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Synchronized tracking control of multiple Euler-Lagrange systems

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***Abstract*—In this paper, we propose a distributed robust control method for synchronized tracking of multiple Euler-Lagrange systems, where the time-varying reference trajectory is sent to only a subset of the agents. It is assumed that the agents can exchange information with their local neighbors on an undirectionally connected communication graph. The controllers are not only distributed on the network, but also decentralized for each generalized coordinate within each agent. Theoretical analysis is performed. And simulation results are provided to support the theoretical results.**

I. Introduction

Motivated by applications in physics, biology and engineering the study of synchronized control of collections of locally connected dynamic systems has become an important topic in control theory. Examples of interesting research directions include coverage control, consensus, formation control, flocking, and leader-follower tracking [1]. In recent years, there have been some remarkable works on synchronized tracking problem for multiple multiple Euler-Lagrange (EL) systems when only a portion of the agents can access the leader. In [2], a method of finite time synchronization tracking control of multirobot systems is proposed. The agent models are assumed to be known and each agent's controller requires its neighbors' control signals. In [3], a model-independent sliding mode control algorithm is proposed. However, the algorithm is discontinuous and requires the availability of the information of both the neighbors and the neighbors' neighbors. In [4], an adaptive robust control algorithm is proposed for multiple uncertain EL systems. In [5], the problem of position synchronization of multiple EL systems is studied. However, the proposed method considers the tracking of a stationary leader which sends a piece-wisely constant reference position signal.

In our recent work [6], we proposed a decentralized adaptive robust controller for trajectory tracking of robot manipulators. In this paper, the work of [6] is modified and extended to develop a new distributed robust control method for synchronized tracking of multiple EL systems, where the time-varying reference trajectory is sent to only a subset of the agents. It is assumed that the agents can exchange information with local neighbors on an undirectionally connected communication graph. In the local controller equipped in each generalized coordinate of each agent, a disturbance observer (DOB) is introduced to compensate for the low-passed coupled uncertainties, and a sliding mode control term is employed to handle the uncertainties that the DOB cannot compensate for sufficiently. By some damping terms, the boundedness of the signals of the overall multiple nonlinear systems is first ensured. Then we show how the DOB and sliding mode control play in a cooperative way in each coordinate to achieve an excellent synchronized tracking performance. Simulation results are provided to support the theoretical results.

II. Background and problem statement

A. Graph Theory

Consider a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with a finite set of N nodes $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ and a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Let i denotes the ith agent. An edge of $\mathcal{E}$ is denoted as $e_{ij} = (v_i, v_j) \in \mathcal{E}$ where agent j can receive information from agent i. In a directed graph, agent j does not send information to agent i, whereas in an undirected graph, if $(v_i, v_j) \in \mathcal{E}$, then $(v_j, v_i) \in \mathcal{E}$. A graph is called connected if there exists a path between any two distinct agents. Denote the adjacency matrix as $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{N\times N}$ with $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. Note $a_{ii} = 0$. For an undirected graph, we have $a_{ij} = a_{ji}$. The set of neighbors of a node v_i is $\mathcal{N}_i = \{v_j \in \mathcal{V} | (v_j, v_i) \in \mathcal{E}\}$, i.e., the set of nodes with information incoming to v_i. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathcal{R}^{N\times N}$ is then defined as $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$, and $l_{ij} = -a_{ij}, i \neq j$.

B. EL System models

Consider N agents governed by the following EL vector equations for $i = 1, \cdots, N$.

$$M_i(\theta_i)\ddot{\theta}_i + C_i(\theta_i, \dot{\theta}_i)\dot{\theta}_i + g_i(\theta_i) + f_i(\dot{\theta}_i) = u_i \qquad (1)$$

where $\theta_i = \theta_i(t) \in \Re^n$ is the generalized coordinate vector; $u_i \in \Re^n$ is the input torque vector; $M_i(\theta) = M_i^T(\theta) \in \Re^{n\times n}, M_i(\theta_i) > 0$ is the inertia matrix; $C_i(\theta_i, \dot{\theta}_i)\dot{\theta}_i \in \Re^n$ is the centrifugal and Coriolis torque; $g_i(\theta_i) \in \Re^n$ is the grativity torque; $f_i(\dot{\theta}) \in \Re^n$ is the friction force torque.

We first impose the following assumption.

Assumption 1: The reference trajectory $\theta_d(t) \in \mathcal{R}^n$ and the time derivatives $\dot{\theta}_d(t)$ and $\ddot{\theta}_d(t)$ are bounded signals.

Define an auxiliary error vector $r_i = [r_{i1}, \cdots, r_{in}]^T$ as

$$r_i = \dot{e}_i + \phi_i e_i \qquad (2)$$

where $e_i = \theta_i - \theta_d$ is the local tracking error vector, $\phi_i = \mathrm{diag}(\phi_{i1}, \cdots, \phi_{in}) > 0$ with constant entries.

