DECENTRALIZED CONTROL: SOME ASPECTS OF STABILITY AND PERFORMANCE

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Abstract: The paper studies some basic aspects of decentralized control design concerning stability and performance. Control structure selection based on performance relative gain array (PRGA, Hovd, Skogestad, 1992) is used and its ability to evaluate the achievable performance is discussed. Robust stability condition for decentralized control is considered, which provides the upper level on subsystems, thus limiting the performance. The compromise between robust stability and performance is illustrated on example and provides material which can be used in teaching complex systems control.

Keywords: control system design, decentralized control, robust stability

1 INTRODUCTION

Several important aspects of decentralized control design are studied in this paper. The main aim is to design appropriate decentralized control, so that the overall system stability is kept and the required performance specifications are achieved.

Decentralized control design comprises several steps and tasks (Skogestad, 2005):
- study the plant – system to be controlled and formulate the control objective;
- find a plant model, simplify it if necessary;
- analyze the model properties, scale the variables;
- decide which variables are to be controlled and which variables are to be the manipulated ones;
- select the control configuration: for decentralized control structure it means to choose the input – output pairing;
- specify the performance requirements respective to the control objective;
- determine the type of controller and design its parameters;
- examine the resulting control system, if the specified requirements are not met, redesign;
- analyze simulation resluts, if necessary repeat the whole procedure;
- realize the designed controller.

In this paper we concentrate on key aspects of decentralized control design: control structure selection with appropriate input-output pairing and resulting single loops design so that it guarantees stability as well as required performance of the overall system including interactions. Two forms of stability condition for decentralized control structure are used and discussed on the example: one based on small gain theorem and one for systems with no RHP (right half plane) zeros. Standard interaction measure used for control structure selection is the relative gain array (RGA), nevertheless, performance relative gain array PRGA (performance relative gain array - closely related to RGA) is used in this paper, since besides input-output pairing it enables to evaluate the achievable performance (Hovd, Skogestad, 1992). The use of these design tools is illustrated and analysed on case study (decentralized control design for quadruple tank model). The interesting part of simulation experiments is connected with non-minimum phase case, where the single loops are minimum phase, however, the interconnected system includes transmission RHP zero, which inherently limits the required performance. The obtained results show the limitations following from overall stability condition and the influence of interconnections on overall system stability and performance. Presented material provides simple illustration of stability versus performance relationship in decentralized control design and can be used in teaching complex systems control.

2 PROBLEM FORMULATION AND PRELIMINARIES

Consider a multi-input multi-output plant described by linear MIMO system model

$$y(s) = G(s)u(s) \tag{1}$$

where complex vectors $y(s), u(s)$ are Laplace images of output and input signal of dimensions p and m respectively, $G(s)$ is transfer function matrix of dimensions $p \times m$. In the following we assume the square system model, i.e. $p = m$ and stable plant G.

Our aim is to design appropriate decentralized control, so that the overall system stability is kept (including possible uncertainties) and the required performance is achieved. In this paper we focus on two most important steps in decentralized control design:

1. determining control configuration, which means to choose appropriate input-output pairing;
2. the respective single control loops design so that the overall requirements are kept.

After completing step 1, the inputs or outputs can be reordered, so that the respective transfer system matrix G with reordered columns or rows has the paired elements on the main diagonal. Then the decentralized controller can be represented by the diagonal matrix $K(s) = diag(k_{ij})$. To find $K(s)$ (step 2), the so called independent design is considered, where individual loops are designed "independently" (simultaneously). In other words, local controllers $k_{ij}(s)$ are designed so that they:

a) stabilize individual loops

b) satisfy the overall system stability condition

c) satisfy the bounds obtained from performance requirements.

Note that conditions in b) and c) may be contradictory (as illustrated later in the presented example).

In the following, sensitivity is denoted as $S(s) = (I + G(s)K(s))^{-1}$ and closed loop transfer function is denoted as $T(s) = G(s)K(s)(I + G(s)K(s))^{-1}$. Argument s is often omitted for better readability.

2.1 Control configuration (pairing) selection

To choose appropriate pairing, several interaction measures have been proposed in literature (RGA, dRGA, PRGA, etc.), more details can be found e.g. in (Schmidt, 2002). Relative gain array (RGA), frequently used in practice, is defined as

$$RGA(G) = G(s) \circ \left(G(s)^{-1}\right)^{T} \tag{2}$$

where $\circ$ denotes entrywise matrix product (Hadamard product).

