# Discrete Flight Path Angle Tracking Control of Hypersonic Flight Vehicles via Multi-rate Sampling 

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#### Abstract

This paper presents the flight path angle tracking control of the longitudinal dynamics of a generic hypersonic flight vehicle(HFV). Due to the use of digital computers and microprocessors for controls applications, the discrete hypersonic flight control is investigated. The altitude command is transformed into the flight path angle information. The back-stepping scheme is applied for the attitude subsystem which includes flight path angle, pitch angle and pitch rate. The virtual control is designed with nominal feedback and Neural Network (NN) approximation. To use the information of throttle setting, the multi-rate sampling method is employed for the two subsystems where the velocity subsystem is considered as slow dynamics. Under the proposed controller, the semiglobal uniform ultimate boundedness (SGUUB) stability is guaranteed. The simulation is presented to show the effectiveness of the proposed control approach.


## I. Introduction

Given the widespread use of digital computers and microprocessors for controls applications, the discrete-time case is certainly warranted [1]. For the control of flight vehicle and spacecraft, controller on the basis of continuous system is usually implemented by a digital computer with a certain sampling interval [2], [3]. Since modern aircraft are equipped with digital computers, the controller should be designed in discrete-time form [4]. There are two methods for designing the digital controller. One method, called emulation, designs a controller with the continuous-time system, and then discretizing the controller. The other is to design the controllers directly based on the discrete system. In contrast to the emulation method, the discrete controller is designed in a discrete domain so that the performance of the controller may not depend on the sampling rate and the upper bounds of the NN weight update rates guaranteeing the convergence can be estimated analytically while emulation method is otherwise [5].
In this paper, the discrete hypersonic flight control is analyzed. Hypersonic Flight vehicles are intended to present a reliable and more cost efficient way to access space with dramatic reductions in flight times. The longitudinal model of the dynamics is known to be unstable, non-minimum phase with respect to the regulated output, and affected by significant model uncertainty. In the last decade, considerable research focused on robust and adaptive hypersonic flight control [6], [7]. In [8], the control structure combines the inputs from the
pilot model, baseline controller and adaptive controller. Based on the input-output linearization using Lie derivative notation, sliding mode control [9] is designed. The sequential loop closure controller design [10] is based on the decomposition of the equations into functional subsystems. The method followed the approach that combined robust adaptive dynamic inversion with back-stepping arguments to obtain control architecture. In [11], one high gain observer based controller is proposed for HFV control with only one NN to compensate the lumped uncertainty. In [12], the attitude states are considered as fast dynamics and the altitude control is transformed into the flight path angle tracking.

It is illustrated that sometimes the controller based on Euler approximate discrete-time model of the plant is superior to back-stepping controllers based on digital control based on continuous-time plant model [13]. In this way, the discrete HFV model is obtained and the dynamic inversion is applied in [14]. By proper assumptions the discrete model is transformed into the strict-feedback form where some theoretical results have been studied in [15]. For the nonlinearity and the coupling, the nominal part of the dynamics should be considered for the feedback control design to provide good performance. The back-stepping neural design is analyzed in [16], [17]. By considering the uncertainty with Gaussian distribution, the Kriging design [18] is studied. In this paper, we focused on the control of the altitude subsystem. Continued with the work in [16], the altitude tracking is done with flight path angle tracking. The controller is designed with nominal value of the control gain. In this way, there is no need to know the upper bound. By multi-rate sampling design, the throttle setting can be viewed as constant during the attitude subsystem controller design. In this way, the assumption of $T \sin \alpha$ in [16] is eliminated.

This paper is organized as follows. Section II describes the longitudinal dynamics of a generic hypersonic flight vehicle. The strict-feedback form is formulated and the discrete analysis model is obtained in Section III. The brief description of HONN is explained in Section IV. Section V presents the adaptive controller design. The simulation result is included in Section VI. Section VII presents several comments and final remarks.