Substituting e_i and r_i into (1), we have

$$M_i(\theta_i)\dot{r}_i + C_i(\theta_i, \dot{\theta}_i)r_i = u_i + \xi_i \tag{3}$$

where $\xi_i = [\xi_{i1}, \cdots, \xi_{in}]^T$ is considered to be an uncertain term of (3), and

$$\xi_i = -M_i(\theta_i)(\ddot{\theta}_d - \phi_i\dot{e}_i) - C_i(\theta_i, \dot{\theta}_i)(\dot{\theta}_d - \phi_i e_i) - g_i(\theta_i) - f_i(\dot{\theta}_i) \tag{4}$$

The global dynamics of the multiple EL systems is

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + g(\theta) + f(\dot{\theta}) = u \tag{5}$$

$$\begin{aligned} M(\theta) &= \mathrm{diag}[M_1(\theta_1), .., M_N(\theta_N)] \\ C(\theta, \dot{\theta}) &= \mathrm{diag}[C_1(\theta_1, \dot{\theta}_1), .., C_N(\theta_N, \dot{\theta}_N)] \\ g(\theta) &= [g_1(\theta_1), \cdots, g_N(\theta_N)]^T \\ f(\dot{\theta}) &= [f_1(\dot{\theta}_1), \cdots, f_N(\dot{\theta}_N)]^T \\ u &= [u_1, \cdots, u_N]^T \end{aligned} \tag{6}$$

where $\theta = [\theta_1^T, \cdots, \theta_N^T]^T$.

And the global version of (3) is given as

$$M(\theta)\dot{r} + C(\theta, \dot{\theta})r = u + \xi \tag{7}$$

$$\xi = -M(\theta)(1_N \otimes \ddot{\theta}_d - \Phi\dot{e}) - C(\theta, \dot{\theta})(1_N \otimes \dot{\theta}_d - \Phi e) - g(\theta) - f(\dot{\theta}) \tag{8}$$

where $\Phi = \mathrm{diag}(\phi_1, .., \phi_N)$, $\xi = [\xi_1, \cdots, \xi_N]^T$ is the global uncertain term, $r = [r_1, \cdots, r_N]^T$ is the global auxiliary error vector, and $e = \theta - 1_N \otimes \theta_d$ is the global tracking error vector.

Then the following properties hold [7].

Property 1:

$$\mu_{\min}(M)I \le M(\theta) \le \mu_{\max}(M)I \tag{9}$$

where $\mu_{\max}(\cdot), \mu_{\min}(\cdot) > 0$ denote respectively the maximal and minimal eigenvalues of a matrix.

Property 2:

$$\| C(\theta, \dot{\theta}) \|_2 \le c_H \| \dot{\theta} \|_2 \tag{10}$$

for some constant $c_H > 0$.

Property 3:

$$\| g(\theta) \|_2 \le c_g, \quad \| f(\dot{\theta}_i) \|_2 \le c_{f1} + c_{f2} \| \dot{\theta} \|_2 \tag{11}$$

for some constants $c_g, c_{f1}, c_{f2} > 0$.

Property 4:

$$x^T \left[\frac{1}{2}\dot{M}(\theta) - C(\theta, \dot{\theta})\right] x = 0, \quad \forall x \neq 0 \tag{12}$$

C. Synchronized tracking problem

The control objective is to design a controller for each agent to track a time-varying reference trajectory exerted by a leader, with the aid of the neighbor agents' information obtained by certain communication protocol. That is, $\| e_i \|_2$ and $\| \dot{e}_i \|_2$ $(i = 1, \cdots, N)$ should be controlled to be small.

To construct a feedback controller, we define the following local synchronization error vector of agent i which will be used as a feedback signal:

$$e_{si} = \sum_{j \in \mathcal{N}_i} a_{ij}(\theta_i - \theta_j) + b_i(\theta_i - \theta_d) \tag{13}$$

where the scalar pinning gain $b_i \ge 0$. If agent i receives information directly from the leader then $b_i > 0$, otherwise $b_i = 0$. a_{ij} is the (i, j) entry of the adjacency matrix $\mathcal{A}$ which defines the communication topology of the network.

Then the global synchronization error vector is given as $e_s = \mathcal{H}e$, where $\mathcal{H} = (\mathcal{L} + B) \otimes I_n$, $e_s = [e_{s1}^T, \cdots, e_{sN}^T]^T$, $B = \mathrm{diag}(b)$, $b = [b_1, \cdots, b_N]^T$.

We impose the following assumption on the communication topology of the network [4], [3].

Assumption 2: The communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ of the multiple EL systems is undirected and connected.

Then, the following lemma is useful [3], [4].

Lemma 1: If the information interchange graph $\mathcal{G}$ is undirected and connected, and if at least one of the elements of b is nonzero, then $\mathcal{L} + B$ is a positive definite symmetric matrix.

According to the aforementioned definitions, the global auxiliary synchronization error vector r_s is given as

$$r_s = \mathcal{H}r = \mathcal{H}(\dot{e} + \Phi e) = \dot{e}_s + \Phi e_s \tag{14}$$

where $r_s = [r_{s1}, \cdots, r_{sN}]^T$, $r_{si} = [r_{si1}, \cdots, r_{sin}]^T$.

For the EL system model (3), it is well known that the uncertainty term is bounded by the following relation [8]:

$$\| \xi \|_2 \le \alpha_1 + \alpha_2 \| e \|_2 + \alpha_3 \| \dot{e} \|_2 + \alpha_4 \| e \|_2 \| \dot{e} \|_2 \tag{15}$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are some positive constants.

