

# Periodic Disturbance Rejection of a Class of Nonlinear System

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**Abstract**—This study presents a periodic disturbance rejection method for a class of nonlinear systems with the input weighting vector in the proportional nonlinear form. Especially, the periodic disturbance does not match with the system input. A neural network approximator is employed for the estimation of the ideal feedforward control input that tackles the influence brought by disturbances in closed-loop system. Moreover, The adaptive control techniques are applied to deal with nonlinear uncertainties and unknown parameters in the system. The proposed control design ensures the closed-loop convergence of the system, i.e. all states converge to a small set around their equilibrium points. A simulation example is included to support this control approach.

## I. INTRODUCTION

Disturbance rejection of nonlinear systems is a widely discussed issue in the recent decade. Many existing disturbance rejection results are based on the internal model principle. An internal model is constructed to generate a desired feedforward input for the annihilation of disturbance. For the disturbance signal generated by an unknown linear exosystem, the global asymptotic stability of a disturbed nonlinear system is achieved via state feedback with a state observer and an internal model [1]. With an additional filter design, this method is extended to a nonlinear system whose nonlinear uncertainties and disturbance uncertainties are tackled concurrently [2]. Generally, not all disturbance signals are linear in practice. From this point of view, periodic disturbances suppression is investigated. An adaptive feedforward disturbance cancellation scheme is proposed for the rejection of sinusoidal disturbances [3] [4]. In particular, periodic disturbances generated by a nonlinear exosystem are considered. The asymptotic tracking is achieved for an output feedback nonlinear system via incorporating filtered transformation, high gain control and saturation for internal model design [5]. As a special case, the periodic disturbance rejection problem is examined. The nonlinear system disturbed by general periodic disturbances which are half-period alternative is asymptotically suppressed. With known wave profiles of disturbances, an estimate of the feedforward control input is constructed by bringing in new operations of a half-period integration and a delay operator [6]. This method is extended to a nonlinear system with

unmatched disturbances in [7].

The aforementioned publications focus on the reconstruction of feedforward control input via internal model design. In fact, the output of the exosystem is constrained by its structures, i.e. not all periodic disturbances can be generated by the exosystem. In terms of a general periodic disturbance without dynamic limits, new approximation methods are worth being investigated. As a hot topic, Neural Network (NN) is exploited in many control design applications. It is well-known that NN can be used to estimate any nonlinear signals in a compact set. The theory about adaptive NN approximator for the periodically disturbed nonlinear function is established. With different selections of basis functions, two types of three-layer NN approximator are included as FSE-MNN-based approximator and FSE-RBFNN-based approximator [8]. After that, A two-layer NN approximator in [9] is introduced for output feedback stochastic nonlinear stabilisation. Motivated by these NN approximator designs, the two-layer approximator is used to estimate the feedforward control input in [10].

Note that the aforementioned nonlinear system has a constant input vector. Recently, disturbance suppression is established for various types of nonlinear systems with time varying weighting vector. Robust adaptive techniques are used in output feedback control design for a disturbed nonlinear system. It is notified that the system is in output feedback form with time varying input and disturbance vectors. The terminology of flat zone in a neighbourhood of the origin is introduced such that output tracking error converges to the flat zone under that control. It is also specified that, in this case, the element in input vector is coupled with a nonlinear function of output [11]. A novel asymptotic disturbance rejection approach for unknown sinusoidal disturbances is designed for nonlinear systems in output feedback form with an input gain vector whose element is a function of the output [12].

In this paper, a new disturbance rejection method is introduced for a nonlinear system in output feedback form

with an input vector with elements being a production of a constant and a nonlinear function of output. This research is still based on the existence of an invariant manifold which has a zero output under an ideal feedforward control input. To extract the zero dynamics of the system, a filter based transformation is proposed as similar as in [5]. As matter of fact that an NN can be used to estimate any nonlinear signals in a prescribed compact set. Since the desired feedforward control input is regarded as an unknown function of bounded disturbance signal, an NN approximator is applied for its estimation. In addition, a new unknown parameter is introduced to compensate the closed-loop influence from nonlinear uncertainties. Adaptive techniques are then applied to estimate unknown parameters online. Finally, backstepping control design based on the estimate of filters is carried out. Lyapunov stability analysis is presented to confirm the closed-loop stability. All states converge to a small area around their equilibrium points. Compared with the existing results, the information of disturbance are not necessarily needed. Furthermore, the disturbance can be any type of periodic signals.