However, RGA index provides no information e.g. for the system with one way interconnections (when the transfer function matrix is upper or lower triangular). To better evaluate system performance, PRGA index has been introduced and shown to provide performance limits for system with decentralized control (Hovd, Skogestad, 1992). PRGA is defined as

$$PRGA(G) = \Gamma = G_D(s)\, G(s)^{-1} \tag{3}$$

where $G_D(s) \approx diag(G(s))$ denotes the diagonal matrix having diagonal elements of $G(s)$ on its diagonal. The relationship between PRGA and closed loop system performance can be summarized as follows. Let us specify the required closed loop performance by bounds on control error (offset) and disturbance

$$\left|e_i(j\varpi)/r_j(j\varpi)\right| = \left|S_{ij}(j\varpi)\right| < 1/\left|w_{ri}(j\varpi)\right| \quad \forall \varpi, i, j \tag{4a}$$

$$\left|e_i(j\varpi)/z_k(j\varpi)\right| = \left|[SG_z]_{ik}(j\varpi)\right| < 1/\left|w_{zi}(j\varpi)\right| \quad \forall \varpi, i, k \tag{4b}$$

where r_j denotes setpoint change, S_{ij} is the respective element of sensitivity function S, z_k is the expected disturbance and G_z its transfer function; w_{ri} and w_{zi} are scalar performance weights for control error and disturbance respectively.

For frequencies, where a feedback is effective ($\varpi < \varpi_B, \varpi_B$ denotes bandwith), it can be usually assumed $S = (I + GK)^{-1} \approx (GK)^{-1}$ yielding the following bounds for individual loops

$$\left|g_{ii}k_i(j\varpi)\right| > \left|\gamma_{ij} w_{ri}(j\varpi)\right| \quad \forall \varpi < \varpi_B, \forall i,j \text{ , } \gamma_{ij} \text{ are elements of PRGA index } \Gamma \tag{5a}$$

$$\left|g_{ii}k_i(j\varpi)\right| > \left|\delta_{ik} w_{zi}(j\varpi)\right| \quad \forall \varpi < \varpi_B, \forall i,k \qquad \delta_{ik} \text{ are elements of } \Gamma G_z \tag{5b}$$

The above inequalities determine performance limits - lower bounds on single loop modules to achieve the required control error and disturbance attenuation and will be discussed in control design stage.

2.2 Stability condition for decentralized control

After the appropriate pairing has been determined, the decentralized control law is to be designed. There are various approaches to find decentralized control law. We adopt individual design as simple possibility to design single loops so that the overall stability and performance requirements are kept, i.e. that interactions do not introduce instability and do not significantly deteriorate performance. Let us consider stability condition for system with decentralized control. Matrix G can be splitted into its diagonal and off-diagonal parts: $G = G_D + G_M$. For stable open loop system GK, the closed loop system stability condition based on small gain theorem is given in the next Lemma (Veselý, Harsányi, 2008).

Lemma 1

Consider stable system G with decentralized controller K. The respective closed loop system $T(s)$ is stable if

$$\left\|G_D^{-1}W\right\|\left\|G_M\right\| < 1 \tag{6a}$$

or

$$\left\|G_D^{-1}W\right\| < \frac{1}{\left\|G_M\right\|} \tag{6b}$$

where matrix W is given by $R^{-1} + G_D = G_D W^{-1}$ (7)

Inequality (6b) can be reformulated into

$$\left\|G_D^{-1}T_D\right\| < M_0 = \frac{1}{\left\|G_M\right\|} \tag{8}$$

where $T_D = G_D K(I + G_D K)^{-1}$.

Condition (8) can be used for stable system without or with RHP zeros (both for minimum and non-minimum phase case). Note that the above condition can be rather limiting in low frequencies, where $\left\|T_D\right\| \approx 1$, for stable system with no RHP zeros this may be too restrictive. The less restrictive condition for this case can be found in (Skogestad, 2005).

Lemma 2

Consider stable system G with decentralized controller K. Assuming that neither G nor G_D has RHP zeros, the overall closed loop system is stable if and only if $(I - ES_D)^{-1}$ is stable, where $E = (G - G_D)G^{-1} = G_M G^{-1}$, $S_D = (I + G_D K)^{-1}$.

