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An adaptive sliding mode approach to decentralized control of uncertain systems

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***Abstract*—This paper proposes a systematic adaptive sliding mode controller design for the decentralized system with nonlinear interactions and unmatched uncertainties. An adaptive tuning approach is developed to deal with unknown but bounded uncertainties/interactions. The sliding surface is designed which obviates the use of regular transformation, by solving a simple LMI-based optimization problem. The feasibility of the LMIs is also discussed in this paper. Finally, a numerical example is used to illustrate this method.**

Keywords-Decentralized control, sliding mode, unmatched unccertainties/interactions, adaptive control

I. INTRODUCTION

Decentralized control has gained considerable attention in the literature for two decades. [1]-[9]. The main idea of decentralized control is to use only local information at the level of each subsystem in the controller for large-scale interconnected systems. This feature can overcome the limitations of the traditional centralized control or partial decentralized control [10] that requires large communication bandwidth to exchange information between subsystems and controller. The decentralized control has much simpler control structure and more practical approach than centralized controller.

On the other hand, sliding mode control (SMC), as a powerful robust control method, has been widely researched,[11], [12]. When using sliding mode, there are two steps i). sliding surface design and ii). control law design. The system states will be driven to the sliding surface and be maintained on it. Once the system is running in the sliding surface, the system is insensitive to the matched perturbations (perturbations coming from input channels). Edwards and Spurgeon [11] develop their approach to the classical sliding mode control design algorithm by introducing a so called *"regular form"* to set up a decomposition comprising matched/unmatched state space components. This approach can be considered too complex for a single SMC system and this motivates the use of an alternative approach obviating a need for transformations, whilst still satisfying matching condition properties.

Choi [12],[13] proposed another SMC approach in which the sliding surface can be designed by solving a simple LMI problem. Although no transformations are used Choi's work is focused on SMC of single rather than decentralized control systems. The proposal here is to further develop Choi's approach with application to uncertain decentralized systems.

When applying sliding mode in the decentralized system, researchers consider the interactions as perturbations and try to eliminate or at least attenuate these perturbations. [9], [14]-[15]. However, the sliding mode can only deal with matched perturbations and the upper bound of perturbations should also be known. In this case, researchers start trying to combine the sliding mode with other robust control methods to overcome the unmatched problem limitation [9]. Šiljak [3] proposes a feedback control using an LMI approach that can deal with the interactions no matter whether or not they are matched. Also the assumption of unknown interactions in [3] is more general which includes both linear and non-linear types of interactions. Kalsi [5], [6] uses the control method of [3] to develop a sliding approach to observer-based control. Although a lot of research has been done in this area [9], [14]-[16], the main contribution of this paper is to use the LMI-based work of Choi [12], [13] to construct a simpler and more general approach to decentralized control. This will also provide an efficient strategy for accounting for modeling uncertainty and subsystem interactions. Also the known upper bound limitation is removed by an adaptive mechanism in our work. Although this paper focuses on the state feedback strategy, output feedback control can be easily formulated as an extension based on the proposed method. Moreover, other robust improvements can be extended based on this approach.

This paper is organized as follows. The basic assumptions are given and the design objective is proposed in Section II. Then the main results are given in Section III, where both the sliding surface and control law designs are represented. Also in Section III, the feasibility of LMIs is discussed and more constraints are proposed to improve the methods. A numerical example is given in Section V which demonstrates the efficacy of the techniques developed in this paper. Finally, the concluding remarks and further work are given in Section V.