The structure of this paper is organised as follows. The problem description is presented in section II. Section III introduces a filtered transformation and extracts system zero dynamics. The NN approximator design is addressed in section IV. Adaptive backstepping control design procedures are presented in section V. Lyapunov stability analysis is then followed to ensure the convergence of the closed-loop system in section VI. A simulation example is included in section VII to demonstrate a standard control approach. Finally, section VIII concludes this paper.

## II. PROBLEM STATEMENT

Consider SISO nonlinear systems which can be transformed to the output feedback form as

$$\begin{cases} \dot{x} &= Ax + \phi(y) + b\sigma(y)u + dw \\ y &= C^T x \end{cases} \quad (1)$$

with

$$A = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ \vdots \\ b_\rho \\ \vdots \\ b_n \end{bmatrix}, C = \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix},$$

and  $d = [d_1, \dots, d_n]^T$  where state  $x = [x_1, \dots, x_n]^T$ , output  $y \in \mathbb{R}$ , input  $u \in \mathbb{R}$ , smooth nonlinear function  $\phi(y) = [\phi_1(y), \dots, \phi_n(y)]^T$  with  $\phi(0) = 0$ , smooth function  $\sigma(y) : \mathbb{R} \rightarrow \mathbb{R}$  with  $\sigma(y) \neq 0$ ,  $w \in \mathbb{R}$  is a bounded periodic disturbance signal,  $b_i \neq 0$  for  $i = \rho, \dots, n$ , i.e. this system is a relative degree  $\rho$  system.

The assumption below is necessary for this control design.

**Assumption 2.1:** The system is minimum phase, that is, the zeros of polynomial  $\mathcal{B}(s) = \sum_{i=\rho}^n b_i s^{n-i}$  have negative real parts.

The design objective of disturbance rejection problem is described as: Find out a finite dimensional system

$$\begin{cases} \dot{\mu} &= v(\mu, y, u), \mu \in \mathbb{R} \\ u &= \zeta(\mu, y) \end{cases} \quad (2)$$

The closed-loop system is then stable under this controller.

## III. FILTERED STATE TRANSFORMATION

For system (1) with relative degree  $\rho$ , a filter is introduced as

$$\begin{cases} \dot{\xi}_1 &= -\lambda_1 \xi_1 + \xi_2 \\ \vdots & \\ \dot{\xi}_{\rho-1} &= -\lambda_{\rho-1} \xi_{\rho-1} + \sigma(y)u \end{cases} \quad (3)$$

where  $\lambda_i > 0$  for  $i = 1, \dots, \rho - 1$  are the designed parameters. The filtered transformation is implemented by introducing

$$\bar{z} = x - [\bar{f}_1, \dots, \bar{f}_{\rho-1}] \xi \quad (4)$$

where  $\xi \in \mathbb{R}^{\rho-1}$ ,  $\bar{f}_i \in \mathbb{R}^n$  for  $i = 1, \dots, \rho - 1$ . The value of  $\bar{f}_i$  is given recursively by

$$\begin{cases} \bar{f}_1 &= b \\ \bar{f}_i &= [A + \lambda_{i+1}I] \bar{f}_{i+1} \quad \text{for } i = 2, \dots, \rho - 1 \end{cases} \quad (5)$$

with positive designed  $\lambda_\rho$ . Then, the system (1) is transformed to

$$\begin{cases} \dot{\bar{z}} &= A\bar{z} + \phi(y) + dw + f\xi_1 \\ y &= C^T \bar{z} \end{cases} \quad (6)$$

where  $f = [f_1, \dots, f_n]^T = [A + \lambda_1 I] \bar{f}_1$ . It is noted that  $f_1 = b_\rho$ . Further, it is observed

$$\mathcal{D}(s) := \sum_{i=1}^n f_i s^{n-i} = \mathcal{B}(s) \prod_{i=1}^{\rho-1} (s + \lambda_i) \quad (7)$$

With Assumption 2.1, it implies that all zeros of  $\mathcal{D}(s)$  are located on the left half plane.  $\xi_1$  is considered as the input of system (7), i.e. this system has relative degree 1. For the convenience of analysing zero dynamics, another state  $\bar{z} \in \mathbb{R}^{n-1}$  is brought in as

$$z = \bar{z}_{2:n} - \frac{f_{2:n}}{f_1} y \quad (8)$$

$(\star)_{2:n}$  stands for a new vector or matrix which is constructed by the 2nd to  $n$ th row of  $\star$ .

