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Design of Switching Adaptive Laws for the State Tracking Problem

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Abstract— **In this paper, we address the issue of the state tracking control for model reference adaptive control systems. For a given plant with a known structure and unknown parameters, this problem can be solved by designing an adaptive law for traditional model reference adaptive control designs. In this paper, for a plant with finite fixed adaptive laws where none of them guarantees the states of plant track those of the reference model, there are not allowed to design other adaptive law. In this case, we formulate a switching mechanism between these adaptive laws to track the reference model. A sufficient condition is given for the problem to be solvable via the convex combination technique, and a switching law is designed. The theoretical derivations are illustrated by means of an example.**

Keywords- MRAC; adaptive control; switching law; state tracking; convex combination

I. INTRODUCTION

Model reference adaptive control (MRAC) is one of the main approaches in adaptive control. For MRAC, an adaptive controller, which has adjustable parameters and the same structure as the ideal controller, is usually given first. Then, an adaptive law is designed to adjust the parameters of the controller such that it can approach the ideal one to realize the state tracking. Differing from conventional non-adaptive controllers, the adaptive mechanism can improve steady accuracy and transient performance when the parameters of the system are unknown[1]. Several adaptive law design methods, including Lyapunov theory[2, 3], hyperstability theory[4, 5] and dissipative theory[6, 7], ensure that the boundedness of all the signals and state error asymptotically approaching zero

In general, the adaptive law need to be redesigned once the system changes. However, in practice, owing to the restrictions of hardwires and environment, it is difficult to redesign or modify the adaptive law when it is in use in the system. On the other hand, a single adaptive law, though works theoretically, is sometimes too complicated to implement in reality. In both cases, it is necessary to use multiple pre-given fixed adaptive laws to achieve state tracking.

The switched control strategy is very important. This is mainly due to the following reasons. Firstly, in some cases, a single controller (continuous or discrete) in a conventional control system is usually too complicated for sensors and actuators to realize. Secondly, a switched controller may stabilize the system when none of the controllers can stabilize the system alone [8-10]. At present, as a kind of hybrid control, the switched control strategy has been applied to automatic vehicles [11, 12], robot manipulators [13] and traffic system[14, 15], etc.

For adaptive control systems, the problem of stability with rapid variation of parameters can be solved by a switching strategy. At present, switched adaptive control is mainly focused on the multiple models adaptive control in which the transient performance is enhanced by a switching strategy of multiple adaptive controllers with different initial values. Though the system is non-switched, the closed-loop system of multiple model adaptive control is a switched system[16-19] . Another newly-arisen issue in switching adaptive control is the adaptive control problem of a switched system in which individual adaptive controllers are designed for the subsystems[20-23]. In these issues, a single adaptive controller is effective if the switching signal is fixed. How to design a switching law for given ineffective adaptive controllers to stabilize the error system, to the best of the authors' knowledge, has not been addressed in the existing literatures, which partly motivates our present work.

This paper studies the state asymptotically tracking problem of MRAC. For a system with finite fixed adaptive laws, none of which can guarantee state tracking, a sufficient condition is given to design a switching law between these adaptive laws via the convex combination technique. State tracking is achieved for the closed-loop system.

The result of this paper has two features. First, unlike conventional MRAC adaptive law design, we design a switching law that orchestrates finite fixed adaptive laws to solve the state tracking problem. Additionally, the proposed method can deal with the case where the adaptive law must be designed with new system, which enlarges the applicability of adaptive control theory.

The rest of this paper is organized as follows. The state tracking MRAC problem is formulated in section II. In Section III, a switching strategy is proposed for the adaptive laws to solve the state tracking problem. Section IV gives a simulation to show the effectiveness of the proposed approach and Section V concludes the paper.

II. PROBLEM STATEMENT

Consider a system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \tag{1}$$

where $A(t) \in R^{n\times n}$ and $B(t) \in R^{n\times m}$ are the system matrix and input matrix, respectively, both of which have known structure and unknown parameters; $x(t) \in R^n$ is the system state; and $u(t) \in R^m$ is the control input.

