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IMPLICATIONS OF INTERNAL MODEL CONTROL FOR PID CONTROLLERS

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Abstract

For a large number of single input-single output (SISO) typically used in the process industries the Internal Model Control (IMC) design procedure is shown to lead to PID controllers, occasionally augmented with a first order lag. These PID controllers have as their only tuning parameter the closed loop time constant or equivalently, the closed loop bandwidth. On-line adjustments are therefore much simpler than for general PID controllers. As a special case, PID tuning rules for systems modeled by a first order lag with deadtime are derived analytically. The superiority of these rules in terms of both closed loop performance and robustness is demonstrated.

I. Performance and Robustness Measures

In this paper we will generally use as performance measure the closed loop bandwidth, defined as the range of frequencies over which the closed loop amplitude ratio (See Fig. 1)

$$\left|\frac{y}{y_s}\right| = \left|\frac{gc}{1+gc}\right| \qquad (1)$$

is approximately unity. Occasionally we will also refer to the Integral Square Error (ISE) caused by a setpoint or disturbance step change, to compare the performance of different controllers.

In practice the plant g is not known exactly but only an approximate model $\tilde{g}$ is available. The modelling error can be represented either in additive form

$$gc = \tilde{g}c + e_a \qquad (2)$$

or in multiplicative form

$$g = \tilde{g}(1+e_m), \quad e_m = \frac{g-\tilde{g}}{\tilde{g}} \qquad (3)$$

The multiplicative ("relative") error e_m is in general physically more meaningful. In most practical situations $|e_m|$ approaches a maximum value equal to or greater than 1 for high frequencies. An example would be the error introduced by a Padé approximation for a time delay:

$$g = e^{-s\Theta}$$

$$\tilde{g} = \frac{1-\frac{\Theta}{2}s}{1+\frac{\Theta}{2}s} \qquad \text{1st order Padé}$$

$$\tilde{g} = \frac{1-\frac{\Theta}{2}s+\frac{\Theta^2}{12}s^2}{1+\frac{\Theta}{2}s+\frac{\Theta^2}{12}s^2} \qquad \text{2nd order Padé}$$

As ω varies $|e_m|$ oscillates between 0 and 2 (Fig. 3).

Assuming that g, $\tilde{g}$ and c have no poles in the open right half plane (RHP) the closed loop system in Fig. 1 will be stable for all plants g for which the Nyquist plot of gc does not encircle (-1,0). This implies that the magnitude of the additive error e_a has to be bounded by

$$|e_a| < |1+\tilde{g}c| \qquad (4)$$

corresponding to the following bound for the multiplicative error

$$|\tilde{g}ce_m| < |1+\tilde{g}c|$$

or

$$|e_m| < |1+(\tilde{g}c)^{-1}| = \left|\frac{\tilde{g}c}{1+\tilde{g}c}\right|^{-1} \qquad (5)$$

As a measure of robustness we will use the maximum peak (α) of the closed loop transfer function

$$\alpha = \max_{\omega} \frac{\tilde{g}c}{1+\tilde{g}c}$$

It follows from (5) that for a particular α stability is guaranteed for all perturbations satisfying

$$\max_{\omega} |e_m| < \frac{1}{\alpha} \qquad (7)$$

α is convenient and widely accepted to be more useful than gain and phase margins. Gain and phase margins only measure the robustness with respect to model uncertainties which are independent of ω. Therefore they tend to be overly optimistic.

II. Internal Model Control (IMC)

IMC was introduced by Garcia and Morari (1982) but a similar concept has been used previously and independently by a number of other researchers. The design procedure concentrates on the IMC controller g_c (Fig. 2) and consists of two steps:

Step 1: Factor the model

$$\tilde{g} = \tilde{g}_+ \tilde{g}_- \qquad (8)$$

such that $\tilde{g}_+$ contains all the time delays and RHP zeros and that $\tilde{g}_-^{-1}$ is stable and does not involve predictors

Step 2: The controller is

$$g = \tilde{g}_-^{-1} f \qquad (9)$$

where the low-pass filter f has been selected such that g_c is proper or, if "derivative action" is allowed, such that g_c has a zero excess of at most 1.

