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Robust H_∞ Control for a Class of Uncertain Nonlinear Switched Systems

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***Abstract*—This paper focuses on the robust H_∞ control problem for a class of nonlinear switched systems containing neutral uncertainties with average dwell time (ADT). Uncertainties are assumed to be nonlinearly dependent on state and state derivative and allowed to appear in channels of state, control input and disturbance input. The robust H_∞ control problem of the switched system with stabilizable and unstabilizable subsystems is solvable if the stabilizable and unstabilizable subsystems satisfy certain conditions and admissible switching strategy among them. ADT and piecewise Lyapunov function approaches are applied to achieve the control design. A numerical example is provided to illustrate the effectiveness of the proposed results.**

I. Introduction

The last decades have witnessed a rapidly growing interest from the control field in the study of switched systems [1-7]. More specifically, switched systems belonging to a class of hybrid dynamical systems contain a finite number of subsystems and a switching signal that must be designed in order to orchestrate the switching among the subsystems. Recently, there is increasing growth of interest in applying ADT switching to handle the switched systems [8-10]. As a class of typical controlled switching signals, ADT switching means that the number of switches in a finite interval is bounded and the average time between the consecutive switching is not less than a specified value. It is widely recognized that ADT switching is of practical and theoretical significance to deal with the related stability analyses and control syntheses problems.

As is well known, uncertainties are unavoidable in engineering control and are frequently the source of instability and performance deterioration. Thus during the past decades, the problems of stability analysis and controller synthesis with uncertainties have received much attention [11-13]. [14] studied the robust stabilization problem for a class of uncertain nonlinear cascaded systems, in which the uncertain parameters are from a known compact set. In [15], the problem of robust l_2-l_∞ filtering for switched linear discrete-time systems with polytopic uncertainties and time-varying delays is investigated. Furthermore, neutral uncertainties describing many practical parameter perturbations are often nonlinearly state and nonlinearly state derivative dependent. [16] discussed the robust L_2-gain performance synthesis problem for a class of nonlinear systems with neutral uncertainties. However, few results have focused on switched systems with neutral uncertainties so far.

On the other hand, H_∞ control theory for switched systems has attracted considerable attention by researchers and has been a hot topic in the control area [17-21]. Especially, results about nonlinear H_∞ control of switched systems have progressively appeared to solve robust stabilization and disturbance attenuation issues [22-25]. The nonlinear H_∞ control problem for switched systems can be stated as follows: Find a compensator, either state feedback or more general output feedback and a switching rule (if necessary) such that (1) the internal state of the closed-loop system is stable and (2) the L_2 gain of the mapping from the exogenous input disturbance to the controlled output is minimized or guaranteed to be less than or equal to a prescribed value. In [26], the H_∞ control problem of switched systems has been addressed with ADT in both linear and nonlinear contexts. [27] investigated the H_∞ control problem for a class of switched nonlinear cascade systems using the multiple Lyapunov function method.

In this paper, we discuss the problem of robust H_∞ control for a class of nonlinear switched systems with neutral uncertainties. For the case where states are measurable, sufficient conditions for the switched system to be asymptotically stable with H_∞-norm bound and design of both switching law and state feedback controller are proposed for all admissible uncertainties. ADT switching is used so that the results cover the case where stabilizable and unstabilizable subsystems both exist in the switched system. An numerical example is given to illustrate the applicability of the developed method. As compared to the existing results, this paper deals with neutral uncertainties. Additionally, uncertainties are also allowed to appear in channels of state, control input and disturbance input.

Notation: we use standard notations throughout this paper. R^n denotes the n-dimensional real Euclidean space, and given a matrix $P, P > 0$ denotes that P is positive definite, P^T stands for the transpose of P, I is the identity matrix, $\|\cdot\|$

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represents either the Euclidean vector norm or the induced matrix 2-norm, and $\overline{\sigma}(\cdot)$ denotes the largest singular value of a matrix.

II. Problem Statement And Preliminaries

In this paper, we consider a class of nonlinear switched systems described by equations of the form:

$$\begin{aligned}\dot{x}+\Delta j_{\sigma(t)}(\dot{x},t) =& f_{\sigma(t)}(x)+\Delta f_{\sigma(t)}(x,t)+(c_{\sigma(t)}(x)\\ &+\Delta c_{\sigma(t)}(x,t))\omega_{\sigma(t)},\\ y =& h_{\sigma(t)}(x),\end{aligned} \tag{1}$$

where $\sigma(t):[0,+\infty)\to I_m=\{1,\cdots,m\}$ is the switching signal, which is assumed to be a piecewise constant function depending on time, $x\in R^n$ is the state, $\omega_i\in R^{c_i}$ is the disturbance input belongs to $L_2[0,\infty)$, $u_i\in R^{m_i}$ and $y\in R^{p_i}$ stand for the control input and the measurement output of the ith subsystem respectively. $f_i(x),c_i(x)$ and $h_i(x)$ are known smooth nonlinear function matrices of appropriate dimensions satisfying $f_i(0)=0$ and $h_i(0)=0$, $\Delta j_i(\dot{x},t),\Delta f_i(x,t)$ and $\Delta c_i(x,t)$ represent unknown smooth nonlinear function matrices, $i\in I_m$.