The system dynamics with new coordinates is given by

$$\begin{cases} \dot{z} &= Dz + \psi_z(y) + d_z w \\ \dot{y} &= z_1 + \psi_y(y) + d_1 w + b_\rho \xi_1 \end{cases} \quad (9)$$

with

$$D = \begin{bmatrix} -\frac{f_2}{f_1} & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{f_{n-1}}{f_1} & 0 & \dots & 1 \\ -\frac{f_n}{f_1} & 0 & \dots & 0 \end{bmatrix} \quad (10)$$

where  $D$  is the left companion matrix of  $f$  and

$$\psi_z(y) = D \frac{f_{2:n}}{f_1} y + \phi_{2:n}(y) - \frac{f_{2:n}}{f_1} \phi_1(y) \quad (11)$$

$$\psi_y(y) = \frac{f_2}{f_1} y + \phi_1(y) \quad (12)$$

$$d_w = d_{2:n} - \frac{f_{2:n}}{b_\rho} d_1 \quad (13)$$

$D$  is Hurwitz from equation (7). Thus, there exist positive matrix  $P_z$  and  $Q_z$  which satisfy

$$D^T P_z + P_z D = -Q_z \quad (14)$$

With the property of  $\phi(y)$ , It renders  $\psi_z(0) = 0$  and  $\psi_y(0) = 0$ .

From Isidori's output regulation theory [13], the existence of a controlled invariant manifold is a necessary condition of establishing a solution to nonlinear system output regulation problem. From this point of view, the following assumption is necessary.

**Assumption 3.1:** There exist an invariant manifold  $\pi(w) : \mathbb{R} \rightarrow \mathbb{R}^{n-1}$  and a forwarding control input  $\mu(w) : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$\begin{cases} \dot{\pi} &= B\pi + d_z w \\ 0 &= \pi_1 + d_1 w + b_\rho \mu \end{cases} \quad (15)$$

Let  $\tilde{z}$  denotes the error between  $z$  and  $\pi$ . The final model for control design is obtained as

$$\begin{cases} \dot{\tilde{z}} &= D\tilde{z} + \psi_z(y) \\ \dot{y} &= \tilde{z}_1 + \psi_y(y) + b_\rho (\xi_1 - \mu) \\ \dot{\xi}_1 &= -\lambda_1 \xi_1 + \xi_2 \\ \vdots & \\ \dot{\xi}_{\rho-1} &= -\lambda_{\rho-1} \xi_{\rho-1} + \sigma(y)u \end{cases} \quad (16)$$

#### IV. NEURAL NETWORKS DISTURBANCE APPROXIMATOR

In the section, a Neural Networks approximator based on the output  $y$  is introduced to approximate the desired input  $\mu(\omega)$  on a compact set  $\Omega$  [9].

$$\mu = W^T S(y) + \delta \quad (17)$$

where  $S(y) = [s_1(y), \dots, s_l(y)]^T : \Omega \rightarrow \mathbb{R}^l$  is a known smooth vector function with neural network node number  $l > 1$ . The basis function is given by Gaussian function as

$$s_i(y) = \exp\left[-\frac{(y-a_i)^2}{h^2}\right], \text{ for } i = 1, \dots, l \quad (18)$$

with the centre  $a_i \in \Omega$  and the width  $h > 0$ . The desired weighting vector  $W = [W_1, \dots, W_l]^T$  is defined as

$$W := \arg \min_{\hat{W} \in \mathbb{R}^l} \left\{ \sup_{y \in \Omega} |\mu - \hat{W}^T S(y)| \right\}$$