II. SYSTEM DESCRIPTION AND PROBLEM FORMATION

Consider a class of perturbed large-scale systems which are comprise N-linked subsystems with uncertainties in the interactions. The dynamic equation of each subsystem is represented as:

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$$\dot{x}_i(t) = A_i x_i(t) + B_i(u_i(x_i,t) + f_i(x_i,t)) + h_i(x,t),$$
$$i = 1,2,\dots,N \qquad (1)$$

where $x_i(t) \in R^{n_i}$ is the state variable of the *i*-th subsystem, $u_i(x_i,t) \in R^{m_i}$ is the control input vector of the *i*-th subsystem, the matrix $A_i \in R^{n\times n}$ is the system characteristic matrix, and the matrix $B_i \in R^{n\times m}$ is the input matrix with full rank $m < n$. And $f_i(x,t)$ is any uncertainty or disturbance in the input channel. The term $h_i(x,t)$ reflects the interaction of the *i*-th subsystem with other subsystems and the uncertainty dynamics associated with the *i*-th subsystem itself.

We also assume the followings to be valid:

1) All the pair $(A_i, B_i), i = 1,2,\dots,N$ are controllable.

2) All the state variables x_i, $i = 1,2,\dots,N$ are locally available for measurement for all time.

3) The subsystem interactions are globally bounded, that is, $\|h_i(x,t)\| < \beta_i < \infty$ for some unknown constant $\beta_i > 0$. The interactions satisfy the same quadratic constraint as, for example [3],[5]-[7] , that is:

$$h_i^T(x,t)h_i(x,t) \le \alpha_i^2 x H_i^T H_i x \qquad (2)$$

4) The external disturbance $f_i(t)$ is bounded by a known constant ε_i, i.e. $\|f_i(x,t)\| < \varepsilon_i$

The overall interconnected system can be rewritten in a compact form as:

$$\dot{x}(t) = Ax(t) + B\big(u(x,t) + f(t)\big) + h(x,t) \qquad (3)$$

where $A = diag(A_1,\dots,A_N)$, $B = diag(B_1,\dots,B_N)$, $f(t) = [f_1^T(t),\dots,f_N^T(t)]^{\mathrm{T}}$ and $h(x,t) = [h_1^T(x,t),\dots,h_N^T(x,t)]^{\mathrm{T}}$, and with the third assumption, the interconnections $h(x,t)$ are bounded as follows:

$$h^T(x,t)h(x,t) \le x\left(\sum_{i=1}^{N}\alpha_i^2 H_i^T H_i\right)x = xH^T Hx$$

where α_i is a bounding constant.

Because all of the subsystems are stabilizable, it is easy to verify that the overall system is controllable.

The objective is to design a totally decentralized sliding mode controller that robustly regulates the state of the overall system without any information exchange between the subsystems. And with this type of controllers, the overall system is robust to all the uncertainties and matched perturbations.

III. MAIN RESULT

It is well known that in SMC design, there are two steps: a). Sliding surface design and b) control law design. In this Section, the sliding surface is designed by an LMI and an adaptive control law is realised.

A. Sliding surface design

Define Γ as any basis of the null space of B^T, i.e. Γ is an orthogonal complement of B.

Consider the following LMIs:

Minimize γ_i , subject to $X > 0$,

$$\begin{bmatrix} \Gamma^T(XA^T + AX + I)\Gamma & \Gamma^T XH_1^T & \cdots & \Gamma^T XH_N^T \\ H_1 X\Gamma & -\gamma_1 I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_N X\Gamma & 0 & \cdots & -\gamma_N I \end{bmatrix} < 0 \qquad (4)$$

Assume that the sliding surface σ is given by

$$\sigma(x,t) = B^T X^{-1} x(t) \qquad (5)$$

where X is a solution to the LMIs (4). This sliding surface is proposed in [12]

Remark 1: we should note that $X = diag(X_1,\dots,X_N)$ because that the LMIs (4) is based on the overall system (3). Also the sliding surface $\sigma(x,t) = [\sigma_1^T(x_1,t),\dots,\sigma_N^T(x_N,t)]^T$. The $\sigma_i(x_i,t)$ represents the sliding surface for the *i*-th subsystem.