The switching sequence $\sigma(t)$ associated with the switched system (1) is given by

$$\begin{aligned}\sum=\{x_0;(i_0,t_0),(i_1,t_1),\cdots,(i_k,t_k),\cdots,\\ |i_k\in I_m,k\in N\},\end{aligned} \tag{2}$$

in which t_0 is the initial time, x_0 is the initial state. When $t\in[t_k,t_{k+1}),\sigma(t)=i_k$, the i_kth subsystem is active, and the trajectory $x(t)$ of the switched system (1) is the trajectory x_{i_k} of the i_kth subsystem. As commonly assumed in the literature, we exclude Zeno behavior for all types of switching signal in this paper. In addition, we assume that the state of the switched system (1) does not jump at the switching instants, i.e., the trajectory $x(t)$ is everywhere continuous.

In this paper, we assume all uncertainties in the switched system (1) having the following properties.

Assumption 1. The uncertain functions $\Delta j_i(\dot{x},t),\Delta f_i\ (x,t)$ and $\Delta c_i(x,t)$ are gain bounded smooth functions described as follows:

$$\begin{aligned}\Delta j_i(\dot{x},t)&=e_{j_i}\delta_{j_i}(\dot{x},t),\|\delta_{j_i}\|\le\|W_{j_i}\dot{x}\|,\\ \Delta f_i(x,t)&=e_{f_i}\delta_{f_i}(x,t),\|\delta_{f_i}\|\le\|W_{f_i}(x)\|,\\ \Delta c_i(x,t)&=e_{c_i}\delta_{c_i}(x,t),\|\delta_{c_i}\|\le\|W_{c_i}(x)\|,\end{aligned} \tag{3}$$

where e_{j_i},e_{f_i},e_{c_i} are known constant matrices and $\delta_{j_i},\delta_{f_i},\delta_{c_i}$ are unknown function vectors with $\delta_{j_i}(0,t)=0$ and $\delta_{f_i}(0,t)$ $=0$. W_{j_i},W_{f_i} are known smooth function matrices, W_{c_i} are given weighting matrices, $i\in I_m$.

Now, the robust H_∞ control problem to be addressed in this paper can be represented as: given a constant $\gamma>0$, design a switching law $i=\sigma(t)$ for the switched system (1) such that

(i) The autonomous system (1) is globally asymptotically stable when $\omega_i\equiv 0$.

(ii) System (1) has weighted L_2-gain from ω_i to y for all admissible uncertainties, ie., there holds

$$\int_0^\infty e^{-\lambda\tau}y^T(\tau)y(\tau)d\tau\le\gamma^2\int_0^\infty\omega_i^T(\tau)\omega_i(\tau)d\tau+\beta(x_0)$$

for some real-valued function $\beta(\cdot)$ with $\beta(0)=0$.

Assumption 2. For robust H_∞ control problem, suppose that not all the subsystems of the switched system (1) are stabilizable.

Definition 1. For any $T_2>T_1\ge 0$, let $N_\sigma(T_1,T_2)$ denote the number of switching of $\sigma(t)$ over (T_1,T_2). If $N_\sigma(T_1,T_2)\le N_0+\frac{T_2-T_1}{\tau_a}$ holds for $\tau_a>0,N_0\ge 0$, then τ_a is called average dwell time.

Definition 2. For the switched system (1), suppose that $V_i(t)$ is the corresponding Lyapunov function for the ith subsystem, then $V(t)$ is called a piecewise Lyapunov function candidate if it can be written as $V(t)=V_{\sigma(t)}(x)$, where $V_{\sigma(t)}(x)$ is switched among $V_i(t)$ in accordance with the piecewise constant switching signal $\sigma(t)$.

III. Main Results

For the switched system (1) with stabilizable and unstabilizable subsystems, the robust H_∞ control problem is solvable if the stabilizable and unstabilizable subsystems satisfy certain conditions and admissible switching law among them, respectively. In what folllows, we give the design method for the robust H_∞ control problem of the switched system (1).

Consider the switched system (1). Under Assumption 2, for the robust H_∞ control problem, not all the subsystems are stabilizable, without loss of generality, we assume that the ith subsystem $(1\le i\le s)$ is stabilizable (where the positive integer s satisfies $1\le s<m$), whereas the other subsystems of (1) are unstabilizable.

Then, for any piecewise constant switching signal $\sigma(t)$ and any $0\le t_0<t$, we let $\Pi^-(t_0,t)$ (resp.,$\Pi^+(t_0,t)$) denote the total activation time of stabilizable (resp., unstabilizable) subsystems during (t_0,t). Then, we present the following switching law:

(F): Let $t_0<t_1<t_2<\cdots<t_i$ ($\lim_{i\to\infty}t_i=\infty$) be a specified sequence of time instants satisfying $\max_i(t_{i+1}-t_i)=T<\infty$. Determined the switching signal $\sigma(t)$ so that the inquality

$$\frac{\Pi^-(t_i,t_{i+1})}{\Pi^+(t_i,t_{i+1})}\ge\frac{\beta+\lambda^*}{\alpha-\lambda^*} \tag{4}$$

holds on time every interval $[t_i,t_{i+1})(i=0,1,\cdots)$ with $\alpha>0,\beta>0$ and $\lambda^*\in(0,\alpha)$. Meanwhile, we choose $\lambda^*\le\alpha$ as the average dwell time scheme: for any $t>t_0$,

$$N_\sigma(t_0,t)\le N_0+\frac{t-t_0}{\tau},\tau>\tau^*=\frac{\ln u}{\lambda^*}. \tag{5}$$