For a vector-valued signal $x(t)$, we define a truncated norm for $T > 0$ as $\| x \|_T \equiv \sup_{t \in [0,T]} \| x(t) \|_2$. Then we have [8]

Lemma 2: Let Assumption 1 and Properties 1∼4 hold. If there is a constant T such that $\| r \|_T$ exists, then for all $t \in [0, T]$ we have for some $\beta_1, \beta_2, \beta_3 > 0$,

$$\| \xi \|_2 \le \beta_1 + \beta_2 \| r \|_T + \beta_3 \| r \|_T^2 \tag{16}$$

III. Controller Design

A. Introduction of DOB

Replacing r in (7) by r_s, we have

$$\mathcal{H}^{-1}\dot{r}_s = M^{-1}u + M^{-1}(\xi - C(\theta, \dot{\theta})\mathcal{H}^{-1}r_s) \tag{17}$$

For the sake of design a distributed and decentralized controller u_{ij} for each generalized coordinate, we define $\mathcal{H}_0^{-1}$ and M_0 as some diagonal nominal matrices of $\mathcal{H}^{-1}$ and M respectively, and then we have

$$\begin{aligned} M_0\mathcal{H}_0^{-1}\dot{r}_s &= u + M_0M^{-1}(\xi - C(\theta, \dot{\theta})\mathcal{H}^{-1}r_s) \\ &\quad + (M_0M^{-1} - I)u + M_0(\mathcal{H}_0^{-1} - \mathcal{H}^{-1})\dot{r}_s \end{aligned} \tag{18}$$

Let $M_0\mathcal{H}_0^{-1} = M_{s0} = \mathrm{diag}(m_{s01}, \cdots, m_{s0N})$ with $m_{s0i} = \mathrm{diag}(m_{s0i1}, \cdots, m_{s0in}) > 0$. Then we have the following vector-valued nominal linear system model.

$$M_{s0}\dot{r}_s = u + w \tag{19}$$

where $w = [w_1^T, \cdots, w_N^T]^T$ with $w_i = [w_{i1}, \cdots, w_{in}]^T$ is the global lumped disturbance vector expressed as

$$\begin{aligned} w &= M_0M^{-1}(\xi - C(\theta, \dot{\theta})\mathcal{H}^{-1}r_s) + (M_0M^{-1} - I)u \\ &\quad + M_0(\mathcal{H}_0^{-1} - \mathcal{H}^{-1})\dot{r}_s \end{aligned} \tag{20}$$

Writing the dynamics of the jth generalized coordinate of the ith agent, we have

$$m_{s0ij}\dot{r}_{sij} = u_{ij} + w_{ij} \quad (21)$$

where w_{ij} is considered as a lumped disturbance term. Since calculation of $\dot{r}_{sij}$ by direct differentiation is usually contaminated with high frequency noise, we may pass w_{ij} through a low-pass filter to obtain its estimate as

$$Q_{ij}(s)w_{ij} = Q_{ij}(s)(m_{s0ij}sr_{sij} - u_{ij}) \quad (22)$$

In this paper, for convenience of expression, s denotes not only the Laplace operator, but also a differential operator. This is the so-called DOB studied extensively in the literature. In this a study, we adopt a simple second-order filter

$$Q_{ij}(s) = \frac{1}{(1+\lambda_{ij}s)^2} \quad (23)$$

where $\lambda_{ij} > 0$.

However, we can only expect $Q(s)w_{ij} \approx w_{ij}$ at low-frequencies due to limited pass-band of the DOB. Moreover, the DOBs' outputs $\widehat{w}_{ij}(i = 1, \cdots, n,\ j = 1, \cdots, N)$ may disturb the signals of the other generalized coordinates. To ease the analysis shown later, a straightforward and simple idea is to saturate the output of the DOB as

$$\widehat{w}_{ij} = \begin{cases} \overline{w} & \text{for } |Q_{ij}(s)(m_{s0ij}sr_{ij} - u_{ij})| \geqslant \overline{w} \\ Q_{ij}(s)(m_{s0ij}sr_{ij} - u_{ij}) & \\ & \text{for } |Q_{ij}(s)(m_{s0ij}sr_{ij} - u_{ij})| < \overline{w} \\ -\overline{w} & \text{for } |Q_{ij}(s)(m_{s0ij}sr_{ij} - u_{ij})| \leqslant -\overline{w} \end{cases} \quad (24)$$

where $\overline{w} > 0$ is a selected upper bound of $|\widehat{w}_{ij}|$. Usually, it is recommended to choose a sufficiently large $\overline{w}$. However, even when $\overline{w}$ is not so large such that $\widehat{w}_{ij}$ is really saturated and hence the estimation error $(w_{ij} - \widehat{w}_{ij})$ is not sufficiently small, the control performance is still satisfactory, owing to the sliding mode control term included in the local controller (25) given later. The key point is that the DOB and the sliding mode control term work in a cooperative manner as suggested in [6]. Owing to their cooperative effects, the problems of high-gain or chattering can be avoided. This will be confirmed later by the numerical examples.

B. Description of the controller

Motivated by the aforementioned discussions, we design the following local controller u_{ij} for the jth generalized coordinate of agent i, using only the neighbor information of agent i to ensure the boundedness of the global system signals and to achieve a satisfactory control performance.

$$u_{ij} = -\rho_a l_1^2 r_{sij} - \rho_b \overline{w} r_{sij} - \rho_c \eta_{\max} r_{sij} - \widehat{w}_{ij} - \eta_{ij}\mathrm{sat}(r_{sij}) \quad (25)$$

where $k, \rho_a, \rho_b, \rho_c, l_1, \eta_{ij} > 0$, $\mathrm{sat}(r_{sij}) = r_{sij}/(|r_{sij}| + \delta_{ij})$, $\delta_{ij} > 0$, $\eta_{\max} = \max(\eta_{11}, \cdots, \eta_{1n}, \cdots, \eta_{N1}, \cdots, \eta_{Nn})$.