Theorem 1: Suppose the LMIs (4) have a solution X and the sliding surface is given by Eq. (5). Then once the sliding surface (5) is reached and maintained thereafter, i.e. $\sigma(x,t) = 0$ and $\dot{\sigma}(\mathrm{x},\mathrm{t}) = 0$ and hence the overall system is stable.

Proof: Define a transformation matrix and its inverse matrix as described in [12] as

$$T = \begin{bmatrix}\Gamma^T \\ S\end{bmatrix}, T^{-1} = [X\Gamma(\Gamma^T X\Gamma)^{-1} \quad B(SB)^{-1}] \qquad (6)$$

And the associated vector z is given as

$$z(t) = \begin{bmatrix} z_1(\mathrm{t}) \\ z_2(\mathrm{t})\end{bmatrix} = \begin{bmatrix} z_1(\mathrm{t}) \\ \sigma(t)\end{bmatrix} = Tx(t) \qquad (7)$$

where $z_1 \in \mathrm{R}^{\mathrm{n-m}}$ and $z_2 \in \mathrm{R}^{\mathrm{m}}$. And with the transformation (6), we can obtain a new state equation as:

$$\dot{z}(\mathrm{t}) = TAT^{-1}z(t) + TB(u(t) + f(x,t)) + Th(x,t) \qquad (8)$$

where,

$$TAT^{-1} = \begin{bmatrix}\Gamma^T \mathrm{A} X\Gamma(\Gamma^T X\Gamma)^{-1} & \Gamma^T \mathrm{A}B(SB)^{-1} \\ \mathrm{SA}\Gamma(\Gamma^T X\Gamma)^{-1} & \mathrm{SA}B(SB)^{-1}\end{bmatrix}$$
$$TB = \begin{bmatrix} 0 \\ SB \end{bmatrix}$$

And note that once the system is on the sliding surface and maintained there, $\sigma(t) = 0$, $\dot{\sigma}(\mathrm{t}) = 0$. In this case, the system is insensitive to all the matched uncertainties or disturbances $f(x,t)$. The state equation (8) then becomes:

$$\dot{z}_1(\mathrm{t}) = \Gamma^T A X\Gamma(\Gamma^T X\Gamma)^{-1} z_1(t) + \Gamma^T h(x,t) \quad (9)$$

Now define a Lyapunov function for the system (9):

$$V(z_1) = z_1^T P z_1 \quad (10)$$

where P is a symmetric and positive definite (s.p.d) matrix.

The time derivative of $V(z_1)$ along the trajectories of Eq. (9) is given by:

$$\dot{V}(z_1) = z_1^T((\Gamma^T A X\Gamma(\Gamma^T X\Gamma)^{-1})^T P + P\Gamma^T A X\Gamma(\Gamma^T X\Gamma)^{-1})z_1 + z_1^T P\Gamma^T h + h^T \Gamma P z_1 \quad (11)$$

To obtain a quadratic form of (11), we use the following result in [16]:

$$X^T Y + Y^T X \le X^T X + Y^T Y$$

for any matrices (or vectors) X and Y with appropriate dimension.
It follows that:

$$z_1^T P\Gamma^T h + h^T \Gamma P z_1 \le z_1^T P\Gamma^T \Gamma P z_1 + h^T h \quad (12)$$

Also, from the Assumption 3), the interactions satisfy the quadratic form:

$$h^T(x,t)h(x,t) \le xH^T Hx = z^T(T^{-1})^T H^T H T^{-1} z \quad (13)$$

Because the system is in the sliding surface, $z_2 = \sigma = 0$. We can simplify the above inequality of interactions (13) as

$$h^T(x,t)h(x,t) \le z_1^T(\Gamma^T X\Gamma)^{-1}\Gamma^T X H^T H X\Gamma(\Gamma^T X\Gamma)^{-1} z_1 \quad (14)$$