The above condition can be reformulated: $(I - ES_D)^{-1}$ stable means $\det(I - ES_D)^{-1} \neq 0$. The sufficient stability condition is then $\left\|ES_D\right\| < 1$, or

$$\left\|G^{-1}S_D\right\| < M_0 = \frac{1}{\left\|G_M\right\|}. \tag{9}$$

Note that whichever condition is used, (8) or (9), it must be satisfied for all frequencies.

3 DECENTRALIZED CONTROL DESIGN: CASE STUDY

The decentralized control design approach based on stability condition (8) or, alternatively, (9) and performance bound (5a) is used to find decentralized control guaranteeing overall system stability and bounded control error (4a). The design procedure is illustrated on the case study - decentralized control of quadruple tank process: input variables are two input flows, output variables are water levels in lower two tanks, detailed description of the plant can be found in (Johansson, 2000), (Rosinová and Markech, 2008). The controlled system is described by the linearized model

$$G(s) = \begin{bmatrix} \dfrac{2.69\gamma_1}{47.11s+1} & \dfrac{2.69(1-\gamma_2)}{(47.11s+1)(73.42s+1)} \\ \dfrac{4.39(1-\gamma_1)}{(87.81s+1)(76.05s+1)} & \dfrac{4.39\gamma_2}{87.81s+1} \end{bmatrix} \tag{10}$$

parameters γ_1, γ_2 determines the input flow split between the lower and upper tank, $0 \le \gamma_1, \gamma_2 \le 1$. It is important to note that depending on values of γ_1, γ_2, two different plant configurations can be obtained:

A: minimum phase configuration for $1 < \gamma_1 + \gamma_2 < 2$ (in this case we consider γ_1=0.6, γ_2=0.8) and

B: non-minimum phase configuration for $0 < \gamma_1 + \gamma_2 < 1$ (in this case we consider γ_1=0.2, γ_2=0.3).

In the first step, the appropriate input-output pairing is determined using RGA, or PRGA index.

In steady state we have RGA index dependent only on values of γ_1, γ_2

$$RGA = G(0) \circ \left[G(0)^{-1}\right]^T \cong \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix}, \quad \lambda = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 - 1} \tag{11}$$

From (10), obviously the adequate pairings are: $u_1 - y_1$, $u_2 - y_2$ for configuration A, $u_1 - y_2$, $u_2 - y_1$ for configuration B (λ is negative). This choice is supported by frequency dependent RGA index shown in Fig.2a, 2b. In both cases, decentralized control is designed independently for both loops, decentralized PID controller is

$$K(s) = \begin{bmatrix} k_1(s) & 0 \\ 0 & k_2(s) \end{bmatrix} \tag{12}$$

$$k_i(s) = P_i + I_i / s + D_i s \qquad i=1, 2 \tag{13}$$

Decentralized control design strategy is following:

When designing parameters of $k_i(s)$, in the first step we consider stability criterion (8) or (9) as a bound on loops responses, next step is to shape the loops responses within stability bounds to achieve the required performance specifications "measured" by performance bound (5a).

The application of this strategy on the case study illustrates also the case, when the individual loops design follows only loops criteria without considering overall stability.

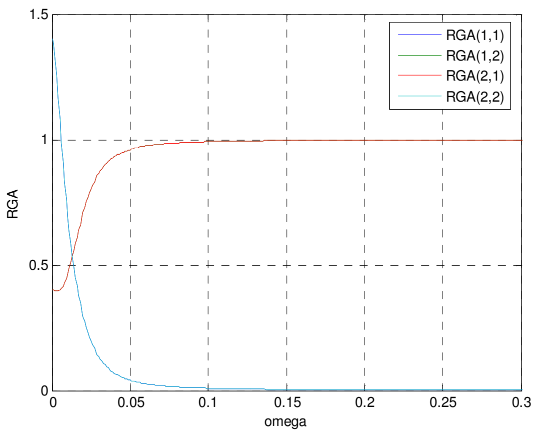

Figure 2a: RGA for minimum phase case (γ_1=0.6, γ_2=0.8)