The closed loop relationships for Fig. 2 are

$$y = \frac{g_c g}{1+g_c(g-\tilde{g})}(y_s-d) + d \qquad (10)$$

$$e = y_s - y = \frac{1-g_c\tilde{g}}{1+g_c(g-\tilde{g})}(y_s-d)$$

$$= \frac{1-\tilde{g}_+ f}{1+\tilde{g}_+ f\, e_m}(y_s-d) \qquad (11)$$

For the special case of a perfect model ($\tilde{g}=g$) (10) and (11) reduce to

$$y = g_c g(y_s-d) + d = g_+ f(y_s-d) + d \qquad (12)$$

$$e = (1-g_+ f)(y_s-d) \qquad (13)$$

The control system obtained in this manner has a number of attractive properties which will be stated in the following. We will assume throughout that $g_c g$ and $\tilde{g}$ have all poles in the open LHP.

P1: Assume $\tilde{g} = g$. Then the system is effectively open loop and "closed-loop stability" is implied by the stability of g_c and g (cf. (12)). While for the classical structure (Fig. 1) it is not at all clear what type of controller c and what parameter choices lead to closed loop stable systems, the IMC structure guarantees closed loop stability for <u>all</u> stable controllers g_c.

P2: Assume $\tilde{g} = g$. Then the closed loop transfer function is $g_+ f$ (cf. (12)). $g_+ f$ is mostly at the designers discretion except that g_+ has to contain all the delays and RHP zeros and f has to be of high enough order such that differentiation is avoided in g_c. Thus the closed loop transfer function can be designed directly and not indirectly via c as in the classic controller design procedure (Fig. 1).

P3: Assume $\tilde{g} = g$. For step inputs in y_s and d the ISE is minimized when g_+ and f are selected such that $|g_+ f| = 1$ (Holt and Morari, 1984). This implies that f has to be unity and g_+ has the form of an allpass.

$$g_+ = e^{-\theta s} \prod_i \frac{-\beta_i s+1}{\beta_i s+1}, \quad \beta_i > 0 \qquad (14)$$

where β_i^{-1} are all the RHP zeros and θ is the time delay present in g.

P4: Select g_c such that

$$g_c(0) = \tilde{g}(0)^{-1} \qquad (15)$$

Then the control error vanishes asymptotically for all asymptotically constant inputs y_s and d. (Follows from (11) via Final Value Theorem). In order to satisfy (15) we will adapt the following convention for $\tilde{g}_+(s)$ and $f(s) = n(s)/d(s)$

$$\tilde{g}_+(0) = n(0) = d(0) = 1 \qquad (16)$$

P5: f satisfying (16) is generally selected to be of the form

$$f(s) = \frac{1}{(\varepsilon s+1)^r} \qquad (17)$$

If $\tilde{g} = g$ and $|g_+| = 1$ then $|y/y_s| = |f|$. Thus, the parameter ε which can be adjusted by the operator, is the closed loop time constant and $\frac{1}{\varepsilon}$ is the closed loop bandwidth. The larger ε is, the slower is the response and the smaller are the actions of the manipulated variable. For (17) the maximum peak $\alpha = 1$, i.e. the robustness characteristics are good.

P6: Select g_c to satisfy (15), (16) and (18)

$$\frac{d}{ds}(g_c\tilde{g})\Big|_{s=0} = 0 \qquad (18)$$

Then the control error vanishes asymptotically for all asymptotically ramp shaped inputs y_s and d (Brosilow, 1983). (Follows from (11) via Final Value Theorem). With the adapted conventions (16), (18) becomes

$$d'(0) - n'(0) = \tilde{g}_+'(0) \qquad (19)$$

where the prime denotes differentiation with respect to s.

P7: An example of a filter satisfying (19) is

$$f = \frac{(2\varepsilon - \tilde{g}_+'(0))s+1}{(\varepsilon s+1)^2} \qquad (20)$$

As explained in P5 the adjustable parameter ε is the closed loop time constant and $1/\varepsilon$ is the closed loop bandwidth.

