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Synthesis of Variable Gain Controllers Based on LQ Optimal Control for a Class of Uncertain Linear Systems

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***Abstract*—This paper proposes a new variable gain controller for a class of uncertain linear systems. The proposed variable gain controller is based on optimal control for the nominal system and consists of the optimal feedback gain and a time-varying adjustable parameter which is designed so as to reduce the effect of uncertainties, i.e. the proposed variable gain controller can achieve good transient performance which is close to LQ optimal control for the nominal system. In this paper, we show sufficient conditions for the existence of the proposed variable gain controller for uncertain linear systems. Finally, numerical examples are presented.**

I. Introduction

Robustness of control systems to uncertainties has always been the central issue in feedback control and therefore for linear systems with unknown parameters, a large number of design methods of robust controllers have been presented (e.g. [1] and references therein). For a system with structured uncertainties, several quadratic stabilizing control laws have also been suggested and a connection between quadratic stabilization and $\mathcal{H}^\infty$ control has been established[2]. It is well known that for robust control for linear dynamical systems with uncertainties, the concept of quadratic stabilization via fixed quadratic Lyapunov functions plays an important role in dealing with the controller design.

By the way in most practical situations, it is desirable to design robust control systems which achieve not only robust stability but also an adequate level of performance. Therefore robust controllers achieving some robust performances such as quadratic cost function, mixed $\mathcal{H}^\infty/\mathcal{H}^2$ control, robust $\mathcal{H}^2$ control and so on have been suggested (e.g.[3], [4], [5]). Additionally, synthesis problems of robust controllers with variable gain have also been tackled (e.g. [6], [7]). Yamamoto and Yamauchi[6] proposed a design method of a robust controller with the ability to adjust control performances adaptively. In [7], an adaptive robust controller with adaptation mechanism has been presented and the adaptive robust controller is tuned on-line based on the information about parameter uncertainties. Besides, we have proposed robust controllers with adaptive compensation inputs[8], [9]. Although the robust controllers in [8] and [9] can achieve not only asymptotical stability but also satisfactory transient behavior, these robust controllers include the additional dynamics of the nominal system. Namely, these robust controllers are dynamic one and their structure is more complex.

From these viewpoints, we propose a variable gain robust controller based on optimal control for a class of uncertain linear systems. The proposed variable gain controller consists of optimal feedback gain designed by using the nominal system and an adjustable time-varying parameter. The adjustable parameter is designed so as to reduce the effect of uncertainties. The proposed variable gain controller can achieve good transient performance which is close to the desirable trajectory generated by the nominal closed-loop system. This paper is organized as follows. In Sec. II, notation and useful lemmas which are used in this paper are shown, and in Sec. III, we introduce the class of uncertain linear systems under consideration. Sec. IV contains the main results. Finally, numerical examples are included to illustrate the results developed in this paper.

II. Preliminaries

In this section, we show notations and useful and well-known lemmas which are used in this paper.

In the sequel, we use the following notation. For a matrix $\mathcal{A}$, The transpose of matrix $\mathcal{A}$ and the inverse of one are denoted by $\mathcal{A}^T$ and $\mathcal{A}^{-1}$ respectively and $\mathrm{rank}\{\mathcal{A}\}$ represents the rank of the matrix $\mathcal{A}$. Also, $H_e\{\mathcal{A}\}$ means $\mathcal{A}+\mathcal{A}^T$ and I_n represents n-dimensional identity matrix and the notation $\mathrm{diag}\,(\mathcal{A}_1,\cdots,\mathcal{A}_N)$ denotes a block diagonal matrix composed of matrices $\mathcal{A}_i$ for $i=1,\cdots,N$. For real symmetric matrices $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A}>\mathcal{B}$ (resp. $\mathcal{A}\geq\mathcal{B}$) means that $\mathcal{A}-\mathcal{B}$ is positive (resp. nonnegative) definite matrix. For a vector $\alpha\in\Re^n$, $||\alpha||$ denotes standard Euclidian norm and for a matrix $\mathcal{A}$, $||\mathcal{A}||$ represents a its induced norm. The symbols "$\stackrel{\triangle}{=}$" and "$\star$" means equality by definition and symmetric blocks in matrix inequalities, respectively.