Under the switching law (F) for any t_0, t satisfying $t_{i-1}<$

$t_0 < t_i < t_{i+1} < \cdots < t_k < t$, we can infer

$$\begin{aligned}
&\beta\Pi^+(t_0,t) - \alpha\Pi^-(t_0,t) \\
\leq&\beta(t_i - t_0) + \sum_{l=i}^{k-1} [\beta\Pi^+(t_l,t_{l+1}) - \alpha\Pi^-(t_l,t_{l+1})] \\
&+ \beta(t - t_k) \\
\leq&\beta(t_i - t_0) - \lambda^*(t_k - t_i) + \beta(t - t_k) \\
\leq&(\beta + \lambda^*)(t_i - t_0) - \lambda^*(t - t_0) + (\beta + \lambda^*)(t - t_k). \quad (6)
\end{aligned}$$

Since on any interval $[t_i, t_{i+1})$, the total activation time period of unstable subsystems satisfies $\Pi^+(t_i, t_{i+1}) \leq \frac{\alpha-\lambda^*}{\alpha+\beta}(t_{i+1} - t_i)$ according to the requirement in (F), we get from (6) that

$$\beta\Pi^+(t_0,t) - \alpha\Pi^-(t_0,t) \leq c - \lambda^*(t - t_0), \quad (7)$$

Where $c = \frac{2(\beta+\lambda^*)(\alpha-\lambda^*)}{(\alpha+\beta)}T$.

The following theorem provides theoretical basis for the robust H_∞ control problem of the switched system (1).

Theorem 1. Given any constant $\gamma > 0$, suppose that there exist radially unbounded positive definite differentiable functions $V_i(x)$, $i = 1, \cdots, m$, constants $\mu \geq 1$, such that the following inequalities

$$\begin{aligned}
&\frac{\partial V_i}{\partial x} f_i + \gamma_i^2 C_i^T C_i + \gamma_i^2 (\frac{1}{2\gamma_i^2} \frac{\partial V_i}{\partial x} B_i + C_i^T D_i) R_i^{-1} (\frac{1}{2\gamma_i^2} \frac{\partial V_i}{\partial x} \\
&\cdot B_i + C_i^T D_i)^T + \alpha V_i < 0, i \leq s, \quad (8)
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial V_i}{\partial x} f_i + \gamma_i^2 C_i^T C_i + \gamma_i^2 (\frac{1}{2\gamma_i^2} \frac{\partial V_i}{\partial x} B_i + C_i^T D_i) R_i^{-1} (\frac{1}{2\gamma_i^2} \frac{\partial V_i}{\partial x} \\
&\cdot B_i + C_i^T D_i)^T - \beta V_i < 0, i > s, \quad (9)
\end{aligned}$$

$$V_i \leq \mu V_j, \quad (10)$$

and

$$\alpha_1^*(\|x\|) \leq V_i(x) \leq \alpha_2^*(\|x\|), \quad i, j = 1, \cdots, m \quad (11)$$

hold, where $\alpha_1^*(x)$ and $\alpha_2^*(x)$ are two class K_∞ functions and

$$\begin{aligned}
&\gamma_i^2 = \frac{\gamma^2}{1 + \bar{\sigma}(W_{c_i})/\lambda_{c_i}^2}, B_i = [c_i, \lambda_{j_i} e_{j_i}, \lambda_{f_i} e_{f_i}, \lambda_{c_i} e_{c_i}], \\
&C_i^T = [(1/\gamma_i) h_i^T, (1/\lambda_{j_i}) f_i^T W_{j_i}^T, (1/\lambda_{f_i}) W_{f_i}^T, 0], \\
&D_i^T = [0, (1/\lambda_{j_i}) B_i^T W_{j_i}^T, 0, 0], R_i = I - D_i^T D_i \quad (12)
\end{aligned}$$

with $\lambda_{j_i}, \lambda_{f_i}$, and $\lambda_{c_i}, i \in I_m$ are positive constants.

Then, the robust H_∞ control problem of the switched system (1) is solvable under the switching condition (F) and the average dwell-time (5).

Proof: From Definition 2, we choose the following piecewise Lyapunov function candidate:

$$V(t) = V_{\sigma(t)}(x) \quad (13)$$

for the switched system (1), where $V_{\sigma(t)}(x)$ is switched among the solution $V_i(x)$'s of (8)-(11) in accordance with the piecewise constant switching signal σ.

Regard neutral uncertainty $\Delta j_i(\dot{x}, t)$ as an exogenous disturbance and make a new extended disturbance input including it. In this case, define

$$d_i^T = [\omega_i^T, -(1/\lambda_{j_i}) \delta_{j_i}^T, (1/\lambda_{f_i}) \delta_{f_i}^T, (1/\lambda_{c_i}) \omega_i^T \delta_{c_i}^T]. \quad (14)$$

Then, we can conclude that

$$\begin{aligned}
d_i^T d_i \leq& \|\omega_i\|^2 + (1/\lambda_{j_i}^2) \delta_{j_i}^T \delta_{j_i} + (\bar{\sigma}(W_{c_i})/\lambda_{c_i}^2) \|\omega_i\|^2 \\
&+ (1/\lambda_{f_i}^2) \delta_{f_i}^T \delta_{f_i} \\
\leq&(1 + \bar{\sigma}(W_{c_i})/\lambda_{c_i}^2) \|\omega_i\|^2 + (1/\lambda_{j_i}^2) \delta_{j_i}^T \delta_{j_i} \\
&+ (1/\lambda_{f_i}^2) \delta_{f_i}^T \delta_{f_i}, \quad (15)
\end{aligned}$$

which means

$$-\gamma^2 \|\omega_i\|^2 \leq \gamma_i^2 d_i^T d_i + (\gamma_i^2/\lambda_{j_i}^2) \delta_{j_i}^T \delta_{j_i} + (\gamma_i^2/\lambda_{f_i}^2) \delta_{f_i}^T \delta_{f_i}.$$