Typical values of $\tilde{g}_+'(0)$ are

$$\frac{d}{ds}(e^{-s\theta})\Big|_{s=0} = -\theta \qquad \frac{d}{ds}(-\beta s+1)\Big|_{s=0} = -\beta$$

$$\frac{d}{ds}\left(\frac{-\beta s+1}{\beta s+1}\right)\Big|_{s=1} = -2\beta$$

Since in general $\tilde{g}_+'(0) < 0$ this implies

$$\alpha = \max_\omega |g_+ f| > 1$$

i.e., α is strictly greater than unity for all f's satisfying (19). This indicates that tighter performance specifications (no offset for ramps) have to be paid for with decreased robustness margins.

P8: It follows from (11) that for closed loop stability the Nyquist plot of $\tilde{g}_+ f e_m$ should not encircle (-1,0). As a sufficient condition (Small Gail Theorem) bounds can be placed on the magnitude

$$|\tilde{g}_+ f e_m| < 1$$

or

$$|f| < \frac{1}{|\tilde{g}_+||e_m|} \qquad (21)$$

Assuming for simplicity $|\tilde{g}_+| = 1$ it becomes obvious that the filter magnitude $|f|$ has to be small wherever the uncertainty $|e_m|$ is large. Because $|e_m|$ approaches or exceeds 1 for high frequencies in all practical situations, the <u>closed loop time constant ε and bandwidth $\frac{1}{\varepsilon}$ is limited by the degree of model uncertainty</u>. The closed loop bandwidth can never be larger than the bandwidth over which the process model is valid. The models used in process control are usually sufficiently good to set ε equal to the open loop time constant or smaller.

In summary, the key advantage of the IMC design procedure is that all controller parameters are related in a straightforward and unique manner to the model parameters. There is only one adjustable parameter ε which has intuitive appeal because it is equal to the closed loop time constant and thus determines the speed of response of the system. $\frac{1}{\varepsilon}$ is the bandwidth of the closed loop system which must always be smaller than the bandwidth over which the process model is valid. Thus, a good initial estimate of ε is available a priori. ε can then be adjusted on-line if necessary.

III. IMC vs. Classical Control

For linear systems the IMC controller g_c can be simply viewed as an alternate parameterization of the classic controller c, albeit a very useful one. Through the transformation

$$c = \frac{g_c}{1-g_c\tilde{g}} = \frac{\tilde{g}_-^{-1}}{f^{-1}-\tilde{g}_+} \qquad (22)$$

Fig 1 and 2 become equivalent. It is obvious from (22) that if there are no delays in g, c is rational and can be implemented as a lead/lag network. While it is generally not at all obvious how to select the proper combination of leads and lags for the best effect, IMC determines all the parameters except one (ε) is a simple fashion from the model. ε is left for on-line adjustment. A further nice feature is that, as long as $\tilde{g} = g$, the closed loop system is stable for all $\varepsilon > 0$. Thus, for the nominal system $\tilde{g}$ at least, ε can be chosen without paying any attention to closed loop stability.

It is natural to expect that for certain classes of process models the lead/lag network c obtained from (23) via the IMC design procedure will be equivalent to a PID controller. Indeed, we found that IMC leads to PID controllers for virtually all models common in industrial practice (Table 1), the only major exception being systems with LHP zeros. Note that Table 1 includes systems with integrators and RHP zeros. Occasionally the PID controllers have to be augmented with a first order lag with time constant τ_F. A few remarks are in order:

R1: When the PID controller of the specified form is applied to the model the closed loop system is stable for all values of $\varepsilon > 0$.

R2: For about one third of the studied cases ε appears only in the denominator of the expression for the controller gain k_c. Thus, qualitatively at least, decreasing the controller gain has as direct an effect on the speed of response and bandwidth as does increasing ε. For most of these cases the tuning of PID controllers is known to be quite simple.

For the majority of process models ε appears in a complex manner in all the parameters (k_c, τ_I, τ_D) of the controller e.g. K and R. It is not surprising that for a number of these processes the trial and error tuning of PID controllers is notoriously difficult. The IMC parametrization shows how all the controller parameters have to be adjusted simultaneously in the most effective manner.

R3: In all cases we required that there should be no offset for step changes. If the process has an integrator and a step disturbance enters through that integrator and thus becomes a ramp, it is logical to require that there should be no offset for ramp changes. This performance specification was met in cases I,K,N,O,R, and S by assuming the filter f to be of the form (20).