Furthermore, the following well-known lemmas are used in this paper.

Lemma 1: For arbitrary vectors λ and ξ and the matrices $\mathcal{G}$ and $\mathcal{H}$ which have appropriate dimensions, the following relation holds.

$$H_e\left\{\lambda^T\mathcal{G}\Delta(t)\mathcal{H}\xi\right\}\leq 2\left\|\mathcal{G}^T\lambda\right\|\left\|\mathcal{H}\xi\right\|$$

where $\Delta(t)\in\mathbb{R}^{p\times q}$ is a time-varying unknown matrix satisfying $\left\|\Delta(t)\right\|\leq 1$.

Proof: The above relation is easily obtained by Schwartz's inequality[10]. ■

$$\begin{pmatrix} -\mathcal{Q} - (1+\tau_1)\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P} + (\delta\tau_1+\tau_2)\, I_n & \Xi(\mathcal{P}) \\ \star & -\tau_2 \Sigma_\sigma^{-1} \end{pmatrix} < 0 \tag{9}$$

Lemma 2: ($\mathcal{S}$-procedure) Let $\mathcal{F}(x)$ and $\mathcal{G}(x)$ be two arbitrary quadratic forms over $\mathbb{R}^n$. Then $\mathcal{F}(x) < 0$ for $\forall x \in \mathbb{R}^n$ satisfying $\mathcal{G}(x) \leq 0$ if and only if there exist a nonnegative scalar τ such that

$$\mathcal{F}(x) - \tau\mathcal{G}(x) \leq 0 \quad \text{for} \quad \forall x \in \mathbb{R}^n$$

Proof: See Boyd et al.[11] ■

Lemma 3: (Schur complement) For a given constant real symmetric matrix Ξ, the following arguments are equivalent.

(i) $\Xi = \begin{pmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & \Xi_{22} \end{pmatrix} > 0$

(ii) $\Xi_{11} > 0$ and $\Xi_{22} - \Xi_{12}^T\Xi_{11}^{-1}\Xi_{12} > 0$

(iii) $\Xi_{22} > 0$ and $\Xi_{11} - \Xi_{12}\Xi_{22}^{-1}\Xi_{12}^T > 0$

Proof: See Boyd et al.[11] ■

III. Problem Formulation

Consider the uncertain linear system described by the following state equation (see **Remark 1**).

$$\frac{d}{dt}x(t) = \left(A + \sum_{k=1}^{\mathcal{N}} \theta_k(t)\mathcal{D}_k\right)x(t) + Bu(t) \tag{1}$$

where $x(t) \in \Re^n$ and $u(t) \in \Re^m$ are the vectors of the state (assumed to be available for feedback) and the control input, respectively. In (1), the matrices A and B denote the nominal values of the uncertain system of (1). The matrices $\mathcal{D}_k$ $(k = 1, \cdots, N)$ which have appropriate dimensions represent the structure of uncertainties and the time-varying parameter vector $\theta(t) \in \Re^N(\theta(t) = (\theta_1(t), \cdots, \theta_{\mathcal{N}}(t))^T)$ shows unknown parameters which belong to the $\mathcal{N}$-dimensional ellipsoidal set expressed as

$$\begin{aligned} &\Delta \stackrel{\triangle}{=} \left\{\theta \in \Re^{\mathcal{N}} \mid \theta^T(t)\Sigma^{-1}\theta(t) \leq 1\right\} \\ &\Sigma = diag\left(\sigma_1^2, \cdots, \sigma_{\mathcal{N}}^2\right) \end{aligned} \tag{2}$$

where $\Sigma \in \Re^{\mathcal{N}\times\mathcal{N}}$ represents the size of the ellipsoid. Beside the nominal system, ignoring the unknown parameters in (1), is given by

$$\frac{d}{dt}\overline{x}(t) = A\overline{x}(t) + B\overline{u}(t). \tag{3}$$

In this paper first of all, we consider the standard linear quadratic control problem for the nominal system of (3) in order to generate the desired response for the uncertain system of (1) systematically. Namely we define the following quadratic cost function for the nominal system of (3).