$\hat{W}$  is the estimate of  $W$ ,  $\delta$  is the NN inherent approximation error satisfying  $|\delta| \leq \bar{\delta}$ , which is the minimum upper bound of  $\delta$ . It can be decreased by increasing the number of  $r$  and  $l$ . The approximation of  $\mu$  can be written as

$$\hat{\mu} = \hat{W}^T S(y). \quad (19)$$

#### V. ADAPTIVE BACKSTEPPING CONTROL DESIGN

In this control design, some designed parameters are introduced. They satisfy

$$\begin{aligned} c_0 &> \frac{1}{2} + \frac{b_\rho^2}{4} \\ c_1 &> -1 \\ c_i &> 0 \quad \text{for } i = 2, \dots, \rho - 1 \\ h_i &> 0 \end{aligned}$$

Define the desired value of  $\xi_i$  as  $\hat{\xi}_i$  for  $i = 1, \dots, \rho - 2$  with error  $\tilde{\xi}_i = \xi_i - \hat{\xi}_i$ . Therefore, from system (9), we have

$$\dot{y} = \tilde{z}_1 + \psi_y(y) + b_\rho (\hat{\xi}_1 + \tilde{\xi}_1 - \mu) \quad (20)$$

The virtual control is designed as

$$\hat{\xi}_1 = \frac{1}{b_\rho} (-c_0 y - \psi_y(y)) + \hat{W}^T S(y) \quad (21)$$

Substituting equation (21) into (20), it gives

$$\dot{y} = -c_0 y + \tilde{z}_1 + b_\rho \tilde{\xi}_1 - b_\rho \tilde{W}^T S(y) - b_\rho \delta \quad (22)$$

with the notation  $\tilde{W} = W - \hat{W}$ .

The backstepping technique is employed to search other virtual controls step by step.

**Step 1:** When  $i = 1$ ,

$$\begin{aligned} \dot{\tilde{\xi}}_1 &= \dot{\xi}_1 - \dot{\hat{\xi}}_1 \\ &= -\lambda_1 \xi_1 + \xi_2 - \frac{\partial \hat{\xi}_1}{\partial \hat{W}} \dot{\hat{W}} \\ &\quad - \frac{\partial \hat{\xi}_1}{\partial y} (\tilde{z}_1 + \psi_y(y) + b_\rho (\xi_1 - \mu)) \\ &= -\lambda_1 \xi_1 + \tilde{\xi}_2 + \hat{\xi}_2 - \frac{\partial \hat{\xi}_1}{\partial \hat{W}} \dot{\hat{W}} \\ &\quad - \frac{\partial \hat{\xi}_1}{\partial y} (\tilde{z}_1 + \psi_y(y) + b_\rho (\xi_1 - \mu)) \end{aligned} \quad (23)$$

Design  $\hat{\xi}_2$  as

$$\begin{aligned} \hat{\xi}_2 &= \lambda_1 \xi_1 - c_1 \tilde{\xi}_1 - h_1 \left( \frac{\partial \hat{\xi}_1}{\partial y} \right)^2 \tilde{\xi}_1 + \frac{\partial \hat{\xi}_1}{\partial \hat{W}} \dot{\hat{W}} \\ &\quad + \frac{\partial \hat{\xi}_1}{\partial y} (\psi_y(y) + b_\rho (\xi_1 - \hat{W}^T S(y))) \end{aligned} \quad (24)$$

**Step i:** Introduce  $\xi_\rho$  for the convenience of notation. Similarly, virtual controls are obtained as:

$$\begin{aligned} \dot{\xi}_i &= -\xi_{i-2} + \lambda_{i-1}\xi_{i-1} - c_{i-1}\xi_{i-1} \\ &\quad + \frac{\partial \hat{\xi}_{i-1}}{\partial y} (\psi_y(y) + b_\rho (\xi_1 - \hat{W}^T S(y))) \\ &\quad + \sum_{i=1}^{i-2} \frac{\partial \hat{\xi}_{i-1}}{\partial \xi_i} \xi_i - h_{i-1} \left( \frac{\partial \hat{\xi}_{i-1}}{\partial y} \right)^2 \xi_{i-1} \\ &\quad + \frac{\partial \hat{\xi}_{i-1}}{\partial \hat{W}} \hat{W} \quad \text{for } i = 2, \dots, \rho \end{aligned} \quad (25)$$