## II. Hypersonic Aircraft Model

The control-oriented model of the longitudinal dynamics of a generic hypersonic aircraft is considered. This model is comprised of five state variables $X=[V, h, \alpha, \gamma, q]^{T}$ and two control inputs $U_{c}=\left[\delta_{e}, \beta\right]^{T}$ where $V$ is the velocity, $\gamma$ is the flight path angle, $h$ is the altitude, $\alpha$ is the attack angle, $q$ is the pitch rate, $\delta_{e}$ is elevator deflection and $\beta$ is the throttle setting.

$$
\begin{align*}
\dot{V} & =\frac{T \cos \alpha-D}{m}-\frac{\mu \sin \gamma}{r^{2}}  \tag{1}\\
\dot{h} & =V \sin \gamma  \tag{2}\\
\dot{\gamma} & =\frac{L+T \sin \alpha}{m V}-\frac{\left(\mu-V^{2} r\right) \cos \gamma}{V r^{2}}  \tag{3}\\
\dot{\alpha} & =q-\dot{\gamma}  \tag{4}\\
\dot{q} & =\frac{M_{y y}}{I_{y y}} \tag{5}
\end{align*}
$$

where $T, D, L$ and $M_{y y}$ represent thrust, drag, lift-force and pitching moment respectively, $m, I_{y y}$ and $\mu$ represent the mass of aircraft, moment of inertia about pitch axis and gravity constant. $r$ is the radial distance from center of the earth and $r=h+R_{E}$. The related definition can be found in [9].

This paper focused on the cruise control with no consideration of the reentry process. The main goal of this paper is to design the altitude and velocity controller separately to follow the tracking reference $h_{d}$ and $V_{d}$.

## III. System Transformation

## A. Strict-Feedback Formulation

Assumption 1: Since $\gamma$ is quite small, we take $\sin \gamma \approx \gamma$ in (2) for simplification.

Remark 1: Similar assumption is made in [10] where the value of the flight path angle is set to be inside $\left[-3^{\circ}, 3^{\circ}\right]$.
The velocity subsystem (1) can be rewritten as

$$
\begin{align*}
& \dot{V}=f_{V}+g_{V} u_{V} \\
& u_{V}=\beta  \tag{6}\\
& y_{V}=V
\end{align*}
$$

where $f_{V}=-\left(D / m+\mu \sin \gamma / r^{2}\right)+\bar{q} S \times 0.0224 \cos \alpha / m, g_{V}=$ $\bar{q} S \times 0.00336 \cos \alpha / m$ if $\beta>1$. Otherwise $f_{V}=-(D / m+$ $\left.\mu \sin \gamma / r^{2}\right), g_{V}=\bar{q} S \times 0.02576 \cos \alpha / m$.
The tracking error of the altitude is defined as $\tilde{h}=h-h_{d}$ and the flight path command is chosen as

$$
\begin{equation*}
\gamma_{d}=\arcsin \left[\frac{-k_{h} \tilde{h}-k_{I} \int \tilde{h} d t+\dot{h}_{d}}{V}\right] \tag{7}
\end{equation*}
$$

if $k_{h}>0$ and $k_{I}>0$ are chosen and the flight-path angle is controlled to follow the reference command $\gamma_{d}$, the altitude tracking error is regulated to zero exponentially.

Define $X_{A}=\left[x_{1}, x_{2}, x_{3}\right]^{T}, x_{1}=\gamma, x_{2}=\theta_{p}, x_{3}=q$ where $\theta_{p}=$ $\alpha+\gamma$.

Then the strict-feedback form equations of the attitude subsystem (3)-(5) are written as

$$
\begin{align*}
\dot{x}_{1} & =f_{1}\left(x_{1}\right)+g_{1}\left(x_{1}\right) x_{2} \\
\dot{x}_{2} & =f_{2}\left(x_{1}, x_{2}\right)+g_{2}\left(x_{1}, x_{2}\right) x_{3} \\
\dot{x}_{3} & =f_{3}\left(x_{1}, x_{2}, x_{3}\right)+g_{3}\left(x_{1}, x_{2}, x_{3}\right) u_{A}  \tag{8}\\
u_{A} & =\delta_{e} \\
y & =x_{1}
\end{align*}
$$

where $f_{1}=-\left(\mu-V^{2} r\right) \cos \gamma /\left(V r^{2}\right)-\bar{q} S \times 0.6203 /(m V) \times$ $\gamma-T \sin \alpha, g_{1}=\bar{q} S \times 0.6203 /(m V), f_{2}=0, g_{2}=1, f_{3}=$ $\bar{q} S \bar{c}\left[C_{M}(\alpha)+C_{M}(q)-0.0292 \alpha\right] / I_{y y}, g_{3}=0.0292 \bar{q} S \bar{c} / I_{y y}$.
In the analysis [11], the altitude is mainly up to elevator deflection while the velocity is controlled by throttle setting. By command transformation, we know attitude subsystem is controlled by elevator deflection. To obtain the information of $\beta$ from the velocity subsystem, the multi-rate sampling design is employed so that velocity and $\beta$ can be considered as constant during the altitude subsystem controller design. The similar idea is studied in [19] where the airspeed, altitude and flight path angle are selected as slow-dynamics variables and considered invariant during the controller design of the fast dynamics.