R4: Whenever the closed loop transfer function is not strictly proper, e.g. D, one has to require that

$$\lim_{\omega\to\infty} |\tilde{g}_+ f| < 1 \qquad (23)$$

Otherwise, (21) will be violated for high frequencies where $|e_m|$ 1 and instability is bound to occur in all practical situations. This explains why $\varepsilon > \beta$ was specified for D,F,L,N,P and R. $\varepsilon = \beta/2$ gives a gain margin of 2.

R5: No systems with LHP zeros are listed in Table 1. For these systems a sequence of lags should be used first to cancel all the zeros. After the zeros have been removed the PID settings for the remaining system can be obtained from the table.

R6: Table 1 is also useful for systems with delays. To obtain a rational model for which controller settings are available in Table 1 a Padé approximation may be used for the delay. This procedure is illustrated in the next two examples.

Example 1: $$g(s) = \frac{e^{-s\theta}}{\tau s+1} \qquad (24)$$

With a first order Padé approximation

$$g(s) = \frac{1-\frac{\theta}{2}s}{(1+\frac{\theta}{2}s)(\tau s+1)} \qquad (25)$$

Entries F and G in Table 1 yield the desired PID controller parameters.

Example 2: $$g(s) = \frac{1-k_1e^{-s\theta}}{s} \qquad (26)$$

With a first order Padé approximation

$$g(s) = \frac{1-k_1+\frac{\theta}{2}(1+k_1)s}{(1+\frac{\theta}{2}s)s} \qquad (26)$$

If $k_1 > 1$ a controller from entries P through S can be selected. If $k_1 < 1$ the LHP zero should be first removed by a simple lag. Then the PID parameters can be obtained from entries J or K.

As is shown by (21) and was explained in P8, the bandwidth $\frac{1}{\varepsilon}$ cannot be larger than the bandwidth over which the model is valid. The Padé approximation introduces a modelling error and therefore ε cannot be made arbitrarily small. $|e_m|$ is shown for Ex. 1 and 2 in Fig. 3. For a first order Padé approximation $|e_m|$ reaches 1 at $\omega\theta \simeq 3$. Therefore (21) requires that $\frac{\theta}{\varepsilon} < 3$ or $\frac{1}{\varepsilon} < 3/\theta$. If the system model with the delays in place is valid over a bandwidth larger than $3/\theta$, closed loop bandwidth has been wasted by approximation. Better control could be obtained by choosing a higher order approximation or equivalently not restricting the controller to be of the PID type. For example a second order Padé approximation would allow a bandwidth up to $\frac{1}{\varepsilon} \simeq 5/\theta$ (Fig. 3). A higher order lead/lag network would be required for the implementation. An alternative is use no approximation at all and to implement a Smith Predictor. In theory this would allow to extend the bandwidth to ∞. However, because of the inherent restrictions on the response from a delay, bandwidth is not that important. Also, in practice, the bandwidth over which the model is valid is limited, and a PID controller yields as good results as a Smith Predictor.

IV. IMC Based PID Control For a First Order Lag With Deadtime

Because the first order lag/deadtime model plays such a dominant role in process control it is worthwhile to discuss Ex. 1 in more detail. The PID parameters obtained from F in Table 1 for the model (25) are listed compactly in Table 2. For system (24) with this controller the closed loop expression is

$$y = \frac{e^{-(\frac{\theta}{\varepsilon})\varepsilon s}}{\frac{(1+\frac{1}{2}\frac{\theta}{\varepsilon})\varepsilon s}{1+\frac{1}{2}\frac{\theta}{\varepsilon}(\varepsilon s)} + e^{-(\frac{\theta}{\varepsilon})\varepsilon s}}(y_s-d)+d \qquad (28)$$

We note three important properties of (28)

- The closed loop response is independent of the system time constant
- Time is scaled by ε
- The shape of the response depends on ε/θ only

This implies that if we specify a value for ε/θ all systems regardless of k, τ and θ will have an identical response except that the time scale will change according to θ. If the deadtime in system A is twice as long as the deadtime in system B, then for a specific ε/θ A will have the identical response as B except that it will take exactly twice as long.

