta_I G^{-1} = \begin{pmatrix} 13.8 & -11.1 \\ 17.2 & -13.8 \end{pmatrix}$$

EXPECT POOR RESPONSE WHEN $\Delta_I \neq 0$!

$\lambda_{11} = 35$

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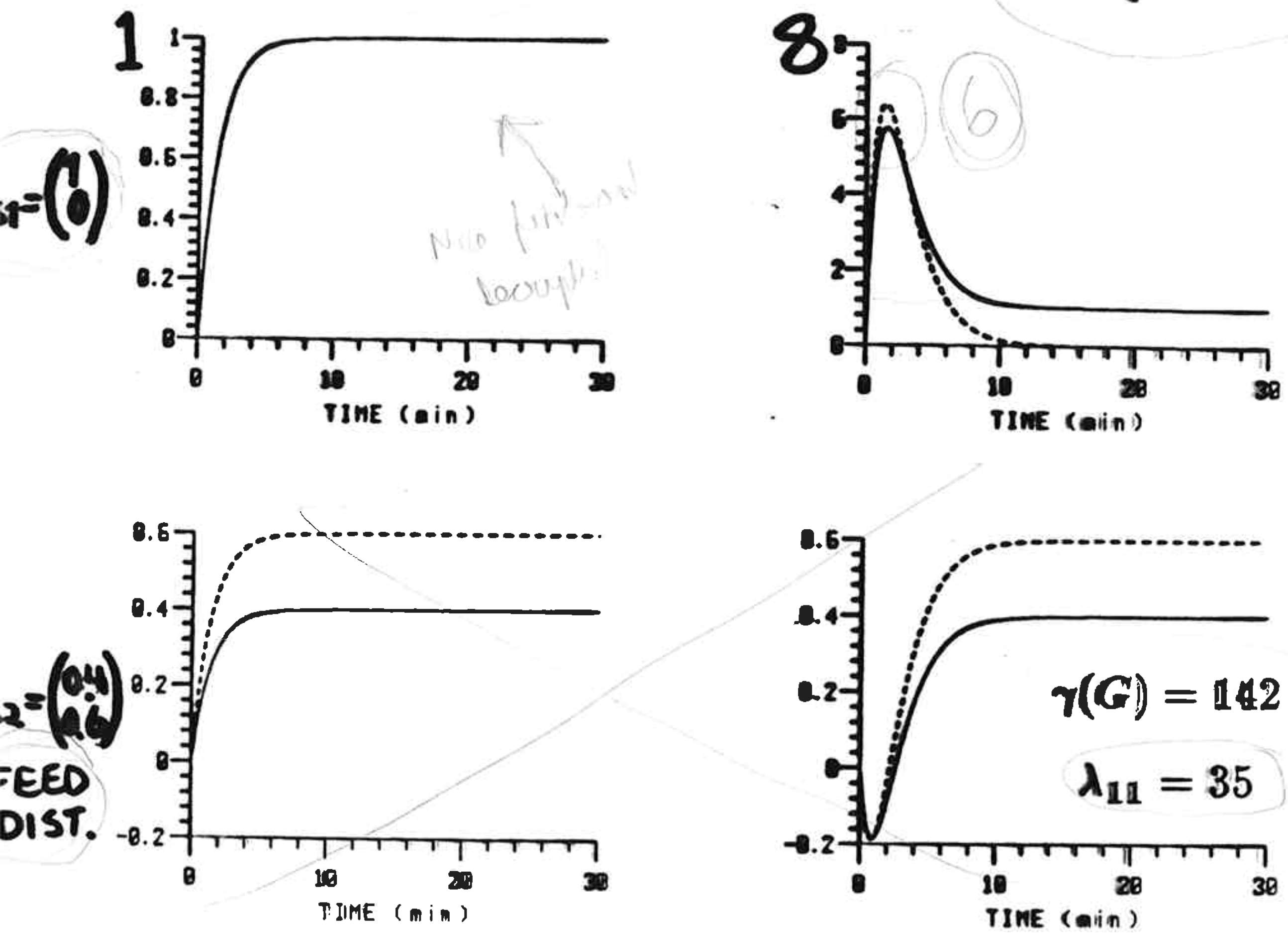
$\Delta_I = 0$ $\qquad$ $\Delta_I = \begin{pmatrix} 0.2 & 0 \\ 0 & -0.2 \end{pmatrix}$



$C = \frac{0.7}{s} G^{-1}$

—— Δy_D

- - - - Δx_B

INVERSE-BASED CONTROLLER NOT ACCEPTABLE IF LARGE RGA ELEMENTS

WHAT ABOUT A DIAGONAL CONTROLLER?