Owing to Assumption 1, it holds that

$$\begin{aligned}
&\dot{V} + \|y\|^2 - \gamma^2 \|\omega_i\|^2 \\
=&\frac{\partial V_i}{\partial x} (f_i + \Delta f_i + c_i \omega_i + \Delta c_i \omega_i - \Delta j_i) + \|y\|^2 - \gamma^2 \|\omega_i\|^2 \\
=&\frac{\partial V_i}{\partial x} (f_i + e_{f_i} \delta_{f_i} + c_i \omega_i + e_{c_i} \delta_{c_i} \omega_i - e_{j_i} \delta_{j_i}) + \|y\|^2 \\
&- \gamma^2 \|\omega_i\|^2 \\
=&\frac{\partial V_i}{\partial x} (f_i + B_i d_i) + h_i^T h_i - \gamma_i^2 d_i^T d_i + \frac{\gamma_i^2}{\lambda_{j_i}^2} \delta_{j_i}^T \delta_{j_i} \\
&+ \frac{\gamma_i^2}{\lambda_{f_i}^2} \delta_{f_i}^T \delta_{f_i}. \quad (16)
\end{aligned}$$

Furthermore

$$\begin{aligned}
\frac{\gamma_i^2}{\lambda_{j_i}^2} \delta_{j_i}^T \delta_{j_i} \leq& \frac{\gamma_i^2}{\lambda_{j_i}^2} (f_i + \Delta f_i + c_i \omega_i + \Delta c_i \omega_i - \Delta j_i)^T W_{j_i}^T \\
&\cdot W_{j_i} (f_i + \Delta f_i + c_i \omega_i + \Delta c_i \omega_i - \Delta j_i) \\
=&\frac{\gamma_i^2}{\lambda_{j_i}^2} (f_i + B_i d_i)^T W_{j_i}^T W_{j_i} (f_i + B_i d_i) \\
=&\frac{\gamma_i^2}{\lambda_{j_i}^2} f_i^T W_{j_i}^T W_{j_i} f_i + \frac{2\gamma_i^2}{\lambda_{j_i}^2} f_i^T W_{j_i}^T W_{j_i} B_i d_i \\
&+ \frac{\gamma_i^2}{\lambda_{j_i}^2} d_i^T B_i^T W_{j_i}^T W_{j_i} B_i d_i. \quad (17)
\end{aligned}$$

Combining the previous two inequalities (16)-(17) and considering (8)-(9), then by completing the squares, there holds

$$\begin{aligned}
&\dot{V}(x(t)) + \|y\|^2 - \gamma^2 \|\omega_i\|^2 \\
=&\frac{\partial V_i}{\partial x} (f_i + B_i d_i) + h_i^T h_i - \gamma_i^2 d_i^T d_i + \frac{\gamma_i^2}{\lambda_{j_i}^2} f_i^T W_{j_i}^T W_{j_i} f_i \\
&+ \frac{2\gamma_i^2}{\lambda_{j_i}^2} f_i^T W_{j_i}^T W_{j_i} B_i d_i + \frac{\gamma_i^2}{\lambda_{j_i}^2} d_i^T B_i^T W_{j_i}^T W_{j_i} B_i d_i \\
&+ \frac{\gamma_i^2}{\lambda_{f_i}^2} \delta_{f_i}^T \delta_{f_i} \\
\leq&\frac{\partial V_i}{\partial x} (f_i + B_i d_i) + \gamma_i^2 C_i^T C_i - \gamma_i^2 d_i^T R_i d_i + 2\gamma_i^2 C_i^T D_i d_i
\end{aligned}$$

$$
\begin{aligned}
&=\frac{\partial V_i}{\partial x}f_i-\gamma_i^2\left\|R_i^{\frac{1}{2}}d_i-R_i^{-\frac{1}{2}}(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i)^T\right\|^2\\
&\quad+\gamma_i^2(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i)R_i^{-1}(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i)^T\\
&\quad+\gamma_i^2C_i^TC_i\\
&\le\frac{\partial V_i}{\partial x}f_i+\gamma_i^2C_i^TC_i+\gamma_i^2(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i)R_i^{-1}\\
&\quad\cdot(\frac{1}{2\gamma_i^2}\frac{\partial V_i}{\partial x}B_i+C_i^TD_i)\\
&\le\begin{cases}-\alpha V_i, i\le s,\\ \beta V_i,\ \ i>s.\end{cases}
\end{aligned}\tag{18}
$$

Note that when $\omega(t)\equiv 0$, we know from (18) that for any $t\in[t_k,t_{k+1})\,(t_0\le k\le N_\sigma(t_0,t))$, the piecewise Lyapunov function candidate (13) satisfies

$$
V(t)=V_{\sigma(t)}(t)\le\begin{cases}e^{-\alpha(t-t_0)}V_{\sigma(t_k)}(t_k), & if\ \ i\le s,\\ e^{\beta(t-t_0)}V_{\sigma(t_k)}(t_k), & if\ \ i>s.\end{cases}\tag{19}
$$

From (10), $V_{\sigma(t_k)}(t_k)\le\mu V_{\sigma(t_k^-)}(t_k^-)$ is true at the switching point t_k. Therefore, we obtain by induction that

$$
\begin{aligned}
V(t)&\le e^{\beta\Pi^+(t_k,t)-\alpha\Pi^-(t_k,t)}V_{\sigma(t_k)}(t_k)\\
&\le\mu e^{\beta\Pi^+(t_k,t)-\alpha\Pi^-(t_k,t)}V_{\sigma(t_k^-)}(t_k^-)\\
&\le\mu e^{\beta\Pi^+(t_k,t)-\alpha\Pi^-(t_k,t)}V_{\sigma(t_{k-1})}(t_{k-1})\\
&\le\cdots\le\mu^k e^{\beta\Pi^+(t_k,t)-\alpha\Pi^-(t_k,t)}V_{\sigma(t_0)}(t_0)\\
&\le\mu^{N(t_0,t)}e^{\beta\Pi^+(t_k,t)-\alpha\Pi^-(t_k,t)}V_{\sigma(t_0)}(t_0),
\end{aligned}\tag{20}
$$

where $N(t_0,t)$ is the switching numbers in the time interval (t_0,t).