$$\mathcal{J} = \int_0^\infty \left(\overline{x}^T(t)\mathcal{Q}\overline{x}(t) + \overline{u}^T\mathcal{R}\overline{u}(t)\right)dt \tag{4}$$

where the matrices $\mathcal{Q} \in \Re^{n\times n}$ and $\mathcal{R} \in \Re^{m\times m}$ are positive definite. It is well-known that the optimal control input minimizing the quadratic cost function of (4) is given by $\overline{u}(t) = -K\overline{x}(t)$, where $K \in \Re^{m\times n}$ represent the optimal control gain matrix. Note that the closed-loop system matrix $A_K \stackrel{\triangle}{=} A - BK$ is stable and the optimal feedback gain matrix $K \in \Re^{m\times n}$ is derived as $K = \mathcal{R}^{-1}B^T\mathcal{P}$ where $\mathcal{P} \in \Re^{n\times n}$ is unique solution of the algebraic Riccati equation

$$H_e\left\{A^T\mathcal{P}\right\} - \mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P} + \mathcal{Q} = 0. \tag{5}$$

Now by using the optimal feedback gain matrix $K \in \Re^{m\times n}$ for the nominal system of (3), we consider the following control input.

$$u(t) \stackrel{\triangle}{=} \gamma(x,t)Kx(t) \tag{6}$$

where $\gamma(x,t) \in \Re^1$ is a time-varying adjustable parameter so as to compensate the effect of unknown parameters.

From eqs.(1) and (6), we have the closed-loop system

$$\frac{d}{dt}x(t) = Ax(t) + \Gamma(x,t)\theta(t) + \gamma(x,t)Kx(t). \tag{7}$$

In (7), $\Gamma(x,t)$ is a matrix expressed as

$$\Gamma(x,t) = \left(\ \mathcal{D}_1x(t), \mathcal{D}_2x(t), \cdots, \mathcal{D}_{\mathcal{N}}x(t)\ \right). \tag{8}$$

From the above discussion, our control objective in this paper is to design the robust stabilizing controller which achieves good transient performance for the uncertain closed-loop system of (7). That is to design the time-varying adjustable parameter $\gamma(x,t) \in \Re^1$ such that the closed-loop system of (7) is robustly stable and achieves satisfactory transient performance close to LQ optimal control for the nominal system of (3).

Remark 1: In this paper, we consider the uncertain dynamical system of (1) which has uncertainties in the state matrix only. The proposed design scheme of the variable controller derived in next section can also be applied to the case that the uncertainties are included in both the system matrix and the input matrix. By introducing additional actuator dynamics and constituting an augmented system, uncertainties in the input matrix are embedded in the system matrix of the augmented system[12]. Therefore the same design procedure can be applied.

IV. Main Results

In this section, we show a design method of the proposed variable gain controller such that the uncertain system of (1) is asymptotically stable.

The following theorem gives sufficient conditions for the existence of the proposed controller.

Theorem 1: Consider the uncertain linear system of (1) and the control input of (6).

If there exist the positive scalars τ_1 and τ_2 satisfying the LMI of (9) then the adjustable time-varying parameter

$$\gamma(t)=\begin{cases}-\left(1+\dfrac{\left\|\Sigma^{1/2}\Gamma^T(x,t)\mathcal{P}x(t)\right\|}{\left\|\mathcal{R}^{-1/2}B^T\mathcal{P}x(t)\right\|^2}\right) & \text{if} \quad x^T(t)\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}x(t)\geq\delta x^T(t)x(t)\\ -\left(1+\dfrac{\left\|\Sigma^{1/2}\Gamma^T(x,t)\mathcal{P}x(t)\right\|}{\delta x^T(t)x(t)}\right) & \text{if} \quad x^T(t)\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}x(t)<\delta x^T(t)x(t)\end{cases} \tag{10}$$

$$\frac{d}{dt}\mathcal{V}(x,t)=x^T(t)\left[H_e\left\{A^T\mathcal{P}\right\}\right]x(t)+2x^T(t)\Gamma(x,t)\theta(t)+2\gamma(t)x^T(t)\mathcal{P}BKx(t) \tag{13}$$

$$\frac{d}{dt}\mathcal{V}(x,t)=-x^T(t)\left(\mathcal{Q}-\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}\right)x(t)+2x^T(t)\Gamma(x,t)\theta(t)+2\gamma(t)x^T(t)\mathcal{P}BKx(t) \tag{14}$$