The effect of ε/θ on performance and robustness is shown quantitatively in Fig. 4, where both J = ISE and α have been plotted as a function of ε/θ. J has been normalized by J_{min}, the minimum error which can be achieved with the best possible control system, a Smith Predictor with a PI controller. J/J_{min} reaches a minimum of 1.092 for $\varepsilon/\theta = 0.68$. At this point $\alpha \simeq 1.3$. For practical purposes a better compromize between performance and robustness is attained for $\varepsilon/\theta = 0.8$ where the ISE is almost minimal but α has dropped to 1. This choice is reasonable if the bandwidth over which the first order lag/deadtime model is valid is at least equal to the bandwidth over which the Padé approximation is valid, i.e. $3/\theta$. Otherwise ε/θ has to be increased.

The obvious question here is if it would ever be worthwhile to use a Smith Predictor. If ISE is the performance measure of choice, the answer is "hardly". At the very best an improvement of less than 10% can be expected. If this improvement is important, one has to ask if it is feasible. It is feasible only if the first order lag/deadtime model is valid over a bandwidth larger than $3/\theta$. For large θ/τ the bandwidth can indeed be larger than that in practice and a Smith Predictor Controller can yield improvements, albeit small ones.

Next we will compare the IMC-PID parameters with the calssic tuning rules by Ziegler and Nichols (1942) and Cohen and Coon (1953) (Fig. 5-7). For all these rules performance and robustness depend strongly on θ/τ while for the IMC rules performance and robustness are independent of θ/τ. The Cohen-Coon rules give reasonable performance ($J/J_{min} < 1.3$) for $0.6 < \frac{\theta}{\tau} < 4$. In this range α varies between 2.5 and 1.1, i.e. the robustness is quite poor especially for small ratios θ/τ. The performance obtained with the closed loop Ziegler-Nichols parameters is good for the range $0.3 < \frac{\theta}{\tau} < 3$ but the robustness is poor except for $\frac{\theta}{\tau} \simeq 0.5$. For $\theta/\tau > 4$ the closed loop system is unstable when the closed loop Ziegler-Nichols parameters are used. In terms of performances the open loop Ziegler-Nichols parameters are only useful in the range $0.2 < \frac{\theta}{\tau} < 1.4$.

Conclusions

We have shown that for most systems commonly encountered and for most models generally used in process control the PID controller is the natural choice. In the absence of nonlinearities, constraints or multivariable interactions it is impossible to improve the performance with more complex controllers unless higher order more accurate process models are available.

In particular, we derived that thepossible ISE improvement offered by a Smith Predictor Controller over a PID controller for a first order lag with deadtime is at most 10% regardless of θ/τ.

At first sight these statements might seem like a negative judgement on IMC for the majority of problems in process control, but just the opposite is the case: IMC formed the basis of all the rules in Tables 1 & 2. If we used IMC directly and if we did not insist on the traditional PID parameters, no rules and no involved tables would be needed in the first place. The IMC design technique is generally applicable regardless of the system involved. No special provisions are required to deal with every single type of system. The complexity of the rules in Tables 1 and 2 demonstrates that the PID parameters K_c, τ_I and τ_D are the consequence of a long hardware tradition rather than because they represent the most practical tuning tools. The unfortunate parameterization of the PID controller might also explain why some of the "IMC clones" have claimed improvements in control quality over PID for simple systems where properly tuned PID controller might also explain why some of the "IMC clones" have claimed improvements in control quality over PID for simple systems where properly tuned PID controllers would have yielded an equally good result.