Taking (5) and (7) into account, we get

$$
\begin{aligned}
V(t)&\le\mu^{N(t_0,t)}e^{\beta\Pi^+(t_k,t)-\alpha\Pi^-(t_k,t)}V_{\sigma(t_0)}(t_0)\\
&\le e^{N_0\ln\mu+c}e^{-(\lambda^*-\frac{\ln\mu}{\tau})(t-t_0)}V_{\sigma(t_0)}(t_0)\\
&\le c_0e^{-\lambda(t-t_0)}V_{\sigma(t_0)}(t_0),
\end{aligned}\tag{21}
$$

where $c_0=e^{N_0\ln\mu+c}$, $\lambda=(\lambda^*-\frac{\ln\mu}{\tau})$.

According to (11), we have

$$
\alpha_1^*(\|x\|)\le V_i(x)\le\alpha_2^*(\|x\|).\tag{22}
$$

Combining (20)-(22) gives

$$
\|x(t)\|\le\alpha_1^{*-1}(c_0e^{-\lambda(t-t_0)}\alpha_2^*(\|x(t_0)\|),\tag{23}
$$

which means global asymptotic stability of the switched system (1) with $\omega(t)\equiv 0$. The proof of internal stability is completed.

It can be easily seen from (18) that for any $t\in[t_k,t_{k+1})\,(t_0\le k\le N_\sigma(t_0,t))$, the piecewise Lyapunov function candidate (13) satisfies

$$
V(t)\le\begin{cases}e^{-\alpha(t-t_k)}V_{\sigma(t_k)}(t_k)-\int_{t_k}^t e^{-\alpha(t-\tau)}\Gamma(\tau)d\tau,\\ if\ \ \sigma(t_k)=i\le s,\\ e^{\beta(t-t_k)}V_{\sigma(t_k)}(t_k)-\int_{t_k}^t e^{\beta(t-\tau)}\Gamma(\tau)d\tau,\\ if\ \ \sigma(t_k)=i>s.\end{cases}
$$

From (10), $V_{\sigma(t_k)}(t_k)\le\mu V_{\sigma(t_k^-)}(t_k^-)$ is true at the switching point t_k. Therefore, we obtain by induction that

$$
\begin{aligned}
V(t)\le&e^{\beta\Pi^+(t_k,t)-\alpha\Pi^-(t_k,t)}V_{\sigma(t_k)}(t_k)\\
&-\int_{t_k}^t e^{\beta\Pi^+(\tau,t)-\alpha\Pi^-(\tau,t)}\Gamma(\tau)d\tau\\
\le&\mu e^{\beta\Pi^+(t_k,t)-\alpha\Pi^-(t_k,t)}V_{\sigma(t_k^-)}(t_k^-)\\
&-\int_{t_k}^t e^{\beta\Pi^+(\tau,t)-\alpha\Pi^-(\tau,t)}\Gamma(\tau)d\tau\le\cdots\\
\le&\mu^k e^{\beta\Pi^+(t_0,t)-\alpha\Pi^-(t_0,t)}V_{\sigma(t_0)}(t_0)\\
&-\mu^k\int_{t_0}^{t_1}e^{\beta\Pi^+(\tau,t)-\alpha\Pi^-(\tau,t)}\Gamma(\tau)d\tau\\
&-\mu^{k-1}\int_{t_1}^{t_2}e^{\beta\Pi^+(\tau,t)-\alpha\Pi^-(\tau,t)}\Gamma(\tau)d\tau-\cdots\\
&-\mu^0\int_{t_k}^{t}e^{\beta\Pi^+(\tau,t)-\alpha\Pi^-(\tau,t)}\Gamma(\tau)d\tau\\
=&\mu^k e^{\beta\Pi^+(t_0,t)-\alpha\Pi^-(t_0,t)}V_{\sigma(t_0)}(t_0)\\
&-\int_{t_0}^t\mu^{N_\sigma(\tau,t)}e^{\beta\Pi^+(\tau,t)-\alpha\Pi^-(\tau,t)}\Gamma(\tau)d\tau\\
=&e^{\beta\Pi^+(t_0,t)-\alpha\Pi^-(t_0,t)+N_\sigma(t_0,t)\ln\mu}V_{\sigma(t_0)}(t_0)\\
&-\int_{t_0}^t e^{\beta\Pi^+(\tau,t)-\alpha\Pi^-(\tau,t)+N_\sigma(\tau,t)\ln\mu}\Gamma(\tau)d\tau.
\end{aligned}\tag{24}
$$

Multiplying both sides of the above inequality by $e^{-N_\sigma(t_0,t)\ln\mu}$ leads to

$$
\begin{aligned}
&e^{-N_\sigma(t_0,t)\ln u}V(t)\\
&+\int_{t_0}^t e^{\beta\Pi^+(\tau,t)-\alpha\Pi^-(\tau,t)-N_\sigma(t_0,\tau)\ln u}y^T(\tau)y(\tau)d\tau\\
\le&e^{\beta\Pi^+(t_0,t)-\alpha\Pi^-(t_0,t)}V_{\sigma(t_0)}(t_0)\\
&+\gamma^2\int_{t_0}^t e^{\beta\Pi^+(\tau,t)-\alpha\Pi^-(\tau,t)-N_\sigma(t_0,\tau)\ln\mu}\omega^T(\tau)\omega(\tau)d\tau.
\end{aligned}
$$