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(x,t)&=-x^T(t)\left(\mathcal{Q}-\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}\right)x(t)+2x^T(t)\Gamma(x,t)\Sigma^{1/2}\Sigma^{-1/2}\theta(t)+2\gamma(t)x^T(t)\mathcal{P}BKx(t)\\ &=-x^T(t)\left(\mathcal{Q}-\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}\right)x(t)+2\left\|\Sigma^{1/2}\Gamma^T(x,t)x(t)\right\|+2\gamma(t)x^T(t)\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}x(t)\end{aligned} \tag{15}$$

$$\frac{d}{dt}\mathcal{V}(x,t)=\begin{pmatrix}x(t)\\ \xi(t)\end{pmatrix}^T\begin{pmatrix}-\mathcal{Q}+\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P} & \Xi(\mathcal{P})\\ \star & 0\end{pmatrix}\begin{pmatrix}x(t)\\ \xi(t)\end{pmatrix}+2\gamma(t)x^T(t)\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}x(t) \tag{17}$$

$$\begin{aligned}&\begin{pmatrix}x(t)\\ \xi(t)\end{pmatrix}^T\begin{pmatrix}-\mathcal{Q}+\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P} & \Xi(\mathcal{P})\\ \star & 0\end{pmatrix}\begin{pmatrix}x(t)\\ \xi(t)\end{pmatrix}+2\gamma(t)x^T(t)\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}x(t)<0\\ &\quad \text{s.t.} \quad x^T(t)\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}x(t)<\delta x^T(t)x(t) \text{ and } \xi^T(t)\Sigma_\sigma^{-1}\xi(t)\leq x^T(t)x(t)\end{aligned} \tag{20}$$

$$\begin{pmatrix}-\mathcal{Q}-\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P} & \Xi(\mathcal{P})\\ \star & 0\end{pmatrix}<0 \;\; \text{s.t.} \;\; x^T(t)\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}x(t)<\delta x^T(t)x(t) \text{ and } \xi^T(t)\Sigma_\sigma^{-1}\xi(t)\leq x^T(t)x(t) \tag{22}$$

$\gamma(t)\in\mathbb{R}^1$ is determined as (10). In (9), δ is a positive constant selected by designers and $\Xi(\mathcal{P})$ is a matrix given by

$$\Xi(\mathcal{P})\stackrel{\triangle}{=}\begin{pmatrix}\mathcal{P}\mathcal{D}_1 & \mathcal{P}\mathcal{D}_2 & \cdots & \mathcal{P}\mathcal{D}_\mathcal{N}\end{pmatrix}. \tag{11}$$

Then the uncertain closed-loop system of (7) is robustly stable.

Proof: By using the unique solution $\mathcal{P}\in\mathbb{R}^{n\times n}$ of the algebraic Riccati equation of (5), we consider the following quadratic function.

$$\mathcal{V}(x,t)\stackrel{\triangle}{=}x^T(t)\mathcal{P}x(t) \tag{12}$$

The time derivative of the quadratic function $\mathcal{V}(x,t)$ can be written as (13). Additionally since the matrix $\mathcal{P}$ is the unique solution of the algebraic Riccati equation of (5), The time derivative of the quadratic function $\mathcal{V}(x,t)$ can be rewritten as (14).

Now, we consider the case of $x^T(t)\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}x(t)\geq\delta x^T(t)x(t)$. In this case using **Lemma 1**, we obtain (15). Here we have used the relation of (2) and $K=\mathcal{R}^{-1}B^T\mathcal{P}$. Besides, by using the adjustable time-varying parameter $\gamma(t)$ of (10), we find that the following relation holds.