Finally, the control input  $u$  is designed as

$$u = \frac{\hat{\xi}_\rho - \hat{\Theta} r(\hat{\xi}_{\rho-1}, y)}{\sigma(y)} \quad (26)$$

where  $\hat{\Theta}$  is the estimate of  $\Theta \in \mathbb{R}$  that is an unknown constant, Smooth nonlinear function  $r(\hat{\xi}_{\rho-1}, y)$  is given by

$$r(\hat{\xi}_{\rho-1}, y) = \frac{1}{\hat{\xi}_{\rho-1}} (\|\psi_z(y)\|^2) \quad (27)$$

where  $\Theta$  and  $r(\hat{\xi}_{\rho-1}, y)$  are introduced for the sake of compensating the closed-loop influence brought by the nonlinear function  $\psi_z(y)$ . The desired value of  $\Theta$  is given by

$$\Theta = \beta \|P_z\|^2 \quad (28)$$

where  $\beta$  is a positive constant satisfying inequality below

$$\beta(1 - \lambda_{\min}) + \frac{1}{2} + \sum_{i=1}^{\rho-1} \frac{1}{4h_i} < 0 \quad (29)$$

where  $\lambda_{\min}$  denotes the minimum eigenvalue of positive definite matrix  $P_z$ .

Estimates of unknown parameters  $\Theta$  and  $W$  are generated by following adaptive laws.

$$\begin{cases} \dot{\hat{\Theta}} &= \gamma_\theta (\hat{\xi}_{\rho-1} r(\hat{\xi}_{\rho-1}, y) - \sigma_\theta \hat{\Theta}) \\ \dot{\hat{W}} &= -\Gamma_w \left( b_\rho y S(y) - \sum_{i=1}^{\rho-1} b_\rho \xi_i \frac{\partial \hat{\xi}_i}{\partial y} S(y) + \sigma_w \hat{W} \right) \end{cases} \quad (30)$$

where  $\gamma_\theta \in \mathbb{R}$  and  $\Gamma_w \in \mathbb{R}^l$  are positive definite designed adaptive gain matrix,  $\sigma_\theta \in \mathbb{R}$  and  $\sigma_w \in \mathbb{R}$  are  $\sigma$  modification gains which are selected to be small positive constants.

The dynamics of  $\tilde{\xi}$  is given by

$$\left\{ \begin{aligned} \dot{\tilde{\xi}}_1 &= \tilde{\xi}_2 - c_1 \tilde{\xi}_1 - h_1 \left( \frac{\partial \hat{\xi}_1}{\partial y} \right)^2 \tilde{\xi}_1 - \frac{\partial \hat{\xi}_1}{\partial y} \tilde{z}_1 \\ &\quad + b_\rho \frac{\partial \hat{\xi}_1}{\partial y} \tilde{W}^T S(y) + b_\rho \frac{\partial \hat{\xi}_1}{\partial y} \delta \\ \dot{\tilde{\xi}}_i &= -\tilde{\xi}_{i-1} + \tilde{\xi}_{i+1} - c_i \tilde{\xi}_i - h_i \left( \frac{\partial \hat{\xi}_i}{\partial y} \right)^2 \tilde{\xi}_i \\ &\quad - \frac{\partial \hat{\xi}_i}{\partial y} \tilde{z}_1 + b_\rho \frac{\partial \hat{\xi}_i}{\partial y} \tilde{W}^T S(y) + b_\rho \frac{\partial \hat{\xi}_i}{\partial y} \delta \\ &\quad \text{for } i = 2, \dots, \rho - 2 \\ \dot{\tilde{\xi}}_{\rho-1} &= -\tilde{\xi}_{\rho-2} - c_{\rho-1} \tilde{\xi}_{\rho-1} - \frac{\partial \hat{\xi}_{\rho-1}}{\partial y} \tilde{z}_1 \\ &\quad - h_{\rho-1} \left( \frac{\partial \hat{\xi}_{\rho-1}}{\partial y} \right)^2 \tilde{\xi}_{\rho-1} + b_\rho \frac{\partial \hat{\xi}_{\rho-1}}{\partial y} \delta \\ &\quad + b_\rho \frac{\partial \hat{\xi}_{\rho-1}}{\partial y} \tilde{W}^T S(y) - \hat{\Theta} r(\hat{\xi}_{\rho-1}, y) \end{aligned} \right. \quad (31)$$