Under the switching law (F) and the average dwell time scheme (5) with $\sigma<\lambda^*$, we can obtain

$$
\begin{aligned}
&\int_{t_0}^t e^{-\alpha(t-\tau)-\sigma\tau}y^T(\tau)y(\tau)d\tau\\
\le&e^{c-\lambda^*}V_{\sigma(t_0)}(t_0)+\gamma^2\int_{t_0}^t e^{c-\lambda^*(t-\tau)}\omega^T(\tau)\omega(\tau)d\tau.
\end{aligned}\tag{25}
$$

Integrating both sides of the foregoing inequality from t_0 to ∞ and rearranging the double-integral area, we obtain

$$
\begin{aligned}
&\int_{t_0}^\infty e^{-\sigma\tau}y^T(\tau)y(\tau)d\tau\\
\le&\frac{\alpha e^c}{\lambda^*}V_{\sigma(t_0)}(t_0)+\frac{\alpha e^c}{\lambda^*}\gamma^2\int_{t_0}^\infty\omega^T(\tau)\omega(\tau)d\tau,
\end{aligned}\tag{26}
$$

which means that the switched system achieves the weighted disturbance attenuation level $\sqrt{\frac{\alpha e^c}{\lambda^*}}\gamma$ under the average dwell time scheme (5) and the switching law (F).

When the switched system (1) is in the following linear form:

$$
\begin{aligned}
[I+E_{j_i}\sum\nolimits_{j_i}(t)F_{j_i}]\dot{x} =& [A_i+E_{a_i}\sum\nolimits_{a_i}(t)F_{a_i}] \\
&+[H_i+E_{h_i}\sum\nolimits_{h_i}(t)F_{h_i}]\omega_i, \\
y =& C_i x,
\end{aligned} \tag{27}
$$

where the uncertain matrices satisfy $\sum_{\upsilon}(t)\sum_{\upsilon}(t)\le I, \upsilon\in\{j_i,a_i,h_i, i\in I_m\}$. Let $\delta_{j_i}=\sum_{j_i}(t)F_{j_i}\dot{x}, \delta_{f_i}=\sum_{j_i}(t)F_{j_i}x, \delta_{g_i}=\sum_{h_i}(t)F_{h_i}x$, it is clear that $\upsilon\in\{j_i,a_i,h_i, i\in I_m\}$. satisfy Assumption 1 with $M_{j_i}=F_{j_i}, W_{f_i}=F_{a_i}, W_{c_i}=F_{h_i}$. Then, we have the following Theorem.

Theorem 2. Given any constant $\gamma>0$, suppose that there exist a set of positive definite matrices $P_i, i\in I_m$, constants $\alpha>0,\beta>0$ and $\mu\ge 1$, such that the following inequalities

$$
\begin{aligned}
&P_iA_i+A_i^TP_i+\gamma_i^2C_i^TC_i+\gamma_i^2(\frac{1}{2\gamma_i^2}P_iB_i+C_i^TD_i)R_i^{-1} \\
&\cdot(\frac{1}{2\gamma_i^2}P_iB_i+C_i^TD_i)^T+\alpha P_i<0, i\le s,
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
&P_iA_i+A_i^TP_i+\gamma_i^2C_i^TC_i+\gamma_i^2(\frac{1}{2\gamma_i^2}P_iB_i+C_i^TD_i)R_i^{-1} \\
&\cdot(\frac{1}{2\gamma_i^2}P_iB_i+C_i^TD_i)^T-\beta V_i<0, i>s,
\end{aligned} \tag{29}
$$

and

$$
P_i\le\mu P_j,\quad i,j\in I_m \tag{30}
$$

hold, where

$$
\begin{aligned}
&\gamma_i^2=\tfrac{\gamma^2}{1+\bar{\sigma}(F_{h_i})/\lambda_{p_i}^2}, \hat{B}_i=[H_i,\lambda_{j_i}E_{j_i},\lambda_{f_i}E_{a_i},\lambda_{p_i}E_{h_i}],\\
&C_i^T=\begin{bmatrix}(1/\gamma_i)C_i\\(1/\lambda_{j_i})F_{j_i}A_i\\(1/\lambda_{f_i})F_{a_i}\\0\end{bmatrix}, D_i=\begin{bmatrix}0\\(1/\lambda_{j_i})F_{j_i}B_i\\0\\0\end{bmatrix},\\
&R_i=I-D_i^TD_i,
\end{aligned}
$$

with $\lambda_{j_i},\lambda_{f_i}$, and $\lambda_{c_i}, i\in I_m$ are positive constants.

Then, the switching strategy (5) satisfying (F) solve the robust H_∞ control problem of the switched system (27).

Proof: The proof is similar to Theorem 1.

IV. Example

In this section, we give a numerical example to illustrate the performance of the proposed approach.