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(x,t)=&-x^T(t)\left(\mathcal{Q}+\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}\right)x(t)\\ &<0 \quad \text{for} \quad \forall x(t)\neq 0\end{aligned} \tag{16}$$

Next, we consider the case of $x^T(t)\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}x(t)<\delta x^T(t)x(t)$ and then the time derivative of the quadratic function $\mathcal{V}(x,t)$ of (14) can be described as (17). In (17), $\xi(t)$ is a $n\times\mathcal{N}$-dimensional vector given by

$$\xi^T(t)=\begin{pmatrix}\theta_1(t)x^T(t) & \theta_2(t)x^T(t) & \cdots & \theta_\mathcal{N}(t)x^T(t)\end{pmatrix}. \tag{18}$$

Note that from the relation of (2) the following inequality for the vector $\xi(t)\in\mathbb{R}^{n\times\mathcal{N}}$ is satisfied.

$$\xi^T(t)\Sigma_\sigma^{-1}\xi(t)\leq x^T(t)x(t) \tag{19}$$

In (19), $\Sigma_\sigma=\text{diag}\left(\sigma_1^2I_n,\sigma_2^2I_n,\cdots,\sigma_\mathcal{N}^2I_n\right)$. One can see that if the condition of (20) holds, then the following inequality is also satisfied.

$$\frac{d}{dt}\mathcal{V}(x,t)<0 \quad \text{for} \quad \forall x(t)\neq 0 \tag{21}$$

Thus we consider the condition of (20). By using the adjustable time-varying parameter $\gamma(t)$ of (10), we have (22) which is a sufficient condition for the inequality of (20). Namely, if the condition of (22) holds, then the inequality of (20) is also satisfied. Applying **Lemma 2** ($\mathcal{S}$-procedure) to the condition of (22) and some trivial manipulations give the LMI of (9). Therefore for the case of $x^T(t)\mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}x(t)<\delta x^T(t)x(t)$, if the LMI of (9) is feasible then the relation of (21) is satisfied.

From the above discussion, the quadratic function $\mathcal{V}(x,t)$ becomes a Lyapunov function and the uncertain linear system of (1) is ensured to be stable. It follows that the result of the theorem is true. The proof of **Theorem 1** is completed. ■

Remark 2: In this paper, the quadratic function $\mathcal{V}(x,t)$ of (12) is introduced and it becomes a Lyapunov function for the uncertain system of (1). On the other hand, the quadratic function $\mathcal{V}(x,t)$ of (12) is also a Lyapnov function for the nominal closed-loop system, i.e. the standard LQ regulator. Therefore, by selecting the design parameter $\delta\in\mathbb{R}^1$ the proposed controller can achieve the good transient performance and adjust the magnitude of the control input, because the Lyapunov function for the uncertain system of (1) and one of the nominal system of (3) have same level set.

$$\frac{d}{dt}x(t) = \begin{pmatrix} -3.0 & 1.0 \\ 0.0 & 1.0 \end{pmatrix} x(t) + \delta_1(t) \begin{pmatrix} 1.0 & 1.0 \\ 0.0 & 0.0 \end{pmatrix} x(t) + \delta_2(t) \begin{pmatrix} 0.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix} x(t) + \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix} u(t) \quad (23)$$

$$\begin{array}{lll} \bullet \text{ Case 1)} & : \ \delta_1(t) = \sqrt{3.0} & , \ \delta_2(t) = -\sqrt{2.0} \\ \bullet \text{ Case 2)} & : \ \delta_1(t) = \sqrt{3.0} \times \sin(5\pi t) & , \ \delta_2(t) = -\sqrt{2.0} \times \cos(5\pi t) \end{array} \quad (27)$$