The controller is explained as follows.

The damping term $-\rho_a l_1^2 r_{sij}$ is adopted to suppress the effects of neglected uncertainties of the global system model summarized as ξ in (8). The constant l_1 is chosen such that $l_1 > \| r \|_2 / \sqrt{Nn}$ for all $t > 0$. It will be shown that there exists such an l_1. The term $-\widehat{w}_{ij}$ is a compensation term by DOB for each generalized coordinate. Since the DOBs' outputs $\widehat{w}_{ij}(i = 1, \cdots, N,\ j = 1, \cdots, n)$ may disturb mutually, to suppress the interactions due to $\widehat{w}_{ij}$, we employ the damping term term $-\rho_b \overline{w} r_{sij}$. The last term of u_{ij} is a smoothed version of sliding mode control term. The damping term $-\rho_c \eta_{\max} r_{sij}$ is a term to suppress the interactions among the sliding mode control terms.

The global expression is given as below which will be used for analysis of the global system.

$$u = -\rho_a l_1^2 r_s - \rho_b \overline{w} r_s - \rho_c \eta_{\max} r_s - \widehat{w} - \eta \mathrm{Sat}(r_s) \quad (26)$$

where $\widehat{w} = [\widehat{w}_1, \cdots, \widehat{w}_N]^T$, $\widehat{w}_i = [\widehat{w}_{i1}, \cdots, \widehat{w}_{in}]^T$, $\eta = \mathrm{diag}[\eta_1, \cdots, \eta_N]^T$, $\eta_i = \mathrm{diag}[\eta_{i1}, \cdots, \eta_{in}]^T$, $\mathrm{Sat}(r_s) = [\mathrm{sat}(r_{s1}), \cdots, \mathrm{sat}(r_{sN})]^T$, $\mathrm{sat}(r_{si}) = [\mathrm{sat}(r_{si1}), \cdots, \mathrm{sat}(r_{sin})]^T$.

Remark 1: The controller (26) is an extension or modification of the decentralized controller for a single EL system [6]. Compared to the controller in [6] where some nonlinear damping terms with signal dependent gains are used, in the present controller, we have to use some linear damping terms with relatively high constant gains, such as $\rho_a l_1^2 r_s$, $\rho_b \overline{w} r_s$ and $\rho_c \eta_{\max} r_s$. This is mainly due to the presence of the matrix $\mathcal{H}$ in (30) given later. Therefore, as the price of multiple EL system control, the controller design is less flexible as the case of a single agent.

C. Comments and guidelines of parameter design

The guidelines of parameter design are aummarized here based on the theoretical analysis given later.

The constant l_1 in (25) should meet the requirement that $l_1 > \| r \|_2 / \sqrt{Nn}$ for all $t > 0$. That is, we have to guess the upper bound of $\| r \|_2$. See Theorem 1 later.

The entries of ϕ_i that appeared in (2) should not be very large, since large values of them may lead to a very large $\| r \|_2^2$ which may violate the condition imposed on l_1.

A small smoothing factor δ_{ij} for sat(r_{sij}) in (25) leads to a small ultimate tracking error of the corresponding generalized coordinate. However, as well known in the literature, a less smooth switching function may cause the chattering problem. A high sliding mode control gain η_{ij} helps to achieve a small control ultimate tracking error, but it may also cause the chattering problem, and may cause a high gain control term $\rho_c \eta_{\max} r_{si}$.

The saturation level $\overline{w}$ of the DOBs should not be very large to avoid causing a high gain control term $\rho_b \overline{w} r_{si}$.

Usually, a smaller λ_{ij} of the DOB filter (23) leads to a better disturbance performance. However, too small a λ_{ij} may make $\widehat{w}_{ij}$ sensitive to the noise.

When l_1^2, $\overline{w}$, $\eta_{\max}$ and η_{ij} meet the aforementioned requirements, the choice of ρ_a, ρ_b and ρ_c in (25) is trivial. Some moderate values of these parameters are satisfactory.

IV. Performance analysis

Since all of the agents interact with their neighbors, we cannot easily see if the signals of the agents are all bounded. We should first ensure the boundedness of the global system signals. Then provided the boundedness of the global system signals, we can analyze the control performance of each agent. Therefore, the performance analysis includes two phases.

A. Analysis of the global system

The results of analysis are given in Theorem 1. The proof is an extension of [8], [6], but with modifications specified by the newly designed controller in this study.