Example 1. Consider the nonlinear switched system (1) with $\sigma=\{1,2\}$ and

$$
\begin{aligned}
&f_1(x)=\frac{1}{4}x, c_1=1, h_1=-\frac{1}{2}x, f_2(x)=-2x, c_2=-1,\\
&h_2=x, \Delta j_1(\dot{x},t)=a_1\dot{x}\sin t, e_{j_1}=1, \delta_{j_1}=a_1\dot{x}\sin t,\\
&W_{j_1}=1, \Delta j_2(\dot{x},t)=a_2\dot{x}\cos t, e_{j_2}=1, \delta_{j_2}=a_2\dot{x}\cos t,\\
&W_{j_2}=1, \Delta f_1(x,t)=\frac{1}{2}b_1x\cos t, e_{f_1}=1, \delta_{f_1}=\frac{1}{2}b_1x\sin t,\\
&W_{j_2}=\frac{1}{2}x, \Delta f_2(x,t)=b_2x\sin t, e_{f_2}=1, \delta_{f_2}=b_2x\sin t,\\
&W_{f_2}=x, \Delta c_1(x,t)=c_1e^{-t}, e_{c_1}=1, \delta_{c_1}=c_1e^{-t}, W_{c_1}=1,\\
&\Delta c_2(x,t)=c_2e^{-t}, e_{c_2}=1, \delta_{c_2}=c_2e^{-t}, W_{c_2}=1,
\end{aligned} \tag{31}
$$

and $a_i,b_i,c_i, i=1,2$ are unknown constants in the set $[0,1]$.

It is easy to check that the first subsystem is unstabilizable and the second one is stabilizable. Let $\gamma^2=2$ and $\lambda_{j_i}=\lambda_{f_i}=\lambda_{c_i}=1$, then according to Theorem 1, we obtain

$$
\begin{aligned}
&\gamma_1=\gamma_2=1, B_1=[-1,1,1,1], B_2=[1,1,1,1],\\
&C_1=[x,-2x,x,0], C_2=[-\tfrac{1}{2}x,\tfrac{1}{4}x,\tfrac{1}{2}x,0],\\
&D_1^T=\begin{bmatrix}0&-1&0&0\\0&1&0&0\\0&1&0&0\\0&1&0&0\end{bmatrix}, D_2^T=\begin{bmatrix}0&1&0&0\\0&1&0&0\\0&1&0&0\\0&1&0&0\end{bmatrix},
\end{aligned}
$$

We choose $V_1(x)=2x^2, V_2(x)=4x^2$, and $\alpha=0.8,\beta=0.5$. Then following (8)-(9),we can infer

$$
\begin{aligned}
&\frac{\partial V_1}{\partial x}f_1+\gamma_1^2C_1^TC_1+\gamma_1^2(\frac{1}{2\gamma_1^2}\frac{\partial V_1}{\partial x}B_1+C_1^TD_1)R_1^{-1}\\
&\cdot(\frac{1}{2\gamma_1^2}\frac{\partial V_1}{\partial x}B_1+C_1^TD_1)^T-\beta V_2=-\frac{315}{16}x^2\le 0,
\end{aligned} \tag{32}
$$

and

$$
\begin{aligned}
&\frac{\partial V_2}{\partial x}f_2+\gamma_2^2C_2^TC_2+\gamma_2^2(\frac{1}{2\gamma_2^2}\frac{\partial V_2}{\partial x}B_2+C_2^TD_2)R_2^{-1}\\
&\cdot(\frac{1}{2\gamma_2^2}\frac{\partial V_2}{\partial x}B_2+C_2^TD_2)+\alpha V_1=-\frac{182}{15}x^2\le 0,
\end{aligned} \tag{33}
$$

Let $\mu=2,\lambda^*=0.3$, we have $\tau^*=\frac{\ln\mu}{\alpha}=0.8664$ and the activation ratio of stabilizable subsystems to unstabilizable subsystems is $\frac{\Pi^-(t_0,t)}{\Pi^+(t_0,t)}=\frac{\beta+\lambda^*}{\alpha-\lambda^*}=1.6$. Using the switching strategy provided by Theorem 1, we obtained that the robust H_∞ control problem of (1) is solvable, the simulation results are depicted in Figs. 1-2.

V. Conclusion

In this paper, we have investigated the problem of robust H_∞ control for a class of uncertain nonlinear switched systems based on ADT. Uncertainties are considered to be nonlinearly relied on state and state derivative and allowed to appear in the state, control input and disturbance input. Under the condition that the activation time ratio between stabilizable subsystems and unstabilizable ones is not less than a specified constant, we have derived sufficient conditions for the stabilization and weighted L_2-gain property of the switched system. The feasibility of the developed results have been proved by using a numerical example.

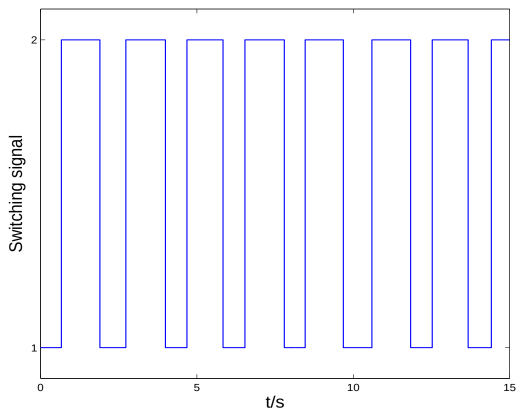

Fig. 1. The switching signals for the switched system (1).

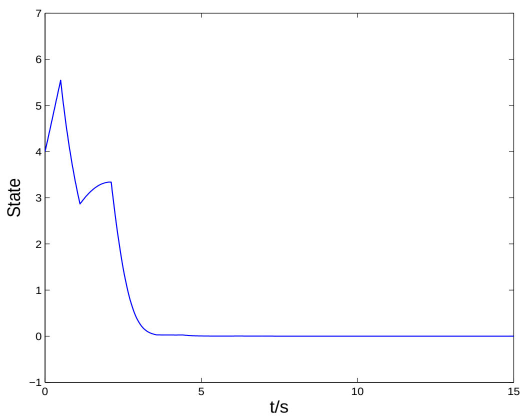

Fig. 2. The state responses of the switched system (1).