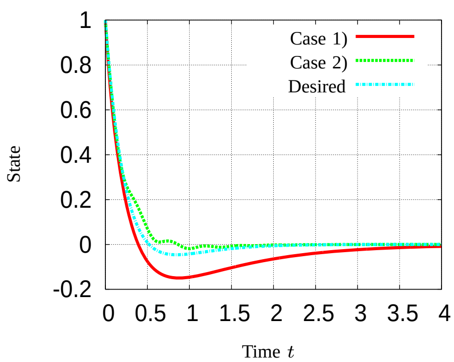

Fig. 1. Time histories of the state $x_1(t)$: Σ_1^*

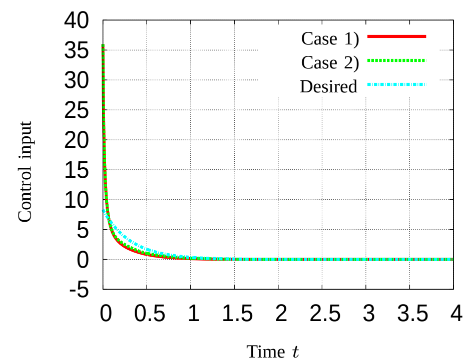

Fig. 3. Time histories of the control input $u(t)$: Σ_1^*

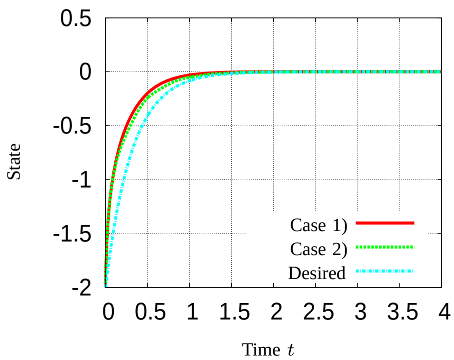

Fig. 2. Time histories of the state $x_2(t)$: Σ_1^*

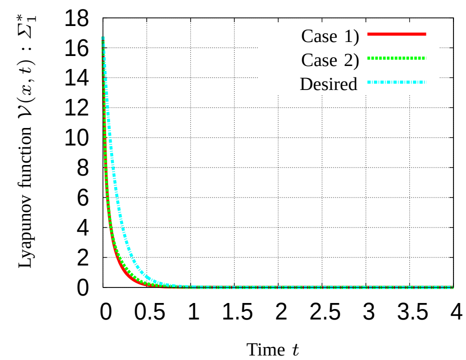

Fig. 4. Time histories of the Lyapunov function $\mathcal{V}(x,t)$: Σ_1^*

V. Illustrative Examples

In order to demonstrate the efficiency of the proposed control scheme, we have run a simple example. Consider the linear system with unknown parameters of (23) and we assume that the parameters σ_1 and σ_2 in the matrix $\Sigma \in \mathbb{R}^{2\times 2}$ in (2) are given by $\sigma_1 = 2.0$ and $\sigma_2 = 5.0 \times 10^{-1}$, respectively.

Firstly we select the weighting matrices $\mathcal{Q}$ and $\mathcal{R}$ such as $\mathcal{Q} = 1.0 \times I_2$ and $\mathcal{R} = 9.0$ for the quadratic cost function for the standard linear quadratic control problem, respectively. Then solving the algebraic Riccati equation of (5), we obtain

$$\begin{aligned} K &= \begin{pmatrix} 2.69892 \times 10^{-2} & -4.17080 \end{pmatrix} \\ \mathcal{P} &= \begin{pmatrix} 1.66545 \times 10^{-1} & 2.69892 \times 10^{-2} \\ \star & 4.17080 \end{pmatrix} \end{aligned} \quad (24)$$

In this example, we consider the following two kinds of the design parameters $\delta \in \mathbb{R}^1$ in (9) .

$$\bullet\ \Sigma_1^* \ : \ \delta = 1.0 \times 10^2, \quad \bullet\ \Sigma_2^* \ : \ \delta = 4.0 \times 10^5 \quad (25)$$

For these design parameters, solving the LMI condition of (9), we have

$$\begin{array}{lll} \bullet\ \Sigma_1^* & : \tau_1 = 1.00121 \times 10^{-7} & , \tau_2 = 3.32679 \times 10^1 \\ \bullet\ \Sigma_2^* & : \tau_1 = 1.0 \times 10^{-7} & , \tau_2 = 4.13381 \times 10^1 \end{array} \quad (26)$$

Now in this example, we consider the two cases for the unknown parameters in (27). Furthermore, the initial value for the uncertain system of (23) and its nominal system are selected as $x(0) = \overline{x}(0) = \begin{pmatrix} 1.0 & -2.0 \end{pmatrix}^T$.