Theorem 1: Let Assumptions 1 and 2 hold. For the multiple EL systems (5) controlled by the proposed distributed robust controller (26), there exists a constant $l_1 > 0$, such that r is bounded as $\| r \|_2 < \sqrt{Nn} l_1$ and hence all the internal signals are bounded, provided the following condition.

$$\sqrt{\frac{\mu_{\min}(M)}{\mu_{\max}(M)}}\sqrt{Nn} l_1 > \left[\frac{2Nn(\beta_1+\beta_2 l_1+\beta_3 l_1^2)}{\rho_a \mu_{\min}(\mathcal{H})}\right]^{1/3} \geq \left(\frac{N^2 n^2 \overline{w}}{2\rho_a\rho_b\mu_{\min}^2(\mathcal{H})}+\frac{N^2n^2\eta_{\max}}{2\rho_a\rho_c\mu_{\min}^2(\mathcal{H})}\right)^{1/4} \tag{27}$$

Remark 2: The first inequality of (27) is easily satisfied for sufficiently large l_1, ρ_a. The second inequality of (27) can be satisfied for sufficiently large l_1, ρ_b, ρ_c.

Proof. According to Assumption 2, we have Lemma 1 and hence $\mu_{\min}(\mathcal{H}) > 0$. We are now ready to show that there exists a constant $l_1 > 0$, such that r is bounded as $\| r \|_2 < \sqrt{Nn} l_1$. The conclusion is proved by contradiction. To this end, according to (27) we first let a positive constant l_1 satisfy

$$\| r(0) \|_2 < \left[\frac{2Nn(\beta_1+\beta_2 l_1+\beta_3 l_1^2)}{\rho_a \mu_{\min}(\mathcal{H})}\right]^{1/3} < \sqrt{\frac{\mu_{\min}(M)}{\mu_{\max}(M)}}\sqrt{Nn} l_1 \leq \sqrt{Nn} l_1 \tag{28}$$

Now assume the signal $r(t)$ is not bounded. Thus there always exists a smallest time T_1 such that $\| r(T_1) \|_2 = \sqrt{Nn} l_1$. Consider a Lyapunov function candidate with respect to the global tracking error vector.

$$V(t,r) = \frac{1}{2} r^T M(\theta) r \tag{29}$$

Taking the derivative along the trajectory of the closed-loop system, we have

$$\begin{aligned}\dot{V}(t,r) &= r^T\left(u+\xi-C(\theta,\dot{\theta})r+\frac{1}{2}\dot{M}(\theta)r\right)\\ &\leq r^T u + \| r \|_2 \| \xi \|_2\\ &= -\rho_a l_1^2 r^T\mathcal{H}r-\rho_b\overline{w}r^T\mathcal{H}r-r^T\widehat{w}\\ &\quad -\rho_c\eta_{\max}r^T\mathcal{H}r-r^T\eta\mathrm{Sat}(r_s)+\| r \|_2\| \xi \|_2\\ &\leq -\rho_a l_1^2\mu_{\min}(\mathcal{H})\| r \|_2^2+\| r \|_2\| \xi \|_2\\ &\quad -\rho_b\overline{w}\mu_{\min}(\mathcal{H})\| r \|_2^2+\| r \|_2\| \widehat{w} \|_2\\ &\quad -\rho_c\eta_{\max}\mu_{\min}(\mathcal{H})\| r \|_2^2+\sqrt{Nn}\eta_{\max}\| r \|_2\end{aligned} \tag{30}$$

Here, $\eta_{\max}$ is the maximum diagonal element of η. Completing the squares, we have

$$\begin{aligned}\dot{V}(t,r) &\leq -\rho_a l_1^2\mu_{\min}(\mathcal{H})\| r \|_2^2+\| r \|_2\| \xi \|_2\\ &\quad +\frac{\| \widehat{w} \|_2^2}{4\rho_b\overline{w}\mu_{\min}(\mathcal{H})}+\frac{Nn\eta_{\max}^2}{4\rho_c\eta_{\max}\mu_{\min}(\mathcal{H})}\end{aligned} \tag{31}$$

By (28), and the assumption that there exists a smallest time T_1 such that $\| r(T_1) \|_2 = \sqrt{Nn} l_1$, we have $\| r \|_2 < \sqrt{Nn} l_1$ for any $t < T_1$. Then using Lemma 2, we have for $t < T_1$,

$$\begin{aligned}\dot{V}(t,r) &\leq -\frac{\rho_a\mu_{\min}(\mathcal{H})}{Nn}\| r \|_2^4+\| r \|_2(\beta_1+\beta_2 l_1+\beta_3 l_1^2)\\ &\quad +\frac{Nn\overline{w}}{4\rho_b\mu_{\min}(\mathcal{H})}+\frac{Nn\eta_{\max}}{4\rho_c\mu_{\min}(\mathcal{H})}\\ &= -\frac{\rho_a\mu_{\min}(\mathcal{H})}{2Nn}\| r \|_2\left(\| r \|_2^3-\frac{2Nn(\beta_1+\beta_2 l_1+\beta_3 l_1^2)}{\rho_a\mu_{\min}(\mathcal{H})}\right)\\ &\quad -\frac{\rho_a\mu_{\min}(\mathcal{H})}{2Nn}\left(\| r \|_2^4-\frac{N^2n^2\overline{w}}{2\rho_a\rho_b\mu_{\min}^2(\mathcal{H})}-\frac{N^2n^2\eta_{\max}}{2\rho_a\rho_c\mu_{\min}^2(\mathcal{H})}\right)\end{aligned} \tag{32}$$