Substitute the inequalities (12) and (14) into (11), we have a quadratic form of the derivative of the Lyapunov function as

$$\dot{V}(z_1) = z_1^T((\Gamma^T A X\Gamma(\Gamma^T X\Gamma)^{-1})^T P + P\Gamma^T A X\Gamma(\Gamma^T X\Gamma)^{-1} + P\Gamma^T \Gamma P + (\Gamma^T X\Gamma)^{-1}\Gamma^T X H^T H X\Gamma(\Gamma^T X\Gamma)^{-1})z_1 \quad (15)$$

The stabilization of system (9) requires that:

$$\dot{V}(z_1) < 0 \quad (16)$$

for all $z_1 \neq 0$.

The development of (16) leads to

$$(\Gamma^T A X\Gamma(\Gamma^T X\Gamma)^{-1})^T P + P\Gamma^T A X\Gamma(\Gamma^T X\Gamma)^{-1} + P\Gamma^T \Gamma P + (\Gamma^T X\Gamma)^{-1}\Gamma^T X H^T H X\Gamma(\Gamma^T X\Gamma)^{-1} < 0 \quad (17)$$

In this case, the problem is to find an s.p.d matrix P such that the inequality (17) is satisfied. By pre-multiplying and post-multiplying the inverse matrix of P, we can simplify the inequality (17).

Define $Y = P^{-1}$, we have:

$$Y(\Gamma^T A X\Gamma(\Gamma^T X\Gamma)^{-1})^T + \Gamma^T A X\Gamma(\Gamma^T X\Gamma)^{-1} Y + \Gamma^T \Gamma + Y(\Gamma^T X\Gamma)^{-1}\Gamma^T X H^T H X\Gamma(\Gamma^T X\Gamma)^{-1} Y < 0$$

We can easily find a matrix $Y = \Gamma^T X\Gamma$ which satisfies the s.p.d condition so that the above inequality becomes:

$$\Gamma^T(AX + XA^T + I + XH^T HX)\Gamma < 0 \quad (18)$$

With Assumption 3), we rewrite the inequality (18) as:

$$\Gamma^T\left(AX + XA^T + I + \sum_{i=1}^{N}(\alpha_i^2 X H_i^T H_i X)\right)\Gamma < 0$$

By defining $\gamma_i = \frac{1}{\alpha_i^2}$ and using the Schur complement, the above inequality can be rewritten as the form of LMIs (4). □

After solving the LMIs (4), we have the s.p.d matrix X which has the structure of $X = diag(X_1, \dots, X_N)$. In this case, for each subsystem, the local sliding surface could be given by

$$\sigma_i(x_i,t) = S_i x_i(t) = B_i^T X_i^{-1} x_i(t) \quad (19)$$

B. Adaptive Control law design

It has proved above that once the system reaches the sliding surface and is maintained on it thereafter, the control objective is achieved. In the following a sliding control law is designed to drive the system to the sliding surface.

Introducing the control strategy for each subsystem:

$$u_i(x_i,t) = \begin{cases} -(S_i B_i)^{-1} A_i x_i(t) + \Psi_i \frac{\sigma_i}{\|\sigma_i\|}, & \|\sigma_i\| \neq 0 \\ -(S_i B_i)^{-1} A_i x_i(t), & \|\sigma_i\| = 0 \end{cases} \quad (20)$$

where,

$$\Psi_i = -\eta_i - \|(SB)^{-1} S\|\hat{\beta}_i(t), \;\; \dot{\hat{\beta}}_i(t) = \|\sigma_i\| \quad (21)$$

where, $\eta_i > \varepsilon_i$ is a positive constant. $\hat{\beta}_i$ is the estimation of the upper bound of the interaction of *i*-th subsystem.

Theorem 2: For each subsystem of the form (1) and the sliding surface (19), by applying the control law (20) and (21) to each subsystem, the overall system trajectory converges to the sliding surface $\sigma_i(x_i,t) = 0$ in finite time and is maintained on the surface, i.e. $\dot{\sigma}_i(x_i,t) = 0$. Meanwhile, the system is insensitive to all the matched disturbances.