The control objective is that the state $x(t)$ of the system (1) tracks the state $x_m(t) \in R^n$ of a reference model specified by the LTI system

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t), \tag{2}$$

where $A_m \in R^{n\times n}$ is a constant Hurwitz matrix, $B_m(t) \in R^{n\times m}$ is a constant input matrix, $x_m(t)$ is the system state of the reference model, and $r(t) \in R^m$ is the bounded reference input. The reference model and the input r are chosen so that $x_m(t)$ represents a desired trajectory that $x(t)$ has to follow.

In order to achieve state tracking, we introduce a state error vector

$$e = x_m - x. \tag{3}$$

However, as well-understood, the parameters of the system (1) are difficult to be adjusted directly. In order to solve the state tracking problem, we introduce an adjustable feedback compensation matrix $F(t) \in R^{m\times n}$ and an adjustable feedforward gain matrix $K(t) \in R^{m\times m}$. Thus we apply a control law as follows

$$u = K(t)r + F(t)x. \tag{4}$$

Combining (4) with (1), we get the closed-loop system

$$\dot{x} = [A(t) + B(t)F(t)]x + B(t)K(t)r. \tag{5}$$

Furthermore, from (3) and (5), we can get the dynamical equation of the state error for the closed-loop system

$$\dot{e} = A_m e + [A_m - A(t) - B(t)F(t)]x + [B_m - B(t)K(t)]r. \tag{6}$$

When the dynamic response of system (6) is identical to that of the reference model (2) under the input $r(t)$, the system (1) is matched with reference model (2), that is,

$$\begin{aligned} A_m &= A(t) + B(t)F^*, \\ B_m &= B(t)K^*, \end{aligned} \tag{7}$$

where F^*, K^* denote the ideal values of $F(t), K(t)$ when the system matches with the reference model, and $F(t)$, $K(t)$ are the estimate of $F^*(t)$, $K^*(t)$, respectively.

In conventional MRAC system, the adaptive law is expressed as [24]

$$\begin{aligned} \dot{F} &= R_F^{-1}(B_m K^{*-1})^T Pex^T, \\ \dot{K} &= R_K^{-1}(B_m K^{*-1})^T Per^T, \end{aligned} \tag{8}$$

where R_F, R_K and P are the positive definite symmetric matrices.

Then, for the closed-loop system, the dynamical equation of the state error is given by

$$\dot{e} = A_m e + B_m K^{*-1}\tilde{F}x + B_m K^{*-1}\tilde{K}r, \tag{9}$$

where $\tilde{F} = F^* - F$ and $\tilde{K} = K^* - K$ are the parameter errors.

The following problem is considered in this paper. Several fixed adaptive laws are offered and none of them satisfies the matching condition (7), meanwhile, it is not allowed to design any other adaptive law. How to design a switching law between these adaptive laws to achieve state tracking, that is, the adaptive laws are given by

$$\begin{aligned} \dot{F}(t) &= \Gamma_i ex^T, \\ \dot{K}(t) &= \Phi_i er^T, \end{aligned} \quad i = 1, 2, ..., N, \tag{10}$$

where $\Gamma_i \in R^{m\times n}$ and $\Phi_i \in R^{m\times n}$ are the adaptive adjustable matrices, but none of them satisfies the matching condition (7). Again, the parameter error are $\tilde{F} = F^* - F$ and $\tilde{K} = K^* - K$ with F and K given by the i-th adaptive (10). In order to make the state of the closed-loop system (5) track the state of the reference model (2), that is $e \to 0$, $t \to \infty$, we design the switching law for the error system (9) and (10).

III. MAIN RESULT

In this section, we will design a switching law for the adaptive laws by means of the convex combination technology.

Theorem 1. If there exist scalars $\alpha_i \in (0,1), (i = 1, \cdots N)$ satisfying $\sum_{i=1}^{N} \alpha_i = 1$ and some positive definite symmetric matrices R_F, R_K and P satisfying

$$\begin{aligned}
&\sum_{i=1}^{N} \alpha_i \Gamma_i = -R_F^{-1} (B_m K^{*-1})^T P, \\
&\sum_{i=1}^{N} \alpha_i \Phi_i = -R_K^{-1} (B_m K^{*-1})^T P.
\end{aligned} \tag{11}$$

Then, there exists a switching law such that the adaptive controllers (4) and (10) make the state $x(t)$ of the closed-loop system (1) asymptotically track the state $x_m(t)$ of a reference model (2).