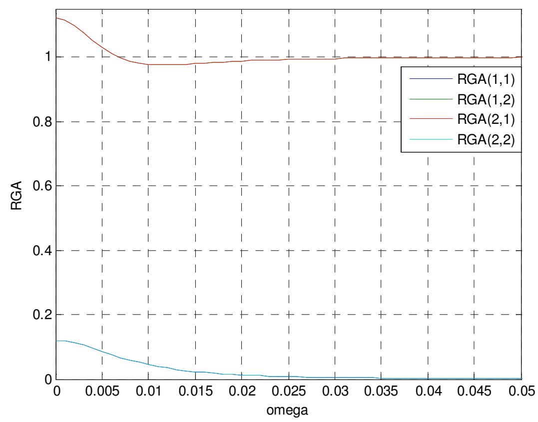

Figure 2b: RGA for non-minimum phase case (γ_1=0.2, γ_2=0.3)

In decentralized control design below, the results are evaluated according to the respective plots. The overall stability condition (8) or (9) is examined from the plots of left hand side and right hand side moduli: red line denotes the upper bound (right hand side of inequality), to satisfy the inequality (8) or (9), the respective blue or green lines should be for all frequencies below the red one.

The performance criterion is checked through the plots respective to w_{ri} from (5a); recall that $1/w_{ri}$ provides the upper bound on control error (within the frequency range, where the feedback is effective), e.g. w_{ri} = 20 corresponds to control error less than 5%.

3.1 Decentralized control for minimum phase configuration A

The results for PI controller parameters designed for individual loops: P1=2.917, I1=0.0619; P2=2.50, I2=0.0285 are shown in Fig.3 and 4. The left part of Fig. 3 shows that stability condition (8) is not satisfied in this case – blue line is for low frequencies above the red one, however, the overall stability is guaranteed by (for this case) the less restrictive condition (9), which is satisfied since green line is below the red one for all frequencies. The respective step responses show minor differences between individual loop (lower plots) and the overall system (upper plots) performance.

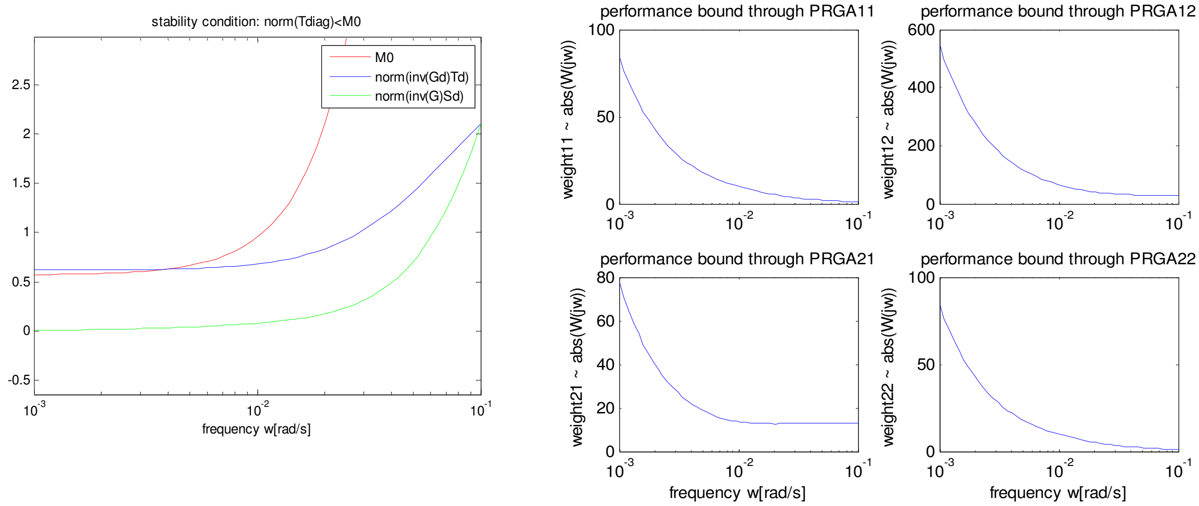

Figure 3: Decentralized control design bounds: stability condition (satisfied), performance bound