Proof: To prove the reachability, we define the Lyapunov function with respect to the sliding surface as:

$$V = \sum_{i=1}^{N}\frac{1}{2}\left[\sigma_i^T(S_i B_i)^{-1}\sigma_i + \|(S_i B_i)^{-1} S\|\bar{\beta}_i^{\,2}\right] \quad (22)$$

The matrix $(S_i B_i)^{-1}$ satisfies the s.p.d condition since that X_i^{-1} is a s.p.d matrix and $(S_i B_i)^{-1} = (B_i^T X_i^{-1} B_i)^{-1} > 0$.
Moreover, we define the estimation error $\bar{\beta}_i(t) = \hat{\beta}(t) - \beta_i$.
Since β_i is a constant, $\dot{\bar{\beta}}_i(t) = \dot{\hat{\beta}}_i(t) = \|\sigma_i\|$.
Differentiating (19) with respect to time yields:

$$\dot{\sigma}_i(x_i,t) = S_iA_ix_i(t) + S_iB_i(u_i(x_i,t) + f_i(t)) + S_ih_i(x,t)$$

Then taking the derivative of the Lyapunov function (22) we have:

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N}\Big\{\sigma_i^T\Big[\Psi_i\frac{\sigma_i}{\|\sigma_i\|} + f_i(t) + (S_iB_i)^{-1}Sh_i(x,t)\Big] \\ &\qquad + \|(S_iB_i)^{-1}S\|\bar{\beta}_i\dot{\hat{\beta}}_i\Big\} \\ &\le \sum_{i=1}^{N}\Big\{\sigma_i^T\Big[\Psi_i\frac{\sigma_i}{\|\sigma_i\|} + \varepsilon_i + (S_iB_i)^{-1}Sh_i(x,t)\Big] \\ &\qquad + \|(S_iB_i)^{-1}S\|\bar{\beta}_i\|\sigma_i\|\Big\}\end{aligned} \tag{23}$$

We can easily verify that $\sigma_i^T\Psi_i\frac{\sigma_i}{\|\sigma_i\|} = \Psi_i\|\sigma_i\|$, and on substituting (21) into (23), we have:

$$\begin{aligned}\dot{V} &\le \sum_{i=1}^{N}\{\Psi_i\|\sigma_i\| + \|\sigma_i\|\varepsilon_i + \|\sigma_i\|\|(S_iB_i)^{-1}S\|\beta_i \\ &\qquad + \|(S_iB_i)^{-1}S\|(\hat{\beta}(t) - \beta_i)\|\sigma_i\|\} \\ &\le \sum_{i=1}^{N}\{-\eta_i\|\sigma_i\| - \|\sigma_i\|\|(SB)^{-1}S\|\hat{\beta}_i(t) + \|\sigma_i\|\varepsilon_i \\ &\qquad + \|(S_iB_i)^{-1}S\|\beta_i \\ &\qquad + \|(S_iB_i)^{-1}S\|(\hat{\beta}(t) - \beta_i)\|\sigma_i\|\} \\ &\le \sum_{i=1}^{N}(-(\eta_i - \varepsilon_i)\|\sigma_i\|) < 0\end{aligned} \tag{24}$$

which implies that the overall system will reach the sliding surface in finite time [11] and be maintained on it.