Proof. Based on the error system (9) and (10), we construct the combined error system

$$\begin{aligned}
&\dot{e} = A_m e + B_m K^{*-1} \tilde{F} x + B_m K^{*-1} \tilde{K} r, \\
&\dot{F} = \sum_{i=1}^{N} (\alpha_i \Gamma_i e x^T), \\
&\dot{K} = \sum_{i=1}^{N} (\alpha_i \Phi_i e r^T),
\end{aligned} \tag{12}$$

where (12) is the convex combination of the adaptive control law (10).

Consider the following Lyapunov function candidate

$$V = \frac{1}{2} [e^T P e + tr(\tilde{F}^T R_F \tilde{F} + \tilde{K}^T R_K \tilde{K})]. \tag{13}$$

The time derivative of the Lyapunov function (13) along the trajectory of the combined error system (12) is

$$\begin{aligned}
\dot{V} &= \frac{1}{2} \Big\{ \dot{e}^T P e + e^T P \dot{e} \\
&+ tr[\sum_{i=1}^{N} (\alpha_i \Gamma_i e x^T)^T R_F \tilde{F} + \tilde{F}^T R_F \sum_{i=1}^{N} (\alpha_i \Gamma_i e x^T) \\
&+ \sum_{i=1}^{N} (\alpha_i \Phi_i e r^T)^T R_K \tilde{K} + \tilde{K}^T R_K \sum_{i=1}^{N} (\alpha_i \Phi_i e r^T)] \Big\} \\
&= \frac{1}{2} [e^T (P A_m + A_m^T P) e] + e^T P B_m K^{*-1} \tilde{F} x + e^T P B_m K^{*-1} \tilde{K} r \\
&+ \frac{1}{2} tr[\sum_{i=1}^{N} (\alpha_i \Gamma_i e x^T)^T R_F \tilde{F} + \tilde{F}^T R_F \sum_{i=1}^{N} (\alpha_i \Gamma_i e x^T) \\
&+ \sum_{i=1}^{N} (\alpha_i \Phi_i e r^T)^T R_K \tilde{K} + \tilde{K}^T R_K \sum_{i=1}^{N} (\alpha_i \Phi_i e r^T)].
\end{aligned} \tag{14}$$

With the help of

$$\begin{aligned}
&e^T P B_m K^{*-1} \tilde{F} x = tr(x e^T P B_m K^{*-1} \tilde{F}), \\
&e^T P B_m K^{*-1} \tilde{K} r = tr(r e^T P B_m K^{*-1} \tilde{K}),
\end{aligned} \tag{15}$$

(14) can be rewritten as

$$\begin{aligned}
\dot{V} &= \frac{1}{2} [e^T (P A_m + A_m^T P) e] \\
&+ tr[\sum_{i=1}^{N} (\alpha_i \Gamma_i e x^T)^T R_F \tilde{F} + x e^T P B_m K^{*-1} \tilde{F}] \\
&+ tr[\sum_{i=1}^{N} (\alpha_i \Phi_i e r^T)^T R_K \tilde{K} + r e^T P B_m K^{*-1} \tilde{K}] \\
&= \frac{1}{2} [\sum_{i=1}^{N} \alpha_i e^T (P A_m + A_m^T P) e] \\
&+ tr[\sum_{i=1}^{N} (\alpha_i \Gamma_i e x^T)^T R_F \tilde{F} + x e^T P B_m K^{*-1} \tilde{F}] \\
&+ tr[\sum_{i=1}^{N} (\alpha_i \Phi_i e r^T)^T R_K \tilde{K} + r e^T P B_m K^{*-1} \tilde{K}].
\end{aligned} \tag{16}$$

According to (11), for the combined error system (12), we get $\dot{V} < 0$,that is,

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{N} \alpha_i \{ \frac{1}{2} [e^T (P A_m + A_m^T P) e] \\
&+ tr\left[(\Gamma_i e x^T)^T R_F \tilde{F} + x e^T P B_m K^{*-1} \tilde{F} \right] \\
&+ tr[(\Phi_i e r^T)^T R_K \tilde{K} + r e^T P B_m K^{*-1} \tilde{K}] \} \\
&< 0.
\end{aligned} \tag{17}$$

Because $\alpha_i > 0$ and $\sum_{i=1}^{N} \alpha_i = 1,$ there exists $l \in N$ at least for (17) such that

$$\begin{aligned}
&\frac{1}{2} [e^T (P A_m + A_m^T P) e] \\
&+ tr\left[(\Gamma_l e x^T)^T R_F \tilde{F} + x e^T P B_m K^{*-1} \tilde{F} \right] \\
&+ tr[(\Phi_l e r^T)^T R_K \tilde{K} + r e^T P B_m K^{*-1} \tilde{K}] \\
&< 0.
\end{aligned} \tag{18}$$

Now, we turn to the error system (9) and (10).