We then can say that there exists a time instant $t_2 = T_1 - t_1 > 0,\ t_1 > 0$ such that

$$\sqrt{Nn} l_1 > \| r(T_1-t_1) \|_2 = \left[\frac{2Nn(\beta_1+\beta_2 l_1+\beta_3 l_1^2)}{\rho_a \mu_{\min}(\mathcal{H})}\right]^{1/3} \geq \left(\frac{N^2 n^2 \overline{w}}{2\rho_a\rho_b\mu_{\min}^2(\mathcal{H})}+\frac{N^2n^2\eta_{\max}}{2\rho_a\rho_c\mu_{\min}^2(\mathcal{H})}\right)^{1/4} \tag{33}$$

However, according to (32), we have $d/dtV(t) \leq 0$, for all $t \in [T_1 - t_1, T_1]$. Therefore, for all $t \in [T_1 - t_1, T_1]$, we have

$$\begin{aligned}V[T_1, r(T_1)] &\leq V[(T_1-t_1), r(T_1-t_1)]\\ &\leq \frac{1}{2}\mu_{\max}(M)\left[\frac{2Nn(\beta_1+\beta_2 l_1+\beta_3 l_1^2)}{\rho_a \mu_{\min}(\mathcal{H})}\right]^{2/3}\end{aligned} \tag{34}$$

But the definition of T_1 leads to

$$V_1[T_1, r(T_1)] \geq \frac{1}{2}\mu_{\min}(M) Nn l_1^2 \tag{35}$$

Clearly, the last two inequalities are in contradiction, according to (28). This implies that the assumption of $\| r(T_1) \|_2 = \sqrt{Nn} l_1$ is false. Thus the error signal vector r is bounded and satisfies $\| r(t) \|_2 < \sqrt{Nn} l_1$ for all $t \geq 0$.

Furthermore, according to Assumption 1, and (2), (15) and (14), we conclude that e, $\dot{e}$, θ, $\dot{\theta}$, ξ and r_s are bounded. And hence each local controller u_{ij} is bounded. Therefore, all the internal signals are bounded. □

Remark 3: The condition (27) is always satisfied for a sufficiently large bound l_1. The results of Theorem 1 only tell us that r and hence all the internal signals can be made to be bounded. It should be emphasized here that at the present stage our purpose is only to ensure the boundedness of the signals. And hence a conservative bound of the signals is acceptable. Later, we will show that the individual synchronization error r_{sij} can be made sufficiently small by virtue of the corresponding local controller.

B. Analysis of each agent

We are now ready to analyze how the DOBs and sliding mode control techniques bring improvement in each generalized coordinate. Substituting the local controller (25) into the subsystem (21), the resultant subsystem of the jth generalized coordinate of the ith agent becomes

$$\begin{aligned} m_{s0ij}\dot{r}_{sij} = &-\rho_a l_1^2 r_{sij} - \rho_b \overline{w} r_{sij} - \rho_c \eta_{\max} r_{sij} \\ &-\widehat{w}_{ij} - \eta_{ij}\frac{r_{sij}}{|r_{sij}|+\delta_{ij}} + w_{ij} \end{aligned} \tag{36}$$

Owing to the results of Theorem 1, $w_{ij}(t)$ and $\widehat{w}_{ij}(t)$ are bounded. Define

$$\eta^*_{ij,0} = \sup_{0\le\tau\le t} |w_{ij}(\tau) - \widehat{w}_{ij}(\tau)| \tag{37}$$

Theorem 2: Let the assumptions and results of Theorem 1 hold. The synchronization error of the jth generalized coordinate of the ith agent satisfies

$$|r_{sij}(t)| \le |r_{sij}(0)| e^{-\frac{c}{m_{s0ij}}t} + \sqrt{\frac{\delta_{ij}\eta^*_{ij,0}}{c}} \tag{38}$$

if $\eta_{ij} \ge \eta^*_{ij,0}$, or

$$|r_{sij}(t)| \le |r_{sij}(0)| e^{-\frac{c}{2m_{s0ij}}t} + \frac{(\eta^*_{ij,0} - \eta_{ij})}{c} + \sqrt{\frac{2\delta_{ij}\eta_{ij}}{c}} \tag{39}$$

if $0 \le \eta_{ij} < \eta^*_{ij,0}$, where $c = \rho_a l_1^2 + \rho_b \overline{w} + \rho_c \eta_{\max}$.

Proof. We first consider the case of $\eta_{ij} \ge \eta^*_{ij,0}$, i.e., the sliding mode control gain exceeds the maximum amplitude of $(w_{ij} - \widehat{w}_{ij})$. From (36), we have

$$\frac{d}{dt}\left(\frac{m_{s0ij} r_{sij}^2}{2}\right) \le -c r_{sij}^2 + \delta_{ij}\eta^*_{ij,0} \tag{40}$$

and hence

$$r_{sij}^2(t) \le e^{-\frac{2c}{m_{s0ij}}t} r_{sij}^2(0) + \frac{\delta_{ij}\eta^*_{ij,0}}{c} \tag{41}$$

This leads to (38).