## VI. STABILITY ANALYSIS

The stability analysis is based on Lyapunov stabilising theory. The Lyapunov function candidate of this system is chosen as

$$V = V_y + \beta V_z + \sum_{i=1}^{\rho-1} V_{\xi_i} + V_w + V_\theta \quad (32)$$

with

$$\left\{ \begin{aligned} V_y &= \frac{1}{2} y^2 \\ V_z &= \tilde{z}^T P_z \tilde{z} \\ \sum_{i=1}^{\rho-1} V_{\xi_i} &= \frac{1}{2} \sum_{i=1}^{\rho-1} \tilde{\xi}_i^2 \\ V_w &= \frac{1}{2} \tilde{W}^T \Gamma_w^{-1} \tilde{W} \\ V_\theta &= \frac{1}{2} \gamma_\theta^{-1} \tilde{\Theta}^2 \end{aligned} \right. \quad (33)$$

with the notation  $\tilde{\Theta} = \Theta - \hat{\Theta}$ .

From equation (22), it shows

$$\begin{aligned} \dot{V}_y &= y\dot{y} \\ &= -c_0 y^2 + y\tilde{z}_1 + b_\rho y \tilde{\xi}_1 - b_\rho y \tilde{W}^T S(y) - b_\rho y \delta \\ &\leq -c_0 y^2 + \frac{1}{2} y^2 + \frac{1}{2} \|\tilde{z}\|^2 + \frac{b_\rho^2}{4} y^2 + \tilde{\xi}_1^2 \\ &\quad - b_\rho y \tilde{W}^T S(y) - b_\rho y \delta \\ &\leq \left( -c_0 + \frac{1}{2} + \frac{b_\rho^2}{4} \right) y^2 + \frac{1}{2} \|\tilde{z}\|^2 + \tilde{\xi}_1^2 \\ &\quad - b_\rho y \tilde{W}^T S(y) - b_\rho y \delta \end{aligned} \quad (34)$$

then from equation (9), it is obtained

$$\begin{aligned}\dot{V}_z &= \dot{\tilde{z}}^T P_z \tilde{z} + \tilde{z}^T P_z \dot{\tilde{z}} \\ &= \tilde{z}^T (D^T P_z + P_z D) \tilde{z} + 2\tilde{z}^T P_z \psi_z(y) \\ &\leq (1 - \lambda_{\min}) \|\tilde{z}\|^2 + \|P_z\|^2 \|\psi_z(y)\|^2\end{aligned}\quad (35)$$

With dynamics of  $\tilde{\xi}_i$  in equation (31), it is derived that

$$\begin{aligned}\sum_{i=1}^{\rho-1} \dot{V}_{\xi_i} &= -\sum_{i=1}^{\rho-1} \left( c_i \tilde{\xi}_i^2 + h_i \left( \frac{\partial \hat{\xi}_i}{\partial y} \right)^2 \tilde{\xi}_i + \frac{\partial \hat{\xi}_i}{\partial y} \tilde{z}_i \right) \\ &\quad + \sum_{i=1}^{\rho-1} b_\rho \tilde{\xi}_i \frac{\partial \hat{\xi}_i}{\partial y} \tilde{W}^T S(y) + \sum_{i=1}^{\rho-1} b_\rho \tilde{\xi}_i \frac{\partial \hat{\xi}_i}{\partial y} \delta \\ &\quad - \tilde{\xi}_{\rho-1} \hat{\Theta} r(\tilde{\xi}_{\rho-1}, y) \\ &\leq -\sum_{i=1}^{\rho-1} \left( c_i \tilde{\xi}_i^2 - \frac{1}{4h_i} \|\tilde{z}\|^2 \right) + \sum_{i=1}^{\rho-1} b_\rho \tilde{\xi}_i \frac{\partial \hat{\xi}_i}{\partial y} \delta \\ &\quad + \sum_{i=1}^{\rho-1} b_\rho \tilde{\xi}_i \frac{\partial \hat{\xi}_i}{\partial y} \tilde{W}^T S(y) - \tilde{\xi}_{\rho-1} \hat{\Theta} r(\tilde{\xi}_{\rho-1}, y)\end{aligned}\quad (36)$$