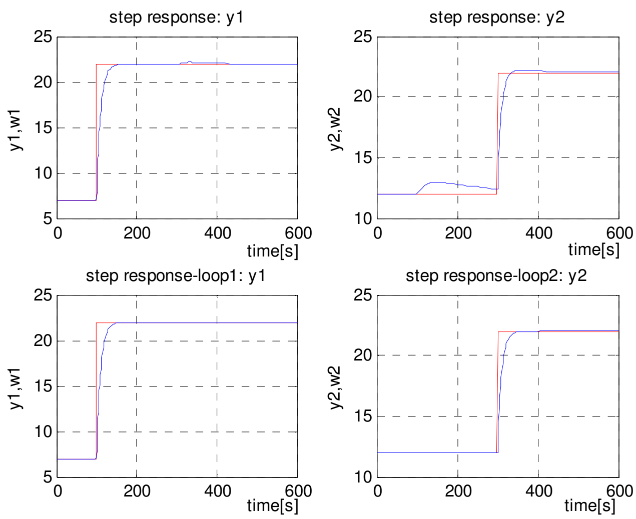

Figure 4: Step responses: interconnected system – upper plots
individual loops – lower plots

3.2 Decentralized control for non-minimum phase configuration B

This configuration is characterized by the existence of transient RHP zeros (while individual transfer functions have no RHP zeros), which complicates the decentralized controller design. We illustrate the impact of interactions on three different designs of control loops.

In the first case the overall stability condition is not satisfied, though the individual loops have satisfactory performance. PI controller parameters: P1=0.9360, I1=0.0156; P2=2.7725, I2=0.0210. As shown in Fig. 5, the overall system stability condition (8) is not satisfied (around the bandwidth frequency), therefore performance indicators have no reason. Step responses in Fig. 6 show the significant differences between individual loops (both are stable and damped) and the overall system, which is unstable.

The next case (P1= 0.4680, I1= 0.0052; P2= 1.1090, I2= 0.0084) shows that as soon as condition (8) is satisfied (Fig. 7: blue line below red one), the overall system responses are similar to single loops ones – Fig. 8; performance indicators are still satisfactory for low frequencies (the right part of Fig. 7).

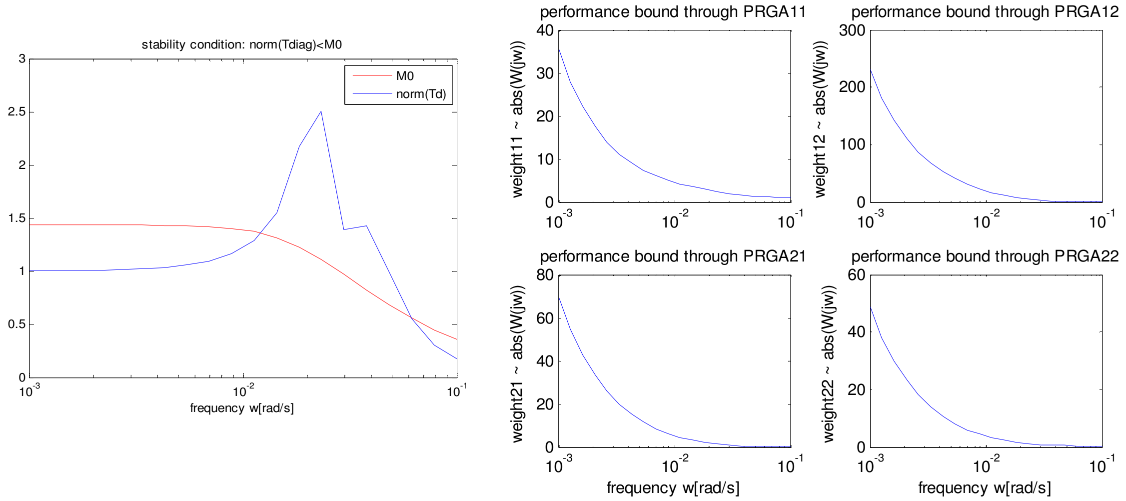

Figure 5: Decentralized control design bounds: stability condition (not satisfied), performance bound indicates satisfactory result