Consider the following Lyapunov function candidate

$$V = \frac{1}{2} [e^T P e + tr(\tilde{F}^T R_F \tilde{F} + \tilde{K}^T R_K \tilde{K})]. \tag{19}$$

When the j -th adaptive control law is active, the time derivative of the Lyapunov function (19) along the trajectory of the error system (9) and (10) is

$$\begin{aligned}
\dot{V} &= \frac{1}{2} [e^T (P A_m + A_m^T P) e] \\
&+ tr\left[(\Gamma_j e x^T)^T R_F \tilde{F} + x e^T P B_m K^{*-1} \tilde{F} \right] \\
&+ tr[(\Phi_j e r^T)^T R_K \tilde{K} + r e^T P B_m K^{*-1} \tilde{K}].
\end{aligned} \tag{20}$$

Therefore, for the error system (9) and (10), the switching law can be designed as

$$\sigma(e,t) = \arg\min_i \{\frac{1}{2}[e^T(PA_m + A_m^T P)e] + tr[(\Gamma_i ex^T)^T R_F \tilde{F} + xe^T PB_m K^{*-1}\tilde{F}] + tr[(\Phi_i er^T)^T R_K \tilde{K} + re^T PB_m K^{*-1}\tilde{K}]\}. \quad (21)$$

Owing to (18) and (21), adaptive laws (10) is orchestrated so that V decreases along the solutions of the error system (9) and (10). Then, the state $x(t)$ of the system (1) asymptotically track the state $x_m(t)$ of the reference model (2) with the given controller (4) and (10).

Remark 1. For the given adaptive law (10), if there exists $h \in N$, such that the h-th adaptive control law satisfying (8), the state tracking problem can be solved by the h-th adaptive control law. In this case, we choose $\alpha_h = 1$, $\alpha_i = 0,\ i \neq h$, therefore, Theorem 1 contains the result of [24] as a special case.

Remark 2. In the case of $N = 1$, the issue is degenerated into the design of a single controller.

Remark 3. The switching law is designed when $\tilde{F}$ and $\tilde{K}$ are available. If $\tilde{F}$ and $\tilde{K}$ they are unavailable, an estimator can be used instead[25] .

IV. EXAMPLE

In order to show the effectiveness of the proposed switching adaptive controllers, we consider an example.

Consider the following system

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -6 & -7 \end{pmatrix} x + \begin{pmatrix} 0 \\ 8 \end{pmatrix} u.$$

The reference model is given by

$$\dot{x}_m = \begin{pmatrix} 0 & 1 \\ -10 & -5 \end{pmatrix} x_m + \begin{pmatrix} 0 \\ 2 \end{pmatrix} r.$$

We compare the simulation results of the system under two given adaptive laws none of which satisfies the state tracking respectively to those under the designed switching law.

For controller (4), there are two adaptive laws as follows

$$\dot{F}(t) = \Gamma_i ex^T, \quad \dot{K}(t) = \Phi_i er^T, \quad i = 1, 2, \quad (22)$$

where

$$\Gamma_1 = [49.4 \quad 49.4], \Phi_1 = [4 \quad 4],$$

$$\Gamma_2 = [0.15 \quad 0.15], \Phi_2 = [1.5 \quad 1.5].$$

Simulations are performed for these adaptive laws, respectively.

For $B_m = B(t)K^*$, we have

$$B(t) = B_m K^{*-1} = \begin{pmatrix} 0 \\ 8 \end{pmatrix}.$$

Choose $N = 2, P = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$.

The parameter errors and the state error are shown in Figure 1 and Figure 2.

Obviously, Figure 1 and Figure 2 show that none of these adaptive laws makes the parameter errors and the state error convergent.