Further, from equation (30), we have

$$\begin{aligned}\dot{V}_w &= -\tilde{W}^T \Gamma_w^{-1} \dot{\tilde{W}} \\ &= b_\rho y \tilde{W}^T S(y) - \sum_{i=1}^{\rho-1} b_\rho \tilde{\xi}_i \frac{\partial \hat{\xi}_i}{\partial y} \tilde{W}^T S(y) + \sigma_w \tilde{W}^T \hat{W} \\ \dot{V}_\theta &= -\gamma_\theta^{-1} \tilde{\Theta} \dot{\hat{\Theta}} \\ &= -\tilde{\xi}_{\rho-1} \tilde{\Theta} r(\tilde{\xi}_{\rho-1}, y) + \sigma_\theta \tilde{\Theta} \hat{\Theta}\end{aligned}\quad (38)$$

Therefore

$$\begin{aligned}\dot{V} &\leq \left( -c_0 + \frac{1}{2} + \frac{b_\rho^2}{4} \right) y^2 - (c_1 + 1) \tilde{\xi}_1^2 - \sum_{i=2}^{\rho-1} c_i \tilde{\xi}_i^2 \\ &\quad + \left( \beta(1 - \lambda_{\min}) + \frac{1}{2} + \sum_{i=1}^{\rho-1} \frac{1}{4h_i} \right) \|\tilde{z}\|^2 \\ &\quad + \beta \|P_z\|^2 \|\psi_z(y)\|^2 - \tilde{\xi}_{\rho-1} \Theta r(\tilde{\xi}_{\rho-1}, y) \\ &\quad + \sigma_w \tilde{W}^T \hat{W} + \sigma_\theta \tilde{\Theta} \hat{\Theta} - b_\rho y \delta + \sum_{i=1}^{\rho-1} b_\rho \tilde{\xi}_i \frac{\partial \hat{\xi}_i}{\partial y} \delta \\ &\leq 2 \left( -c_0 + \frac{1}{2} + \frac{b_\rho^2}{4} \right) V_y - 2(c_1 + 1) V_{\xi_1} \\ &\quad + \left( \beta(1 - \lambda_{\min}) + \frac{1}{2} + \sum_{i=1}^{\rho-1} \frac{1}{4h_i} \right) \frac{V_z}{\lambda_p} \\ &\quad - \sum_{i=2}^{\rho-1} 2c_i V_{\xi_i} - \sigma_w \Gamma_w V_w - \sigma_\theta \gamma_\theta V_\theta + \frac{\sigma_w}{2} \|W\|^2 \\ &\quad + \frac{\sigma_\theta}{2} \|\Theta\|^2 - b_\rho y \delta + \sum_{i=1}^{\rho-1} b_\rho \tilde{\xi}_i \frac{\partial \hat{\xi}_i}{\partial y} \delta\end{aligned}\quad (39)$$

Note that the derivative of  $V$  is in a form of  $\dot{V} = -\zeta V + \eta$  where  $\zeta > 0$ , and  $\eta$  is bounded. The system converges to a small region around its equilibrium points when time  $t$  tends to infinity.