In the case of $0 \le \eta_{ij} < \eta^*_{ij,0}$, we have

$$\begin{aligned} \frac{d}{dt}\left(\frac{m_{s0ij} r_{sij}^2}{2}\right) \le &-\frac{c}{2} r_{sij}^2 - \left(\sqrt{\frac{c}{2}}|r_{sij}| - \frac{\eta^*_{ij,0} - \eta_{ij}}{\sqrt{2c}}\right)^2 \\ &+ \frac{(\eta^*_{ij,0} - \eta_{ij})^2}{2c} + \delta_{ij}\eta_{ij} \end{aligned} \tag{42}$$

and hence

$$r_{sij}^2(t) \le e^{-\frac{c}{m_{s0ij}}t} r_{sij}^2(0) + \left(\frac{(\eta^*_{ij,0} - \eta_{ij})^2}{c_i^2} + \frac{2\delta_{ij}\eta_{ij}}{c_i}\right) \tag{43}$$

This leads to (39). □

However, $\eta^*_{ij,0}$ may not be small since the initial value $\widehat{w}_{ij}(0)$ is often set to be zero. To investigate the performance after a short transient phase of DOB. Let $t_{ij}(\lambda_{ij})$ be an effective time-constant of the DOB depending on λ_{ij}, until which the initial value of $(w_{ij} - \widehat{w}_{ij})$ has decayed out sufficiently such that for a relatively small constant $\eta^*_{ij,t_{ij}}$ we have

$$\eta^*_{ij,t_{ij}} = \sup_{t_{ij}(\lambda_{ij})\le\tau\le t} |w_{ij}(\tau) - \widehat{w}_{ij}(\tau)| \tag{44}$$

Comparing (37) and (44), it is expected that $\eta^*_{ij,t_{ij}}$ can be much smaller than $\eta^*_{ij,0}$. Then we have

Corollary 1: For $t \ge t_{ij}(\lambda_{ij})$, the synchronization error of the jth generalized coordinate of the ith agent satisfies

$$|r_{sij}(t)| \le |r_{sij}(t_{ij})| e^{-\frac{c}{m_{s0ij}}(t-t_{ij})} + \sqrt{\frac{\delta_{ij}\eta^*_{ij,t_{ij}}}{c}} \tag{45}$$

if $\eta_{ij} \ge \eta^*_{ij,t_{ij}}$, or

$$\begin{aligned} |r_{sij}(t)| \le\ & |r_{sij}(t_{ij})| e^{-\frac{c}{2m_{s0ij}}(t-t_{ij})} \\ &+ \frac{(\eta^*_{ij,t_{ij}} - \eta_{ij})}{c} + \sqrt{\frac{2\delta_{ij}\eta_{ij}}{c}} \end{aligned} \tag{46}$$

if $0 \le \eta_{ij} < \eta^*_{ij,t_{ij}}$, where $c = \rho_a l_1^2 + \rho_b \overline{w} + \rho_c \eta_{\max}$.

Theorem 2 and Corollary 1 imply that the auxiliary synchronization error r_s is uniformly ultimately bounded (UUB), and hence the auxiliary tracking error $r = \mathcal{H}^{-1} r_s$ is UUB.

V. Simulation Studies

For the sake of comparison, we borrow the example in [3] and carry out the numerical simulations under the same conditions as possible. Consider a group of 6 two-DOF planar robot arms:

$$\begin{aligned} &\begin{bmatrix} m_{i11}(\theta_i) & m_{i12}(\theta_i) \\ m_{i21}(\theta_i) & m_{i22}(\theta_i) \end{bmatrix} \begin{bmatrix} \ddot{\theta}_{i1} \\ \ddot{\theta}_{i2} \end{bmatrix} + \begin{bmatrix} h_{i1}(\theta_i, \dot{\theta}_i) \\ h_{i2}(\theta_i, \dot{\theta}_i) \end{bmatrix} \\ &\qquad + \begin{bmatrix} g_{i1}(\theta_i) \\ g_{i2}(\theta_i) \end{bmatrix} = \begin{bmatrix} u_{i1} \\ u_{i2} \end{bmatrix} \end{aligned} \tag{47}$$

where $i = 1, \cdots, 6$, and

$$\begin{aligned} m_{i11}(\theta_i) &= m_{i1} l_{ci1}^2 + m_{i2}(l_{i1}^2 + l_{ci2}^2) + I_{i1} + I_{i2} \\ &\quad + 2m_{i2} l_{i1} l_{ci2}\cos(\theta_{i2}) \\ m_{i12}(\theta_i) &= m_{i2} l_{ci2}^2 + I_{i2} + m_{i2} l_{i1} l_{ci2}\cos(\theta_{i2}) \\ m_{i21}(\theta_i) &= m_{i12} \\ m_{i22}(\theta_i) &= m_{i2} l_{ci2}^2 + I_{i2} \end{aligned} \tag{48}$$

$$\begin{aligned} h_{i1}(\theta_i, \dot{\theta}_i) &= -m_{i2} l_{i1} l_{ci2}(2\dot{\theta}_{i1}\dot{\theta}_{i2} + \dot{\theta}_{i2}^2)\sin(\theta_{i2}) \\ h_{i2}(\theta_i, \dot{\theta}_i) &= m_{i2} l_{i1} l_{ci2}\dot{\theta}_{i1}^2 \sin(\theta_{i2}) \\ g_{i1}(\theta_i) &= g(m_{i1} l_{ci1} + m_{i2} l_{i1})\cos(\theta_{i1}) \\ &\quad + m_{i2} g l_{ci2}\cos(\theta_{i1} + \theta_{i2}) \\ g_{i2}(\theta_i) &= m_{i2} g l_{ci2}\cos(\theta_{i1} + \theta_{i2}) \end{aligned} \tag{49}$$

where $g = 9.807$[m/s^2], and the physical parameters are given in Table I.