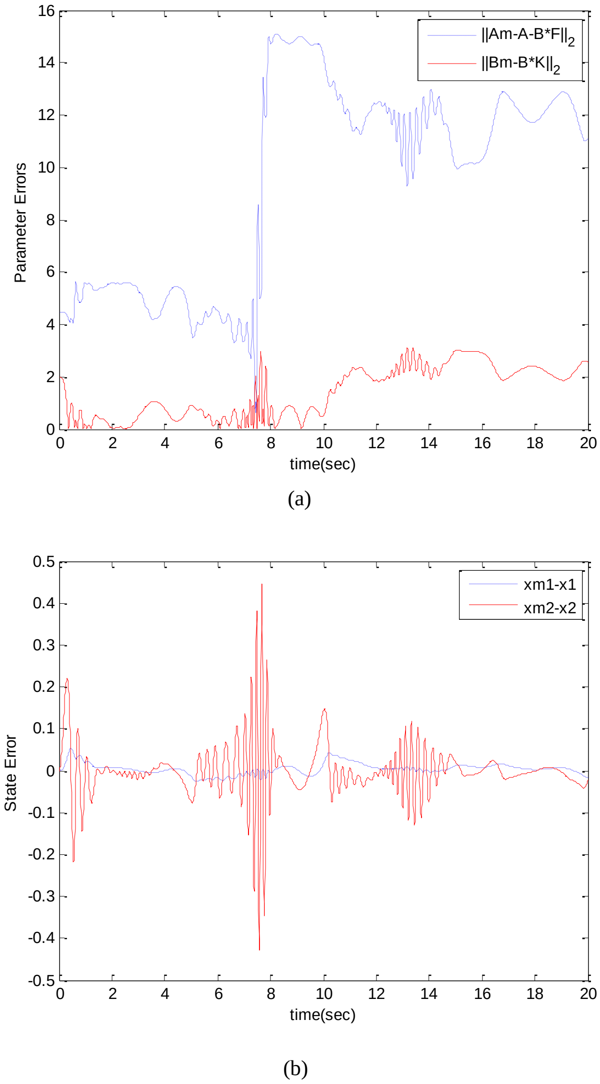
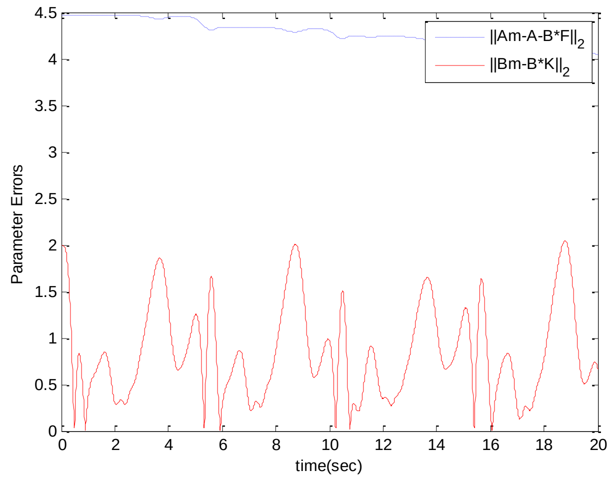
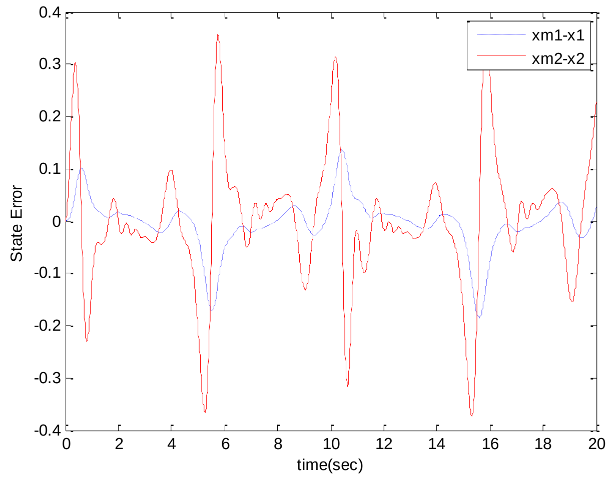


Figure 1. The simulation result ($i = 1$).
(a): The parameter errors. (b): The state error



(a)



(b)

Figure 2. The simulation result ($i = 2$).