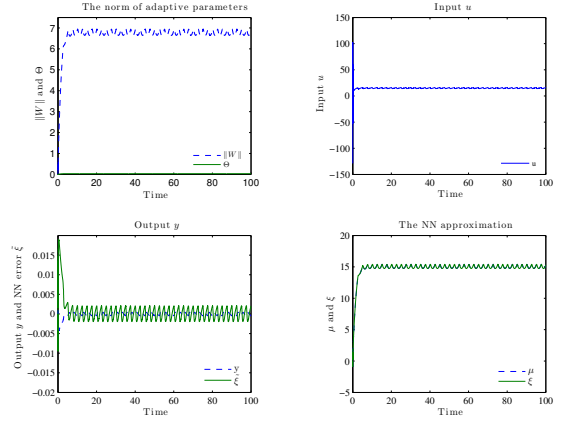


Fig. 1: The simulation result

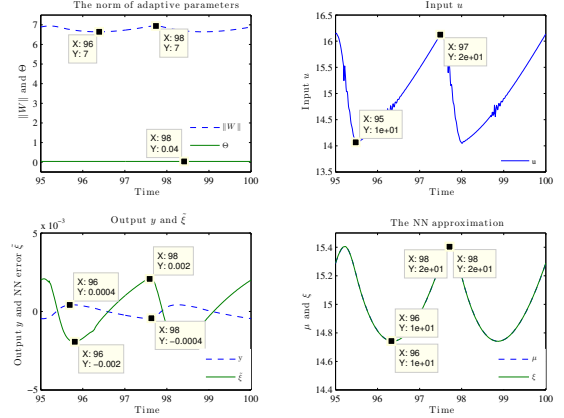


Fig. 2: An enlargement of Figure 1

## VII. SIMULATION STUDY

Consider a three order disturbed nonlinear system as

$$\begin{cases} \dot{x}_1 = x_2 + y^2 \\ \dot{x}_2 = x_3 + y^3 + 2w + (|y| + 0.1)u \\ \dot{x}_3 = 3w + (|y| + 0.1)u \end{cases}$$

The filter is selected as

$$\dot{\xi} = -\xi + (|y| + 0.1)u$$

with the virtual control given by

$$\hat{\xi} = -3y - 2y - y^2 + W^T S(y)$$

It is easy to find

$$D = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}$$

Adaptive laws are designed as

$$\begin{cases} \dot{\hat{W}} = -\Gamma_w(y)S(y) - \xi \frac{\partial \hat{\xi}}{\partial y} S(y) + \sigma_w W \\ \dot{\hat{\Theta}} = \gamma_\theta ((-3y + y^3 - 2y^2)^2 + (-2y - y^2)^2 - \sigma_\theta \Theta) \end{cases}$$

with adaptive gains  $\Gamma_w = 100I$ ,  $\gamma_\theta = 100$ , and  $\sigma$ -modification gains  $\sigma_w = 0.00001$ ,  $\sigma_\theta = 0.00003$ . Following our backstepping design procedure, the control input  $u$  is designed as

$$u = \frac{1}{|y|+0.1} \left( \xi - \hat{\xi} - \left( \frac{\partial \hat{\xi}_1}{\partial y} \right)^2 \xi + \frac{\partial \hat{\xi}}{\partial \hat{W}} \hat{W} \right) + \frac{1}{|y|+0.1} \frac{\partial \hat{\xi}}{\partial y} (2y+y^2+\xi - \hat{W}^T S(y)) - \frac{1}{|y|+0.1} \frac{\hat{\Theta}}{\hat{\xi}} \left( (-3y+y^3-2y^2)^2 + (-2y-y^2)^2 \right)$$

where

$$\begin{aligned} \frac{\partial \hat{\xi}_1}{\partial y} &= -\frac{1}{5+2y} + W^T S'(y) \\ \frac{\partial \hat{\xi}}{\partial \hat{W}} &= S^T(y) \end{aligned}$$

The simulation result is illustrated in Fig. 1 and Fig. 2. Fig. 2 shows the steady state of systems. It can be observed that all states are going to be stable. The first subfigure illustrates the norm of adaptive parameters, the second subfigure presents the system control input which has a little transient attenuation that is caused by the small NN approximation error, the third subfigure shows the convergence of output ( $|y| < 0.0001$ ) and NN approximation error ( $|\hat{\xi}| < 0.002$ ), to the end, it is also observed that the NN approximator works well from last subfigure.

## VIII. CONCLUSIONS

In this paper, disturbance rejection is achieved for a periodically disturbed nonlinear system in output feedback form with time varying input weighting vector. NN is applied to approximate the feedforward control input  $\mu$ . Compared with the results in [7], it is noticed the control design method is in the same way with different disturbance approximation methods. The proposed NN method estimates the feedforward control input without any pre-known information. While in [7], the wave profile of the disturbance signal is assumed to be half-period alternative. The disturbance period is known for the regeneration of the real amplitude and phase shift. Nevertheless, the method in [7] has a quick and nice time response. Generally, it is not easy to have the period of the disturbance in the real practice. The NN approximation method is a good choice when the information of disturbance signals are not enough.

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