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TABLE 1

	Model	$\frac{y}{y_s} = g_+ f$	Controller	$k_c k$	τ_I	τ_D	τ_F
A	$\frac{k}{\tau s+1}$	$\frac{1}{\varepsilon s+1}$	$\frac{1}{k}\frac{\tau s+1}{\varepsilon s}$	$\frac{\tau}{\varepsilon}$	τ	–	–
B	$\frac{k}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{1}{\varepsilon s+1}$	$\frac{(\tau_1 s+1)(\tau_2 s+1)}{k\varepsilon s}$	$\frac{\tau_1+\tau_2}{\varepsilon}$	$\tau_1+\tau_2$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$	–
C	$\frac{k}{\tau^2 s^2+2\xi\tau s+1}$	$\frac{1}{\varepsilon s+1}$	$\frac{\varepsilon^2 s^2+2\xi\tau s+1}{k\varepsilon s}$	$\frac{2\xi\tau}{\varepsilon}$	$2\xi\tau$	$\frac{\tau}{2\xi}$	–
D	$k\frac{-\beta s+1}{\tau s+1}$	(1) $\frac{-\beta s+1}{\varepsilon s+1}$	$\frac{\tau s+1}{k(\beta+\varepsilon)s}$	$\frac{\tau}{\beta+\varepsilon}$	τ	–	–
E	$k\frac{-\beta s+1}{\tau s+1}$	(2) $\frac{-\beta s+1}{(\beta s+1)(\varepsilon s+1)}$	$\frac{\tau s+1}{ks(\beta\varepsilon s+2\beta+\varepsilon)}$	$\frac{\tau}{2\beta+\varepsilon}$	τ	–	$\frac{\beta\varepsilon}{2\beta+\varepsilon}$
F	$k\frac{-\beta s+1}{\tau^2 s^2+2\xi\tau s+1}$	(1) $\frac{-\beta s+1}{\varepsilon s+1}$	$\frac{\tau^2 s^2+2\xi\tau s+1}{k(\beta+\varepsilon)s}$	$\frac{2\xi\tau}{\beta+\varepsilon}$	$2\xi\tau$	$\frac{\tau}{2\xi}$	–
G	$k\frac{-\beta s+1}{\tau^2 s^2+2\xi\tau s+1}$	(2) $\frac{-\beta s+1}{(\beta s+1)(\varepsilon s+1)}$	$\frac{\tau^2 s^2+2\xi\tau s+1}{ks(\beta\varepsilon s+2\beta+\varepsilon)}$	$\frac{2\xi\tau}{2\beta+\varepsilon}$	$2\xi\tau$	$\frac{\tau}{2\xi}$	$\frac{\beta\varepsilon}{2\beta+\varepsilon}$
H	$\frac{k}{s}$	(3) $\frac{1}{\varepsilon s+1}$	$\frac{1}{k\varepsilon}$	$\frac{1}{\varepsilon}$	–	–	–
I	$\frac{k}{s}$	(4) $\frac{2\varepsilon s+1}{(\varepsilon s+1)^2}$	$\frac{2\varepsilon s+1}{k\varepsilon^2 s}$	$\frac{2}{\varepsilon}$	2ε	–	–
J	$\frac{k}{s(\tau s+1)}$	(3) $\frac{1}{\varepsilon s+1}$	$\frac{\tau s+1}{k\varepsilon}$	$\frac{1}{\varepsilon}$	–	τ	–
K	$\frac{k}{s(\tau s+1)}$	(4) $\frac{2\varepsilon s+1}{(\varepsilon s+1)^2}$	$\frac{(\tau s+1)(2\varepsilon s+1)}{k\varepsilon^2 s}$	$\frac{2\varepsilon+\tau}{\varepsilon^2}$	$2\varepsilon+\tau$	$\frac{2\varepsilon\tau}{2\varepsilon+\tau}$	–
L	$k\frac{-\beta s+1}{s}$	(1,3) $\frac{-\beta s+1}{\varepsilon s+1}$	$\frac{1}{k(\beta+\varepsilon)}$	$\frac{1}{\beta+\varepsilon}$	–	–	–
M	$k\frac{-\beta s+1}{s}$	(2,3) $\frac{-\beta s+1}{(\beta s+1)(2s+1)}$	$\frac{1}{k(\beta\varepsilon s+2\beta+\varepsilon)}$	$\frac{1}{2\beta+\varepsilon}$	–	–	$\frac{\beta\varepsilon}{2\beta+\varepsilon}$
N	$k\frac{-\beta s+1}{s}$	(1,4) $\frac{(-\beta s+1)(\beta 2+\varepsilon)(s+1)}{(\varepsilon s+1)^2}$	$\frac{(\beta+2\varepsilon)s+1}{ks(\beta+\varepsilon)^2}$	$\frac{\beta+2\varepsilon}{(\beta+\varepsilon)^2}$	$\beta 2\varepsilon$	–	–
O	$k\frac{-\beta s+1}{s}$	(4) $\frac{(-\beta s+1)(2\beta+\varepsilon)(s+1)}{(\beta s+1)(\varepsilon s+1)^2}$	$\frac{2(\beta+\varepsilon)s+1}{ks(\beta\varepsilon^3 s+\varepsilon^2+4\beta\varepsilon+2\beta^2)}$	$\frac{2(\beta+\varepsilon)}{2\beta^2+4\beta\varepsilon+\varepsilon^2}$	$2(\beta+\varepsilon)$	–	$\frac{\beta\varepsilon^2}{2\beta^2+4\beta\varepsilon+\varepsilon^2}$
P	$k\frac{-\beta s+1}{s(\tau s+1)}$	(1,3) $\frac{-\beta s+1}{\varepsilon s+1}$	$\frac{\tau s+1}{k(\beta+\varepsilon)}$	$\frac{1}{\beta+\varepsilon}$	–	τ	–
Q	$k\frac{-\beta s+1}{s(\tau s+1)}$	(2,3) $\frac{-\beta s+1}{(\beta s+1)(\varepsilon s+1)}$	$\frac{\tau s+1}{k(\beta\varepsilon s+2\beta+\varepsilon)}$	$\frac{1}{2\beta+\varepsilon}$	–	τ	$\frac{\beta\varepsilon}{2\beta+\varepsilon}$
R	$k\frac{-\beta s+1}{s(\tau s+1)}$	(1,4) $\frac{(-\beta s+1)((\beta+2)s+1)}{(\beta s+1)^2}$	$\frac{(\tau s+1)((\beta+2\varepsilon)s+1)}{k(\beta\varepsilon s+2\beta+\varepsilon)}$	$\frac{\beta+2\varepsilon+\tau}{(\beta+\varepsilon)^2}$	$\beta+2\varepsilon+\tau$	$\frac{\tau(\beta+2\varepsilon)}{\beta+2\varepsilon+\tau}$	–
S	$k\frac{-\beta s+1}{s(\tau s+1)}$	(4) $\frac{(-\beta s+1)(2(\beta+\varepsilon)s+1)}{(\beta s+1)(\varepsilon s+1)^2}$	$\frac{(\tau s+1)(2(\beta+\varepsilon)s+1)}{ks(\beta\varepsilon^2 s+\varepsilon^2+4\beta\varepsilon+2\beta^2)}$	$\frac{2(\beta+\varepsilon)+\tau}{2\beta^2+4\beta\varepsilon+\varepsilon^2}$	$2(\beta+\varepsilon)+\tau$	$\frac{2\tau(\beta+\varepsilon)}{2(\beta+\varepsilon)+\tau}$	$\frac{\beta\varepsilon^2}{2\beta^2+4\beta\varepsilon+\varepsilon^2}$