INSENSITIVE to input uncertainty

Error matrix $C^{-1}\Delta_I C = \Delta_I$

BUT even <u>NOMINAL</u> response ($\Delta_I = 0$) poor because controller does not correct for "directionality" of plant:

Why is that so ... large [illegible]

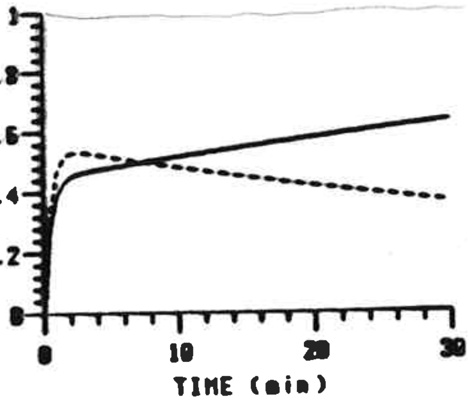
$$\text{In most cases:} \quad \gamma(G) \text{ large} \;\overset{C:\text{ diagonal}}{\Rightarrow}\; \gamma(GC) \text{ large}$$

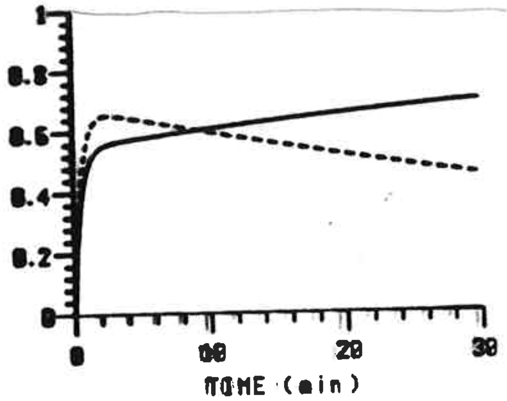
1) not too much affected by unc.

2) but nominal bad

$\Delta_I = 0$ $\qquad\qquad$ $\Delta_I = \begin{pmatrix} 0.2 & 0 \\ 0 & -0.2 \end{pmatrix}$

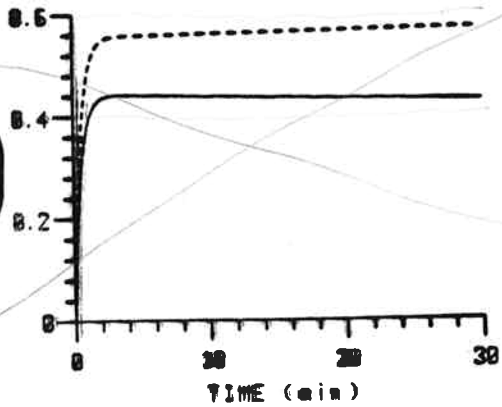
$y_{s1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

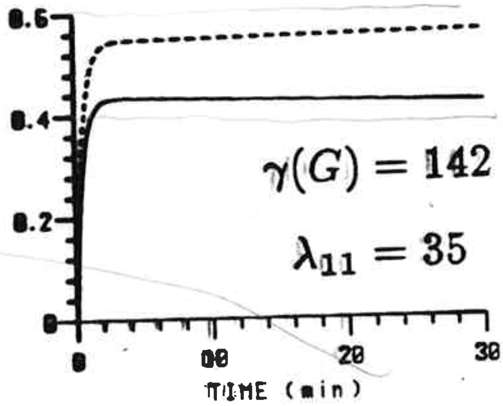




$y_{s2} = \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$

FEED DIST.





PAIRINGS: $y_D \leftrightarrow L$, $x_B \leftrightarrow V$

——— Δy_D

- - - - Δx_B

$C(s) = \frac{1}{s}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\qquad$ $\gamma(GC) = 142$

Does not help

~~Also the~~ o Δ' diagonal controller does not work very well for plants with large RGA elements

CONCLUSIONS

I. RGA as Interaction Measure

$\lambda_{ii} \, (\det \Lambda_{ii}) < 0 \Rightarrow$
- failure sensitive
- independent design impossible

All other interpretations / statements are heuristic

II. RGA as Sensitivity Indicator

LARGE RGA-elements imply:

1) Plant sensitive to <u>un</u>correlated element uncertainty

$$r < \frac{1}{\|RGA\|_1} \quad \text{for stability}$$

2) Inverse-based controller sensitive to input uncertainty

3) Diagonal controller gives poor nominal performance