(a): The parameter errors. (b): The state error

In order to achieve the objective, we will design a switching law by Theorem 1

With the help of (4) and (22), we obtain switching adaptive laws

$$\begin{array}{l} \dot{F}(t) = \Gamma_\sigma e x^T, \\ \dot{K}(t) = \Phi_\sigma e r^T, \end{array} \quad \sigma = 1, 2,$$

where the switching law σ is chosen by (21) as Figure 3.

Here, we choose $\alpha_1 = 0.2$, $\alpha_2 = 0.8$, $R_F = 1.25$, $R_K = 0.25$, such that (11) hold. From Figure 4, it is easy to see that the parameter errors and the state error are convergent under the switching law. Then, state tracking is achieved.

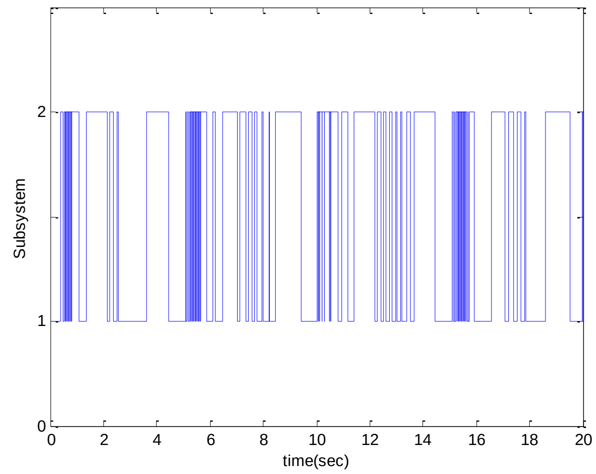
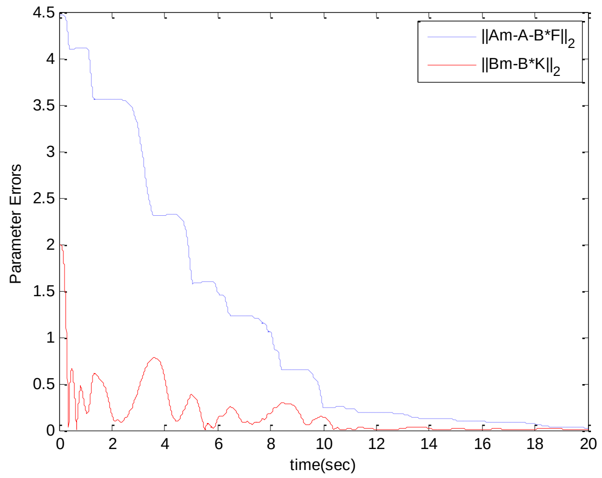
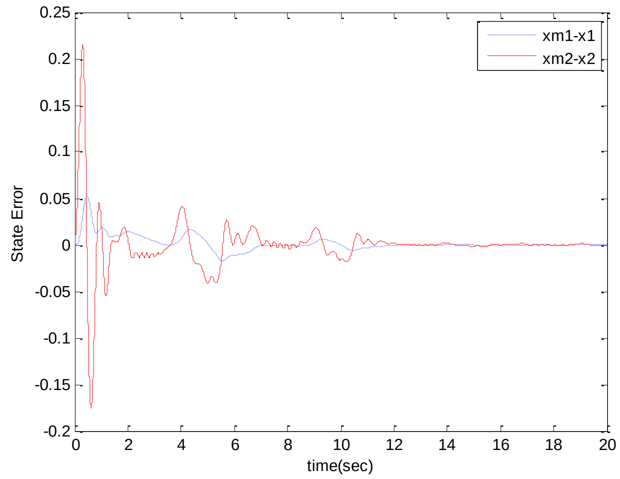


Figure 3. The switching signal



(a)



(b)

Figure 4. The simulation results under switching law.
(a): The parameter errors. (b): The state error

V. Conclusions

We have studied the state asymptotically tracking control problem for model reference adaptive control systems. For a system with several fixed adaptive laws, a sufficient condition has been developed to solve the state tracking problem via the convex combination technique by designing the controller switching strategy. A simulation example has been given to show the effectiveness of the proposed method.

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