Table 1: IMC based PID controller parameter. The adjustable parameter ε is the time constant, $1/\varepsilon$ the bandwidth of the closed loop system. Occasionally a lag $(\tau_F s+1)^{-1}$ has to be added to the PID controller.

PID Controller: $k_c(1+\tau_D s+\frac{1}{\tau_I s}) = k_c\frac{\tau_I\tau_D s^2+\tau_I s+1}{\tau_I s}$

Comments:

1. Practical equipment $\varepsilon > \beta$ ($\varepsilon = 2\beta$ gives gain margin of 2)
2. ISE optimal for setpoint step changes
3. No offset for setpoint step changes
4. No offset for setpoint rays changes/disturbance step changes

$$kk_c = \frac{2(\frac{\tau}{\theta})+1}{2(\frac{\varepsilon}{\theta})+1}$$

$$\frac{\tau_I}{\tau} = 1 + \tfrac{1}{2}(\frac{\theta}{\tau})$$

$$\frac{\tau_D}{\tau} = \frac{1}{2(\frac{\tau}{\theta})+1}$$

Table 2: IMC based PID parameters for $g(s) = k\ e^{-s\Theta}/(\tau s+1)$. $\varepsilon/\Theta = 0.68$ for minimum ISE. As a practical recommendation $\varepsilon/\Theta \geq 0.8$.