The control objective of system (8) is to design an adaptive controller, which makes $\gamma \rightarrow \gamma_{d}$, further $h \rightarrow h_{d}$ and all the signals involved are bounded.
Assumption 2: $f_{i}$ and $g_{i}$ are unknown smooth functions which can be decomposed into the nominal part $f_{i N}, g_{i N}$ and the unknown part $\Delta f_{i}, \Delta g_{i}, i=1,3, V$.

## B. Discrete-time Model

By Euler approximation with different sample time period $T_{V}$ and $T_{s}$, systems (6) and (8) can be approximated as

$$
\begin{align*}
& V(k+1)=V(k)+T_{V}\left[f_{V}(k)+g_{V}(k) u_{V}(k)\right]  \tag{9}\\
& x_{1}(k+1)=x_{1}(k)+T_{s}\left[f_{1}(k)+g_{1}(k) x_{2}(k)\right] \\
& x_{2}(k+1)=x_{2}(k)+T_{s}\left[f_{2}(k)+g_{2}(k) x_{3}(k)\right]  \tag{10}\\
& x_{3}(k+1)=x_{3}(k)+T_{s}\left[f_{3}(k)+g_{3}(k) u_{A}(k)\right]
\end{align*}
$$

Here $T_{V}$ is selected four times as $T_{s}$. So during the design of attitude subsystem, the velocity and throttle setting could be considered as constant.

## IV. HONN Approximation

Higher order neural network (HONN) is one kind of linearly parameterized NNs. The structure of HONN is expressed as follows:

$$
\begin{gather*}
U(\omega, X)=\omega^{T} \theta(X) \quad \omega, \theta(X) \in R^{N}  \tag{11}\\
\theta_{i}(X)=\prod_{j \in I_{i}}\left[s\left(X_{j}\right)\right]^{d_{j i}} \tag{12}
\end{gather*}
$$

where $X \subset R^{m}$ is the input to HONN, $N$ is the NN nodes number, $\left\{I_{1}, I_{2}, \ldots, I_{N}\right\}$ is a collection of $N$ not-ordered subsets of $\{1,2, \ldots, m\}$, specified by the designer, $d_{j i}$ 's are prescribed nonnegative integers, $\omega$ is an adjustable synaptic weight vector, and $s\left(X_{j}\right)$ is a monotonically increasing and differentiable
sigmoidal function. In this paper, it is chosen as a hyperbolic tangent function, i.e., $s\left(X_{j}\right)=\left(e^{X_{j}}-e^{-X_{j}}\right) /\left(e^{X_{j}}+e^{-X_{j}}\right)$.
For a desired function $U^{*}$, it is assumed there exists an ideal weight vector $\omega^{*}$ such that the smooth function vector can be approximated by an ideal NN on a compact set

$$
\begin{equation*}
U^{*}=\omega^{* T} \theta(X)+\varepsilon(X),\|\varepsilon(X)\|<\varepsilon_{M} \tag{13}
\end{equation*}
$$

where $\varepsilon(X)$ is the bounded NN approximation error vector and $\varepsilon_{M}$ is the supreme of $\varepsilon(X)$.

## V. Discrete Control Design

## A. Adaptive NN Control for Attitude Subsystem

The errors are defined as

$$
\begin{align*}
z_{1}(k) & =x_{1}(k)-x_{1 d}(k)  \tag{14}\\
z_{2}(k) & =x_{2}(k)-x_{2 d}(k)  \tag{15}\\
z_{3}(k) & =x_{3}(k)-x_{3 d}(k) \tag{16}
\end{align*}
$$

where $x_{2 d}(k), x_{3 d}(k)$ are the virtual control inputs to be designed.
Step 1. From (14),

$$
\begin{equation*}
z_{1}(k+1)=x_{1}(k)+T_{s}\left[f_{1}(k)+g_{1}(k) x_{2}(k)\right]-x_{1 d}(k+1) \tag{17}
\end{equation*}
$$

where $x_{1 d}(k+1)$ is acquired from (7).
Since $g_{1}(k)$ and $f_{1}(k)$ are unknown, the uncertainty is defined as