Remark 2: Following [15], with the adaptive mechanism, we do not need to know the upper bound of the interaction. It is worthwhile noting that for the purpose of reducing the "chattering" around the switching surface, a commonly used approach is to substitute $\frac{\sigma_i}{\|\sigma_i\|+\theta_i}$ in the SMC for $\frac{\sigma_i}{\|\sigma_i\|}$, where θ_i is a small positive constant (the so called boundary layer) [11]. However, if we use the boundary layer method here, $\hat{\beta}_i(t)$ may keep growing in magnitude during sliding and the chattering will not be reduced since $\dot{\hat{\beta}}_i(t) \ne 0$. In this case, we should modify our controller (20) to:

$$u_i(x_i,t) = \begin{cases} -(S_iB_i)^{-1}A_ix_i(t) + \Psi_i\frac{\sigma_i}{\|\sigma_i\|+\theta_i}, & \|\sigma_i\| > \theta_i \\ -(S_iB_i)^{-1}A_ix_i(t), & \|\sigma_i\| < \theta_i \end{cases} \tag{25}$$

C. Feasibility of LMIs and improvement

A lot of literature only describe the LMIs and do not prove the feasibility of the LMIs. It is always reasonable to prove the feasibility when raising an LMI problem. To prove the LMIs (4), we should introduce a preliminary Lemma:

Lemma 1 [13]: Given a symmetric matrix $G \in R^{n\times n}$ and two matrices $U \in R^{p\times n}$ and $V \in R^{n\times m}$ where $p,m < n$. Consider the problem of finding some matrix *K* such that:

$$G + UKV^T + VK^TU^T < 0$$

Denote by $\tilde{U}$ and $\tilde{V}$ the orthogonal complements of U and V. Then the above inequality is solvable for *K* if and only if

$$\tilde{U}^TG\tilde{U} < 0, \tilde{V}^TG\tilde{V} < 0$$

And the feasibility of the LMIs (4) is given by the following Lemma 2.

Lemma 2: The optimization problem given by LMIs (4) is feasible if the pairs (A_i, B_i) are controllable for all the subsystems.

Proof: Because the LMIs (4) are based on the overall system equation (3), we first need to prove the controllability of the overall system (3) It then follows that the overall system is controllable as all of the overall system eigenvalues are changeable by the local inputs, i.e. there are no fixed modes in this decentralized system. [1] Assume there is no actuator disturbance or no interactions in the overall state equation (3). In this case, there exists a control law $u = Kx$ if and only if there exist an s.p.d matrix *X* such that:

$$XA^T + AX + XK^TB + BKX < 0 \tag{26}$$

As a consequence of the system controllability, we can always find a gain matrix *K* and an s.p.d matrix *X* satisfying (25), Moreover, we can restrict (26) to:

$$XA^T + AX + I + XK^TB + BKX < 0 \tag{27}$$

There also still exists a *K* and an s.p.d matrix *X* satisfying (27). Then by using *Lemma 1*, the following inequality is feasible:

$$\Gamma^T(XA^T + AX + I)\Gamma < 0 \tag{28}$$

Using the Schur complement in the LMIs (4):

$$\Gamma^T(XA^T + AX + I)\Gamma + \Gamma^T\textstyle\sum_{i=1}^{N}(\frac{1}{\gamma_i}XH_i^TH_iX)\,\Gamma < 0 \tag{29}$$

Therefore, from (28) and (29), the solution of the LMIs (4) are guaranteed by the existence of a set of large enough γ_i □

The LMI optimization problem given by (4) does not pose any restriction on the size of the matrix *X*. Consequently, the results of these two optimization problems may yield inappropriate results. For example, very small γ_i and *X* result in very large *S* values. In this case, we can restrict *X* by posing a further constraint on γ_i as:

$$\gamma_i > \frac{1}{\bar{\alpha}_i^2}, \bar{\alpha}_i > 0 \tag{30}$$

$\bar{\alpha}_i$ are given constants for $i = 1,2,\ldots,N$. Or equivalently:

$$X > \Lambda \tag{31}$$

where, $\Lambda = \mathrm{diag}(\kappa_1 I_{n_1}, \ldots, \kappa_N I_{n_N})$, $\kappa_i > 0, i = 1,\ldots,N$ are given constants. The upper bound of the parameters could also be defined in the same way.