The results of the simulation of this example are depicted in Figures 1 – 8. In these figures, "Case 1)" and "Case 2)" represent the time-histories of the state variables $x_1(t)$ and $x_2(t)$, the control input $u(t)$ and the Lyapunov function $\mathcal{V}(x,t)$. Besides, "Desired" represents the desirable transient behavior, the control input and the time-histories of the Lyapunov function $\mathcal{V}(x,t)$ generated by the nominal system.

From Figures 1 – 4, we find that the proposed variable gain controller (Σ_1^*) achieves good transient performance. However, the proposed control input is excessive comparing with the

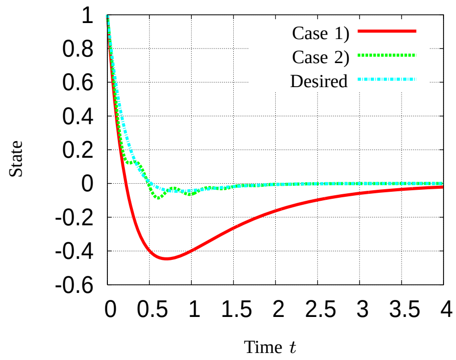

Fig. 5. Time histories of the state $x_1(t)$: Σ_2^*

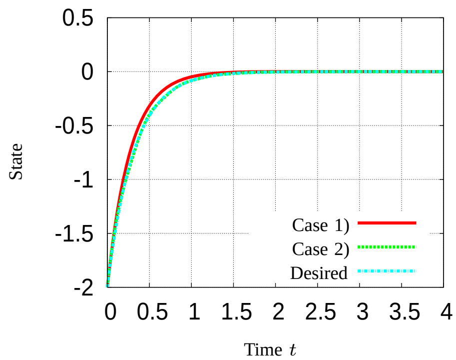

Fig. 6. Time histories of the state $x_2(t)$: Σ_2^*

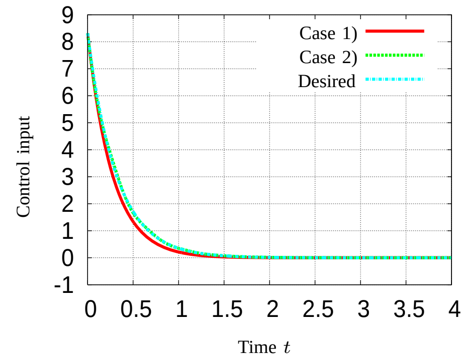

Fig. 7. Time histories of the control input $u(t)$: Σ_2^*

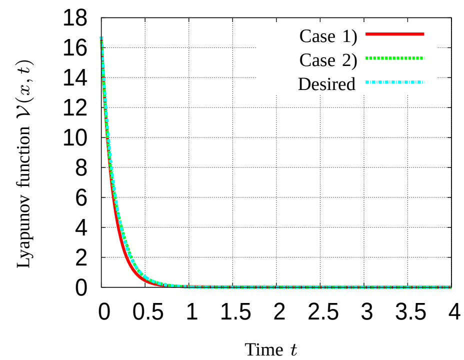

Fig. 8. Time histories of the Lyapunov function $\mathcal{V}(x,t)$: Σ_2^*

nominal system. On the other hand, one can see from Figures 5 – 8 that although the error between the transient response for the proposed controller (Σ_2^*) and the one of the nominal system is large, the control input in Σ_2^* is close to the desired one. Namely, the proposed controller can adjust the transient performance and the control input by means of selecting the design parameter $\delta \in \mathbb{R}^1$ in (10). Therefore the effectiveness of the proposed variable gain controller is shown.

VI. Conclusions

In this paper we have proposed a new variable gain controller for a class of uncertain linear systems. Besides, by numerical simulations, the effectiveness of the proposed controller has been presented. One can see that the crucial difference between the existing results[8], [9] and our new one is that the structure of proposed controller is simple and the proposed variable gain controller can adjust the transient performance and the control input by means of selecting the design parameter.

The future research subjects are an extension of the proposed controller to such a broad class of systems as uncertain large-scale systems, uncertain discrete-time systems, uncertain time-delay systems and so on.

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