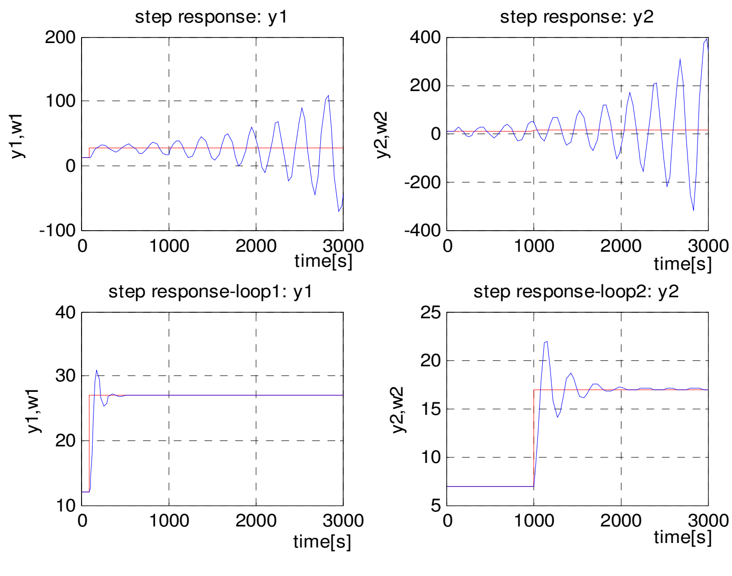

Figure 6: Step responses: interconnected system – upper plots
individual loops – lower plots; big difference between individual loops and overall system

The third case shows results after the redesign of loop controllers parameters, note that the respective performance bounds – Fig. 9 are worse than in previous case – Fig. 7. Corresponding step responses in Fig. 10 are slower than in previous case.

PI controller parameters were designed for individual loops, considering the overall stability condition (8): P1=0.2847, I1=0.0025; P2=0.5308, I2=0.0063.

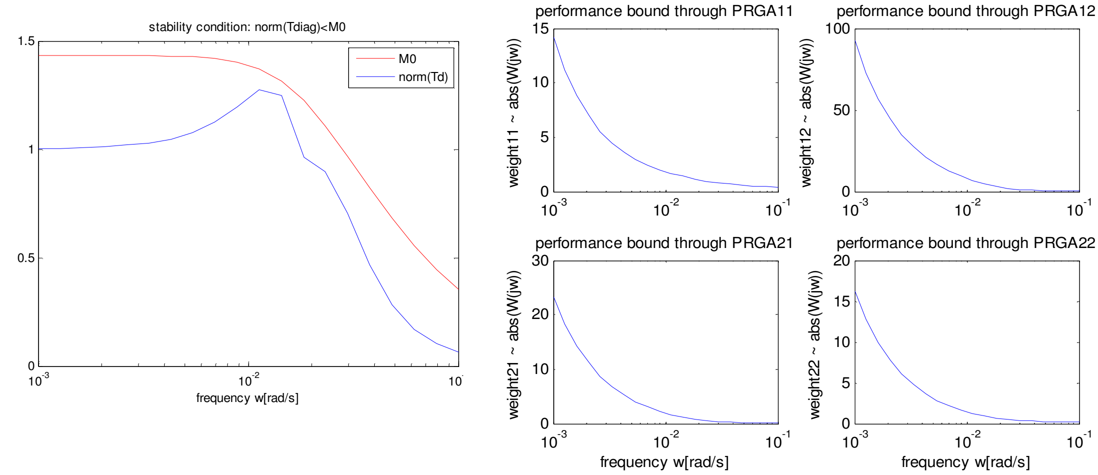

Figure 7: Decentralized control design bounds: stability condition (satisfied), performance bound

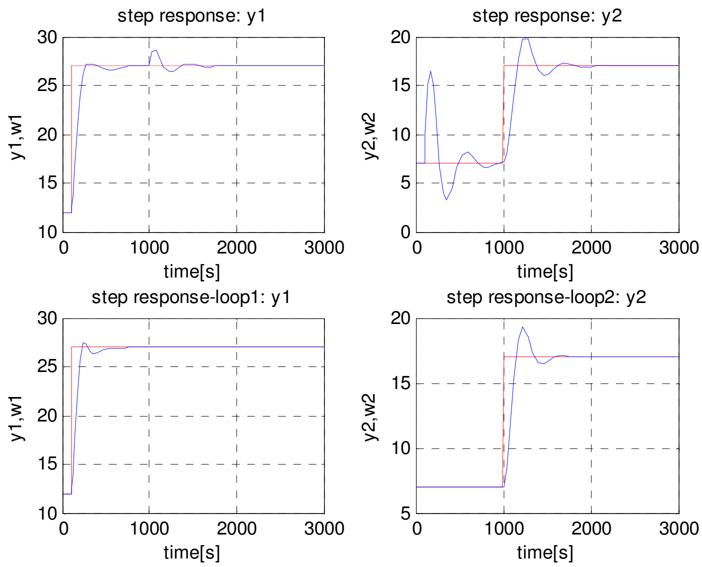

Figure 8: Step responses: interconnected system – upper plots
individual loops – lower plots;