Remark 3: We should note that if the constants $\alpha_i, i = 1,\ldots,N$ are known, the optimization problem (4) becomes feasibility problem. By substituting $\gamma_i = 1/\alpha_i^2$ into (4), the solution of the LMIs is equivalent to finding a feasible solution *X*. However, the use of an optimization algorithm could still be appropriate to find a solution capable of dealing with potentially stronger interactions.

For the purpose of improving the performance, here we introduce a modification to the LMIs in (4). By adding a μ-stability constraint, the system satisfies $lim_{t\to\infty} e^{\mu t}\|x(t)\| = 0$ for all solution trajectories *x*. Therefore, we can ensure a minimum positive decay rate μ >after arriving sliding surface. Moreover, the larger the rate is, the earlier the sliding surface could be reached. Hence, the Eq. (4) could be rewritten as:

Minimize γ_i, subject to $X > 0$,

$$\begin{bmatrix} \Gamma^T(X\bar{A}^T + \bar{A}X + I)\Gamma & \Gamma^T X H_1^T & \cdots & \Gamma^T X H_N^T \\ H_1 X\Gamma & -\gamma_1 I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_N X\Gamma & 0 & \cdots & -\gamma_N I \end{bmatrix} < 0 \tag{32}$$

where, $\bar{A} = A + \mu I$. Other improvement of robust performance for the system (e.g. H_∞, H_2, etc.) could also be constructed based on this LMI approach.

IV. Numerical Example

In this Section, we illustrate the performance of the proposed decentralized controller with an example similar to the one used in [7]. This non-linear interconnected system model consists of three subsystems. The first subsystem is a second-order system and the others are third-order systems. Disturbance signals are also added to each subsystem.

The dynamic subsystems are given by:

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u_1 + f_1(x_1,t)) + h_1(x,t)$$

$$\dot{x}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (u_2 + f_2(x_2,t)) + h_2(x,t)$$

$$\dot{x}_3 = \begin{bmatrix} -3 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & -2 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} (u_3 + f_3(x_2,t)) + h_3(x,t)$$

where, $x_1 = [x_{11} \quad x_{12}]^T$, $x_2 = [x_{21} \quad x_{22} \quad x_{23}]^T$, $x_3 = [x_{31} \quad x_{32} \quad x_{33}]^T$

$$f_1(x_1,t) = 0.4\sin(x_{11}) + 0.5\sin(10t),$$

$$f_2(x_1,t) = 0.3\cos(x_{22}) + 0.6\sin(5t),$$

$$f_3(x_1,t) = 0.5\cos(x_{33}) + 0.6\sin(7t)$$

$$h_1(x,t) = \alpha_1 \cos(x_{22}) H_1 x,$$

$$h_2(x,t) = \alpha_2 \cos(x_{32}) H_2 x,$$

$$h_3(x,t) = \alpha_3 \cos(x_{11}) H_3 x$$

$$\alpha_1 = \alpha_2 = \alpha_3 = 0.1,$$

$$H_1 = \frac{1}{\sqrt{10}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$H_2 = \frac{1}{\sqrt{15}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$H_3 = \frac{1}{\sqrt{13}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Choosing $\bar{\alpha}_1 = \bar{\alpha}_2 = 0.2$ and solving the optimization problem (4) and (29) results in

$$X_1 = \begin{bmatrix} 0.515 & -0.5 \\ -0.5 & 0.515 \end{bmatrix} X_2 = \begin{bmatrix} 1.454 & -0.515 & -0.722 \\ -0.515 & 0.738 & -0.514 \\ -0.722 & -0.514 & 1.451 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} 0.221 & -0.112 & -0.013 \\ -0.112 & 0.184 & -0.138 \\ -0.013 & -0.138 & 0.228 \end{bmatrix}$$

with this solution, we have the sliding surface matrices for each of the subsystems:

$$S_1 = [32.94 \quad 33.94]$$
$$S_2 = [36.143 \quad 50.744 \quad 36.627]$$
$$S_3 = [15.772 \quad 28.942 \quad 18.459]$$