$$
\begin{align*}
U_{1}(k) & =-\frac{1}{T_{s} g_{1}(k)}\left[-T_{s} f_{1}(k)-x_{1}(k)+x_{1 d}(k+1)\right] \\
& +\frac{1}{T_{s} g_{1 N}(k)}\left[-T_{s} f_{1 N}(k)-x_{1}(k)+x_{1 d}(k)\right] \\
& =\omega_{1}^{* T} \theta_{1}\left(X_{1}(k)\right)+\varepsilon_{1}\left(X_{1}(k)\right) \tag{18}
\end{align*}
$$

where $X_{1}(k)=\left[V(k), x_{1}(k), h(k), h_{d}(k), h_{d}(k+1)\right]^{T}, f_{1 N}(k)$ and $g_{1 N}(k)$ are the nominal parts of $f_{1}(k)$ and $g_{1}(k), \omega_{1}^{*}$ is the optimal parameters for NN to approximate $U_{1}(k)$ and $\varepsilon_{1}\left(X_{1}\right)$ is the NN reconstruction error.
Take $x_{2}(k)$ in (17) as the virtual control input and design its desired value as

$$
\begin{align*}
x_{2 d}(k) & =\frac{1}{T_{s} g_{1 N}(k)}\left[c_{1} z_{1}(k)-T_{s} f_{1 N}(k)-x_{1}(k)+x_{1 d}(k)\right] \\
& +\hat{\omega}_{1}^{T}(k) \theta_{1}\left(X_{1}(k)\right) \tag{19}
\end{align*}
$$

where $\hat{\omega}_{1}$ is the estimation of $\omega_{1}^{*}$.
Combining (15), (17) and (19), the following equation can be obtained.

$$
\begin{align*}
z_{1}(k+1) & =x_{1}(k)+T_{s}\left[f_{1}(k)+g_{1}(k) x_{2}(k)\right]-x_{1 d}(k+1) \\
& =T_{s} g_{1}(k) z_{2}(k)+T_{s} g_{1}(k)\left[\tilde{\omega}_{1}^{T}(k) \theta_{1}\left(X_{1}(k)\right)-\varepsilon_{1}\left(X_{1}(k)\right)\right] \\
& +\frac{g_{1}(k)}{g_{1 N}(k)} c_{1} z_{1}(k) \tag{20}
\end{align*}
$$

where $\tilde{\omega}_{1}(k)=\hat{\omega}_{1}(k)-\omega_{1}^{*}$. The robust updating algorithm for the NN weights is

$$
\begin{equation*}
\hat{\omega}_{1}(k+1)=\hat{\omega}_{1}(k)-\lambda_{1} z_{1}(k+1) \theta_{1}\left(X_{1}(k)\right)-\delta_{1} \hat{\omega}_{1}(k) \tag{21}
\end{equation*}
$$

where $\lambda_{1}>0$ and $0<\delta_{1}<1$.
Step 2. From(15),

$$
\begin{equation*}
z_{2}(k+1)=x_{2}(k)+T_{s}\left[f_{2}(k)+g_{2}(k) x_{3}(k)\right]-x_{2 d}(k+1) \tag{22}
\end{equation*}
$$

Define $X_{2}(k)=\left[X_{1}^{T}(k), x_{2}(k), h_{d}(k+2)\right]^{T}$. From (19), $x_{2 d}(k+$ 1) involves $x_{1}(k+1), f_{1}(k+1), z_{1}(k+1)$ and $x_{1 d}(k+2)$. It can be concluded that $x_{2 d}(k+1)$ is the function of $X_{2}(k)$. The uncertainty $U_{2}(k)$ is defined and can be approximated by NN as

$$
\begin{equation*}
U_{2}(k)=x_{2 d}(k)-x_{2 d}(k+1)=\omega_{2}^{* T} \theta_{2}\left(X_{2}(k)\right)+\varepsilon_{2}\left(X_{2}(k)\right) \tag{23}
\end{equation*}
$$

where $\omega_{2}^{*}$ is the optimal parameters and $\varepsilon_{2}\left(X_{2}(k)\right)$ is the NN reconstruction error.

Take $x_{3}(k)$ in (22) as the virtual control input and design its desired value as

$$
\begin{equation*}
x_{3 d}(k)=\frac{1}{T_{s} g_{2}(k)}\left[-x_{2}(k)+x_{2 d}(k)+c_{2} z_{2}(k)\right]+\hat{\omega}_{2}^{T}(k) \theta_{2}\left(X_{2}\right) \tag{24}
\end{equation*}
$$

where $\hat{\omega}_{2}$ is the estimation of $\omega_{2}^{*}$. The result of (23) and (24) is due to the fact that $f_{2}=0$ and $g_{2}=1$.