The network topology for communication among the agents is shown in Fig. 1. It can be verified that agents 1∼6 are undirectionally connected, and only agents 3 and 6 have access

TABLE I
PHYSICAL PARAMETERS OF THE ROBOT MANIPULATORS ($i = 1, \cdots, 6$)

link mass [kg]	$m_{i1} = 1.0 + 0.3i$, $m_{i2} = 1.5 + 0.3i$
link length [m]	$l_{i1} = 0.2 + 0.06i$, $l_{i2} = 0.3 + 0.06i$
mass center [m]	$l_{ic1} = 0.1 + 0.03i$, $l_{ic2} = 0.15 + 0.03i$
inertial tensor [kg·m^2]	$I_{i1} = 0.0073, 0.0137, 0.0229, 0.0355, 0.0521, 0.0732$ $I_{i2} = 0.0194, 0.0309, 0.0461, 0.0656, 0.0900, 0.1198$

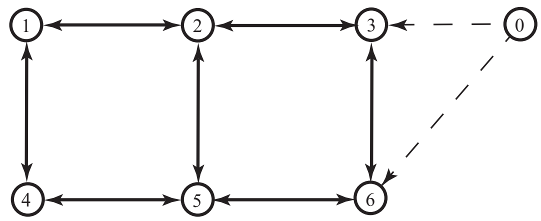

Fig. 1. Information exchange graph of the leader and followers

to the leader (agent 0). The corresponding adjacency matrix and pinning vector are given as follows.

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}, \qquad b = [0, 0, 1, 0, 0, 1]^T \tag{50}$$

We investigate the synchronized tracking performance for the following reference trajectory vector and its derivative generated by the leader.

$$\begin{aligned} \theta_d &= \left[\cos\left(\frac{2\pi}{60}t\right),\ \sin\left(\frac{2\pi}{60}t\right)\right]^T \quad [\mathrm{rad}] \\ \dot{\theta}_d &= \left(\frac{2\pi}{60}\right)\left[-\sin\left(\frac{2\pi}{60}t\right),\ \cos\left(\frac{2\pi}{60}t\right)\right]^T \quad [\mathrm{rad/s}] \end{aligned} \tag{51}$$

And to show that the controllers are robust against nonzero initial tracking errors, the initial conditions are given as

$$\begin{aligned} \theta_i(0) &= \left[\frac{\pi}{7}i,\ \frac{\pi}{8}i\right]^T \quad [\mathrm{rad}] \\ \dot{\theta}_i(0) &= [0.05i - 0.2,\ -0.05i + 0.2]^T \quad [\mathrm{rad/s}] \end{aligned} \tag{52}$$

According to the design guidelines, we choose the design parameters of the local controllers (25) as follows.

$$\begin{aligned} &\phi_{i1} = \phi_{i2} = 0.5 \\ &\rho_a = \rho_b = \rho_c = 1,\ \eta_{i1} = \eta_{i2} = 3 \\ &l_1 = 2,\ \delta_{i1} = \delta_{i2} = 0.05 \end{aligned} \tag{53}$$

where $i = 1, \cdots, 6$. The nominal values in (21) are given as $m_{s0i1} = m_{s0i2} = 1$. The time-constants of the DOB filters are given as $\lambda_{i1} = \lambda_{i2} = 0.02$. And the saturation level of the DOBs is chosen as $\overline{\widehat{w}} = 30$ (see (24)).

The simulation results are shown in Fig. 2, where from the top to the bottom are respectively the position-tracking errors e_{i1}, e_{i2}, auxiliary errors r_{i1}, r_{i2}, control signals u_{i1}, u_{i2} and DOBs' outputs $\widehat{w}_{i1}$, $\widehat{w}_{i2}$, where the lines of magenta, cyan, red, green, black and blue represents the signals of agents 1~6 respectively. It can be found in Fig. 1 that the proposed distributed controllers deliver a very excellent synchronized tracking performance, owing to the cooperative effects by DOBs and sliding mode control terms.

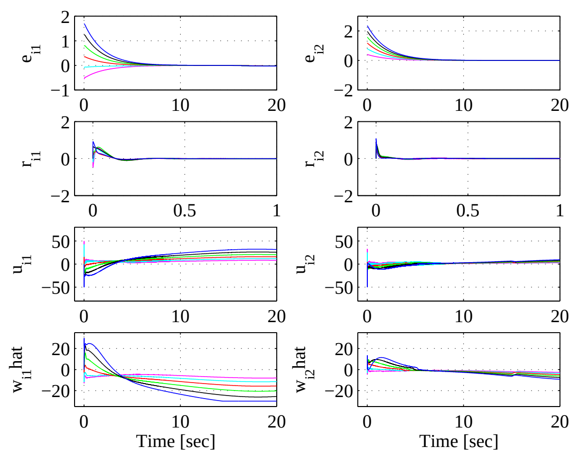

Fig. 2. Synchronized tracking results of 6 two-DOF planar robot arms.

VI. CONCLUSIONS

In this paper, a distributed robust control method for synchronized tracking of multiple EL systems has been proposed. The problem setting is similar to the works of [3], [4], where the time-varying reference trajectory is sent to only a subset of the agents and the network graph is assumed to be undirectionally connected. The proposed distributed controllers while delivering a very excellent control performance, are model-free and require only the neighbors' information. Therefore the proposed method is considered to be simple and requires moderate computational burden.

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