Using the control law (25) and choosing $\theta_i = 0.01$ and $\eta_i = 5$ for $i = 1,2$. The initial conditions $x_1(0) = [0.5 \quad 0.5]^T$, $x_2 = [0.5 \quad 0.5 \quad 0.5]^T$. $x_3 = [0.5 \quad 0.5 \quad 0.5]^T$

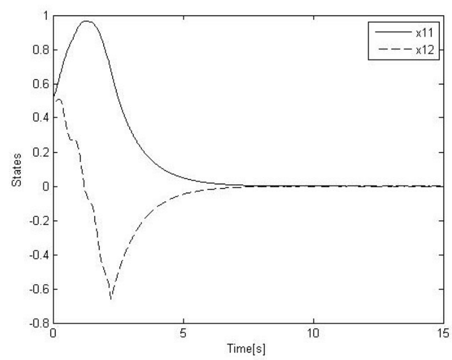

Figure 1. State responses for subsystem 1

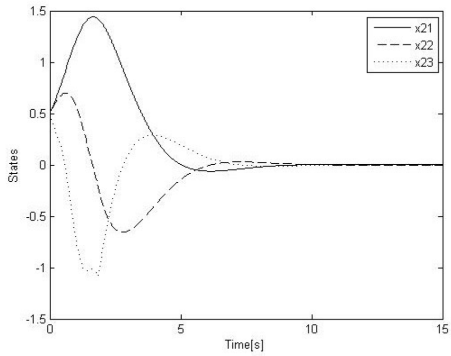

Figure 2. State responses for subsystem 2

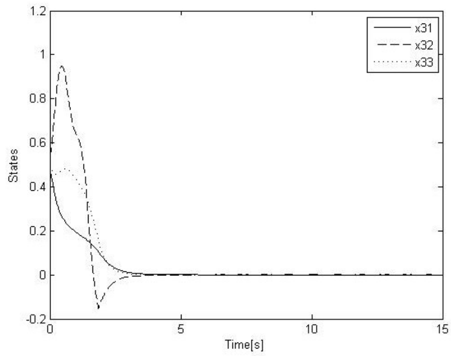

Figure 3. State responses for subsystem 3

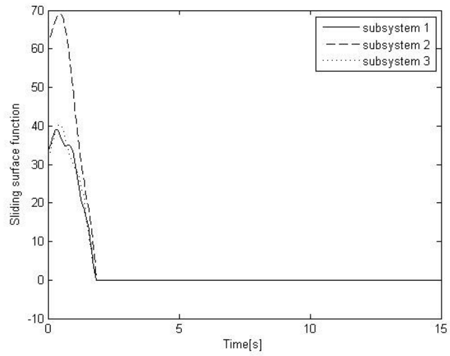

Figure 4. Sliding surface function for both subsystems

Figures 1-2 show the state responses for the system. From Figure 4, the sliding surface is reached in finite time. Moreover, combine with Figure 3, we note that after reaching the sliding surface, the system is stable and insensitive to the disturbances. However, in the reaching phase, the system is sensitive to the disturbances. This is also one of the main disadvantages of sliding mode theory.

V. CONCLUSION

This paper proposes a simple and easy way to design a decentralized sliding mode system. The difference between the proposed approach and other decentralized sliding mode methods is that, this method requires the solution of only one LMI optimization problem in the sliding surface design, without a requirement for any unmatched and matched separation analysis. The mismatched uncertainties and interactions are handled well by the LMI approach. Meanwhile, all of the matched uncertainties and external disturbances are rejected by the SMC. The upper bound limitation of the uncertainties/interaction in the classical sliding mode design is removed by the adaptive mechanism. Other robust control methods could be easily extended based on this LMI approach.

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