The robust updating algorithm for the NN weights is

$$
\begin{equation*}
\hat{\omega}_{2}(k+1)=\hat{\omega}_{2}(k)-\lambda_{2} z_{2}(k+1) \theta_{2}\left(X_{2}(k)\right)-\delta_{2} \hat{\omega}_{2}(k) \tag{25}
\end{equation*}
$$

where $\lambda_{2}>0$ and $0<\delta_{2}<1$.
Step 3. From (16),

$$
\begin{equation*}
z_{3}(k+1)=x_{3}(k)+T_{s}\left[f_{3}(k)+g_{3}(k) u_{A}(k)\right]-x_{3 d}(k+1) \tag{26}
\end{equation*}
$$

Define $X_{3}(k)=\left[X_{2}^{T}(k), x_{3}(k), h_{d}(k+3)\right]^{T}$. Similarly we can deduce that the uncertainty $U_{3}(k)$ is the function of $X_{3}(k)$ and it can be approximated by NN as

$$
\begin{align*}
U_{3}(k) & =-\frac{1}{T_{s} g_{3}(k)}\left[-T_{s} f_{3}(k)-x_{3}(k)+x_{3 d}(k+1)\right] \\
& +\frac{1}{T_{s} g_{3 N}(k)}\left[-T_{s} f_{3 N}(k)-x_{3}(k)+x_{3 d}(k)\right] \\
& =\omega_{3}^{* T} \theta_{3}\left(X_{3}(k)\right)+\varepsilon_{3}\left(X_{3}(k)\right) \tag{27}
\end{align*}
$$

where $\omega_{3}^{*}$ is the optimal parameters for NN to approximate $U_{3}(k)$ and $\varepsilon_{3}\left(X_{3}(k)\right)$ is the NN reconstruction error. The actual control input is designed as

$$
\begin{align*}
u_{A}(k) & =\frac{1}{T_{s} g_{3 N}(k)}\left[-T_{s} f_{3 N}(k)-x_{3}(k)+x_{3 d}(k)+c_{3} z_{3}(k)\right] \\
& +\hat{\omega}_{3}^{T}(k) \theta_{3}\left(X_{3}(k)\right) \tag{28}
\end{align*}
$$

where $\hat{\omega}_{3}$ is the estimation of $\omega_{3}^{*}$. where $\tilde{\omega}_{3}(k)=\hat{\omega}_{3}(k)-\omega_{3}^{*}$. The update law for the NN weights is

$$
\begin{equation*}
\hat{\omega}_{3}(k+1)=\hat{\omega}_{3}(k)-\lambda_{3} z_{3}(k+1) \theta_{3}\left(X_{3}(k)\right)-\delta_{3} \hat{\omega}_{3}(k) \tag{29}
\end{equation*}
$$

where $\lambda_{3}>0$ and $0<\delta_{3}<1$.
Theorem 1: Considering system (10) with the controller (28), virtual design (19), (24) and the update law (21), (25), (29), all the signals involved are semiglobal uniform ultimate bounded.
The proof is quite similar to [16] and thus omitted here to save space.

## B. Adaptive NN Control for Velocity Subsystem

Define $X_{V}(k)=\left[V(k), x_{1}(k), x_{2}(k), x_{3}(k), x_{4}(k), V_{d}(k+1)\right]^{T}$ and $z_{V}(k)=V(k)-V_{d}(k)$

$$
\begin{align*}
& z_{V}(k+1)=V(k+1)-V_{d}(k+1) \\
& =V(k)+T_{V}\left[f_{V}(k)+g_{V}(k) u_{V}(k)\right]-V_{d}(k+1) \tag{30}
\end{align*}
$$

The control input is designed as

$$
\begin{align*}
u_{V}(k) & =\frac{1}{T_{V} g_{V N}}\left[-T_{V} f_{V N}(k)-V(k)+V_{d}(k+1)\right] \\
& +\hat{\omega}_{V}^{T}(k) \theta_{V}\left(X_{V}(k)\right) \tag{31}
\end{align*}
$$

where $\hat{\omega}_{V}$ is the estimation of $\omega_{V}^{*}$.
The robust updating law for NN weights is

$$
\begin{equation*}
\hat{\omega}_{V}(k+1)=\hat{\omega}_{V}(k)-\lambda_{V} z_{V}(k+1) \theta_{V}\left(X_{V}(k)\right)-\delta_{V} \hat{\omega}_{V}(k) \tag{32}
\end{equation*}
$$

where $\lambda_{V}>0$ and $0<\delta_{V}<1$.
Theorem 2: Considering system (9) with the controller (31) and the update law (32), the velocity is semiglobal uniform ultimate bounded. The proof is omitted here.