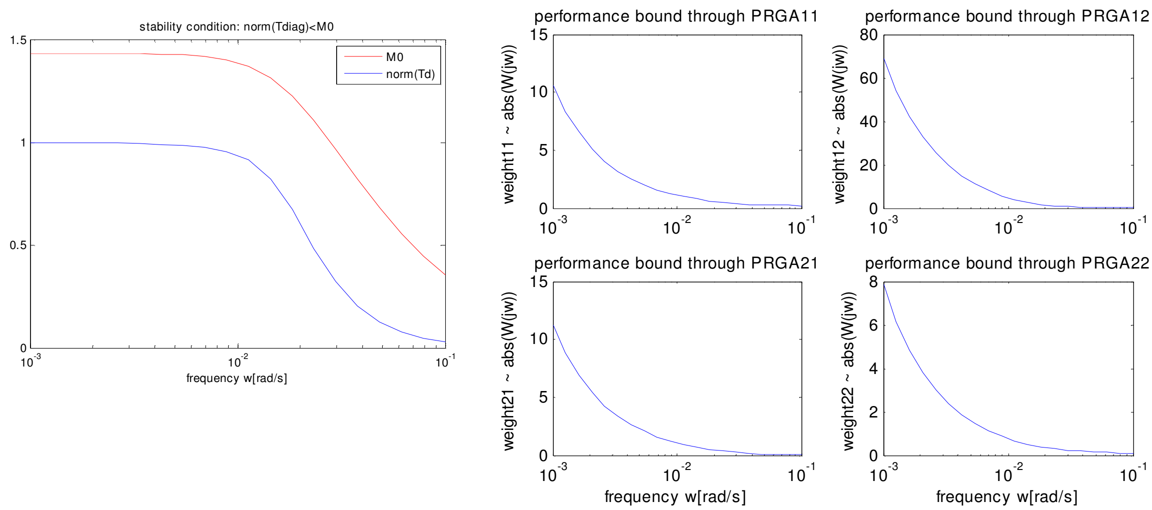

Figure 9: Decentralized control design bounds: stability condition (satisfied), performance bounds

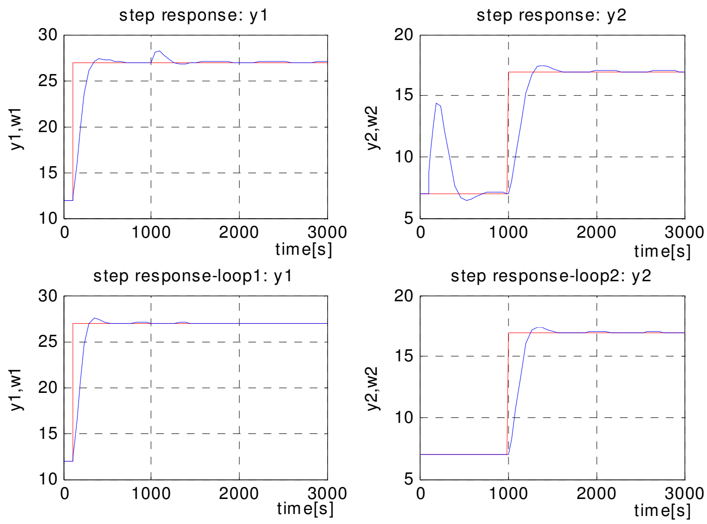

Figure 10: Step responses: interconnected system – upper plots
individual loops – lower plots;

4 CONCLUSION

The paper studies some basic aspects of decentralized control design concerning stability and performance. The decentralized control design strategy includes controller configuration (selection of appropriate pairing based on RGA), and individual control loops design based on overall stability condition and performance evaluation. This strategy is illustrated on example – case study of quadruple tank process. Two forms of overall stability conditions are used and briefly discussed, one of them (based on small gain theorem) formulated for stable systems which may have RHP zeros is less restrictive in higher frequencies, the other formulated for stable systems without RHP zeros is less restrictive in lower frequencies as shown in the example. Presented material provides simple illustration of stability versus performance relationship in decentralized control design and is believed to be useful in teaching complex systems control.

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