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References

[1] D. Liberzon, Switching in Systems and Control, Birkhäuser, Boston, 2003.

[2] D. Liberzon, A. S. Morse, Basic problems in stability and design of switched systems, *IEEE Control Systems Magazine* 19 (5) (1999) 59-70.

[3] P. Mhaskar, N. H. El-Farra, P. D. Christofides, Predictive control of switched nonlinear systems with scheduled mode transitions, *IEEE Transactions on Automatic Control* 50 (11) (2005) 1670-1680.

[4] Z. D. Sun, S. S. Ge, Analysis and synthesis of switched linear control systems, *Automatica* 41 (2) (2005) 181-195.

[5] J. Daafouz, P. Riedinger, C. Iung, Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach, *IEEE Transactions on Automatic Control* 47 (11) (2002) 1883-1887.

[6] M. Chadli, M. Darouach, Robust admissibility of uncertain switched singular systems, *International Journal of control* 84 (10) (2011) 1796-1806.

[7] M. S. Branicky, Multiple Lyapunov functions and other analysis tools for switched and hybrid systems, *IEEE Transaction on Automatic Control* 43 (4) (1998) 475-482.

[8] J. P. Hespanha, A. S. Morse, Stability of switched systems with average dwell-time, *in: Proceedings of the 38th IEEE Conference on Decision and Control*, Phoenix, AZ, 1999, pp. 2655-2660.

[9] X. M. Sun, J. Zhao, G. M. Dimirovski, State feedback control for discrete delay systems with controller failures based on average dwell-time method, *IET Control Theory and Applications* 2 (2) (2008) 126-132.

[10] A. Y. Aleksandrov, A. A. Kosov, A. V. Platonov, On the asymptotic stability of switched homogeneous systems, *Systems and Control Letters* 61 (1) (2012) 127-133.

[11] K. Sofiane, R. Micky, L. Philippe, Combining H_∞ approach and interval tools to design a low order and robust controller for systems with parametric uncertainties: application to piezoelectric actuators, *International Journal of control* 85 (3) (2012) 251-259

[12] J. Kaloust, Z. Qu, Robust control design for nonlinear uncertain systems with an unknown time-varying control direction, *IEEE Transactions on Automatic Control* 42 (3) (1997) 393-399 .

[13] S. M. Hoseini, M. Farrokhi, A. J. Koshkouei, Robust adaptive control of nonlinear non-minimum phase systems with uncertainties, *Automatica* 47 (1) (2011) 1-13.

[14] W. Z. Su, M. Y. Fu, Robust stabilization of nonlinear cascaded systems, *Automatica* 42 (4) (2006) 645-651

[15] L. Zhang, P. Shi, E. K. Boukas, C. Wang, Robust l_2-l_∞ filtering for switched linear discrete time-delay systems with polytopic uncertainties, *IET Control Theory and Applications* 1 (3) (2007) 722-730.

[16] Y. S. Fu, Z. H. Tian, S. J. Shi, Robust H_∞ control of uncertain nonlinear systems, Automatica 42 (9) (2006) 1547-1552.

[17] G. S. Deaecto, J. C. Geromel, J. Daafouz, Dynamic output feedback H_∞ control of switched linear systems, *Automatica* 47 (8) (2011) 1713-1720.

[18] L. Rodrigues, E. K. Boukas, Piecewise-linear H_∞ controller synthesis with applications to inventory control of switched production systems, *Automatica* 42 (8) (2006) 1245-1254.

[19] Z. R. Xiang, C. H. Qiao, M. S. Mahmoud, Finite-time analysis and Robust H_∞ control of uncertain nonlinear systems control for switched stochastic systems, *Journal of the Franklin Institute* 349 (3) (2012) 915-927.

[20] K. Hu, J. Yuan, Finite sum equality approach to H_∞ output-feedback control for switched linear discrete-time systems with time-varying delay, *IET Control Theory and Applications* 3 (8) (2009) 1006-1016.

[21] P. Niamsup, Controllability approach to H_∞ control problem of linear time-varying switched systems, *Nonlinear Analysis: Hybrid Systems* 2 (3) (2008) 875-886.

[22] W. M. Xiang, J. Xiao, H_∞ finite-time control for switched nonlinear discrete-time systems with norm-bounded disturbance, *Journal of the Franklin Institute* 348 (2) (2011) 331-352.

[23] X. Z. Liu, S. Yuan, On designing H_∞ estimator for switched nonlinear systems of neutral type, *Communications in Nonlinear Science and Numerical Simulation* 16 (11) (2011) 4379-4389.

[24] B. Niu, J. Zhao, Stabilization and L2-gain analysis for a class of cascade switched nonlinear systems: An average dwell-time method, *Nonlinear Analysis: Hybrid Systems* 5 (4) (2011) 671-680.

[25] B. Niu, J. Zhao, Robust H_∞ control for a class of switched nonlinear cascade systems via multiple Lyapunov functions approach, *Applied Mathematics and Computation* 218 (11) (2012) 6330-6339.

[26] G. S. Zhai, B. Hu, K. Yasuda, A. N. Michel, Disturbance attenuation properties of time-controlled switched systems, *J. Franklin Inst.* 338 (7) (2001) 765-779.

[27] M. Wang, J. Zhao, H_∞ control for a class of cascade switched nonlinear systems, *Asian Journal of Control* 10 (6) (2008) 724-729.