## VI. Simulations

In this section, we verify the effectiveness and performance of the proposed adaptive neural controller. The flight of the vehicle is at trimmed cruise condition $M=15, V=15,060 \mathrm{ft} / \mathrm{s}$, $h=110,000 \mathrm{ft}$. Reference commands are generated by the filter:

$$
\begin{gather*}
\frac{h_{d}}{h_{c}}=\frac{\omega_{n 1} \omega_{n 2}^{2}}{\left(s+\omega_{n 1}\right)\left(s^{2}+2 \varepsilon_{\mathrm{c}} \omega_{n 2} s+\omega_{n 2}^{2}\right)}  \tag{33}\\
\frac{V_{d}}{V_{c}}=\frac{\omega_{n 3}}{\left(s+\omega_{n 3}\right)} \tag{34}
\end{gather*}
$$

where $\omega_{n 1}=0.2, \omega_{n 2}=0.2, \varepsilon_{c}=0.7, \omega_{n 3}=0.1$.
The parameters for the controller are selected as $k_{h}=0.2$, $k_{I}=0.1, \lambda_{1}=0.05, \lambda_{2}=0.05, \lambda_{3}=0.05, \delta_{1}=0.02, \delta_{2}=0.02$, $\delta_{3}=0.02, T_{s}=0.03 \mathrm{~s}, T_{V}=0.12 \mathrm{~s}, c_{1}=0.9, c_{2}=0.8, c_{3}=0.2$, $c_{V}=0.6$.


Fig. 1. Step Tracking: Altitude and Velocity Response


Fig. 2. Step Tracking: Control Inputs


Fig. 3. Step Tracking: Flight Path Angle Tracking


Fig. 4. Step Tracking: Attack Angle


Fig. 5. Step Tracking: Pitch Rate


Fig. 6. Step Tracking: Trajectories of NN Weights


Fig. 7. Square Signal Tracking: Altitude and Velocity Response


Fig. 8. Square Signal Tracking: Control Inputs


Fig. 9. Square Signal Tracking: Flight Path Angle Tracking


Fig. 10. Square Signal Tracking: Attack Angle


Fig. 11. Square Signal Tracking: Pitch Rate


Fig. 12. Square Signal Tracking: Trajectories of NN weights

## A. Step Tracking

Fig. 1 depicts the response performance that the altitude controller tracks the step change with magnitude 500ft while the velocity steps from $15060 \mathrm{ft} / \mathrm{s}$ to $15160 \mathrm{ft} / \mathrm{s}$. The control inputs of the elevator deflection and the throttle setting are shown in Fig.2. Flight path angle tracks the reference command very well in Fig. 3 so that the system tracks the altitude step change. From the pitch angle in Fig. 4 and pitch rate response in Fig.5, we know the related system states are bounded. Also we find the NN weights are bounded in Fig.6.

## B. Square Signal Tracking

The system tracks the square signal with amplitudes 500 ft and period 80 seconds while velocity is maintained in the neighborhood of $15060 \mathrm{ft} / \mathrm{s}$. The results are referred to Fig.712. It can be observed that all the system states are bounded and the velocity is regulated with a small error around the initial value. Also it is noted that the system states flight path angle, pitch angle, pitch rate perform good tracking response of altitude command, $x_{2 d}, x_{3 d}$ separately.

## VII. CONCLUSIONS

The altitude control of HFV is transformed into the flight path angle tracking. The two dynamics are sampled by different rate. In this way, the velocity and throttle setting from the slow dynamics can be employed for the design of attitude
subsystem. Simulation results show the effectiveness of the method.

## Acknowledgements

This work was supported by DSO National Laboratories of Singapore through a Strategic Project Grant (Project No. DSOCL10004), National Science Foundation of China (Grants No: 61134004,61004002), NWPU Basic Research Funding (Grant No: JC20120236) and Aeronautical Science Foundation of China(Grant No: 20110184).

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