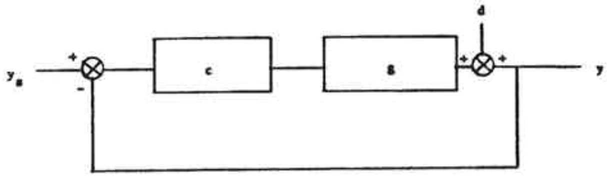

Figure 1. Classical Feedback Controller

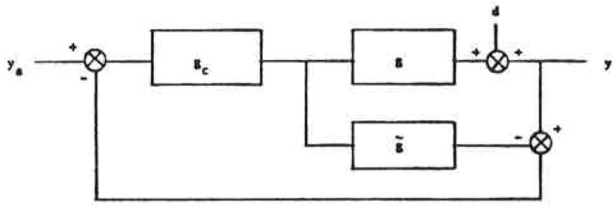

Figure 2. Internal Model Control Structure

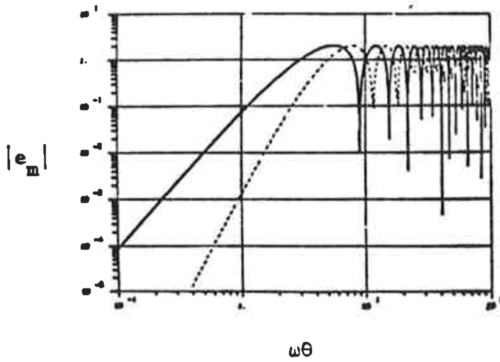

Figure 3. $|e_m|$ for 1st order (solid) and 2nd order (dashed) Padé approximation.

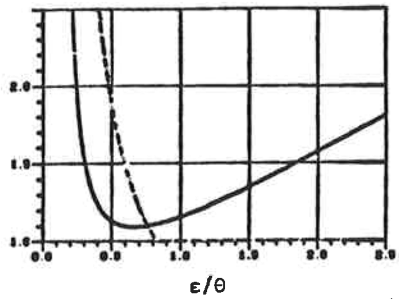

Figure 4. IMC based PID controller for $g(s) = k\ e^{-s\Theta}/(\tau s+1)$. Effect of ε/Θ on the ISE and α. Solid line: J/JMIN; dashed line: α.

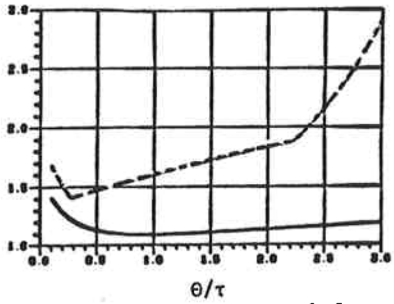

Figure 5. Closed loop Ziegler-Nichols settings applied to $g(s) = k\ e^{-\Theta s}(\tau s+1)$. Solid: J/JMIN; dashed: α

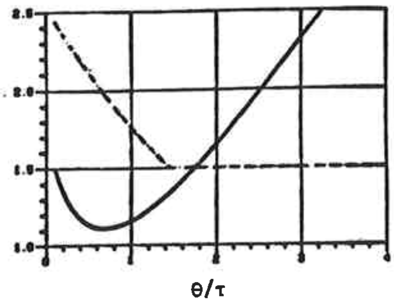

Figure 6. Open-loop Ziegler-Nichols settings applied to $g(s) = k\ e^{-\Theta s}/(\tau s+1)$. Solid: J/JMIN; dashed: α.

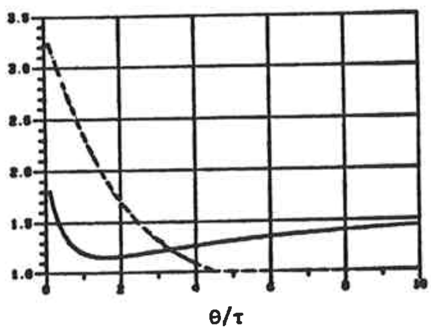

Figure 7. Cohen-Coon settings applied to $g(s) = k\ e^{-\Theta s}/(\tau s+1)$. Solid: J/JMIN; dashed: α.