PID CONTROLLER DESIGN FOR MAGNETIC LEVITATION MODEL

Mária Hypiusová and Jakub Osuský

Institute of Control and Industrial Informatics, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology
Ilkovičova 3, 812 19 Bratislava, Slovak Republic
e-mail: maria.hypiusova@stuba.sk

Abstract: The paper deals with design of PID controller for unstable SISO systems in the frequency domain. The method is accomplished with performance specification in terms of phase margin and the modification of Neimark D-partition method which ensures desired phase margin. The practical application is illustrated by the PID controller design for the Magnetic Levitation Model.

Keywords: PID controller, unstable system, D-partition, phase margin, Magnetic Levitation (maglev)

1 INTRODUCTION

The frequency domain PID controller tuning is a topic of great interest in the industries. The wide-spread use of simple methods of the Ziegler-Nichols type (Ziegler and Nichols, 1942; Aström and Hägglund, 1995; Isermann, 1989; Vítečková *et al*., 2000) clearly indicates the need for simple methods that use minimal process information but provide required closed-loop performance.

From the point of view of control engineering, magnetic levitation (maglev) systems are challenging because of the nonlinear nature of the plant dynamics, the very small degree of natural damping in the process, the strict positioning specifications often required by the application and the system dynamics are open-loop unstable.

Maglev technology has a wide range of applications, for instance, high-speed transportation systems (Holmer, 2003), seismic attenuators for gravitational wave antennas (Varvella *et al.*, 2004), self-bearing blood pumps (Masuzawa *et al*., 2003) for use in artificial hearts, haptic interfaces (Berkelman and Hollis, 2000), photolithography devices for semiconductor manufacturing (Kim and Trummer, 1998), and microrobots (Khamesee *et al.*, 2002).

The design method presented in this paper is a graphical approach based on the D-partition method (Neimark, 1992). To achieve the desired phase margin, controllers are usually designed using Bode characteristics (Fung, *et al.*, 1998), (Ho, *et al.*, 1995). The modification consists in ensuring the desired phase margin.

2 MAGNETIC LEVITATION MODEL

Levitation is the stable equilibrium of an object without contact and can be achieved using electric or magnetic forces. In a magnetic levitation, or maglev, system a ferromagnetic object is suspended in air using electromagnetic forces. These forces cancel the effect of gravity, effectively levitating the object and achieving stable equilibrium.

The model of magnetic levitation shown in Figure 1 consists of a coil levitating a steel ball in magnetic field. The position of the steel ball is sensed by an inductive linear position sensor connected to A/D converter. The coil is driven by a power amplifier connected to D/A

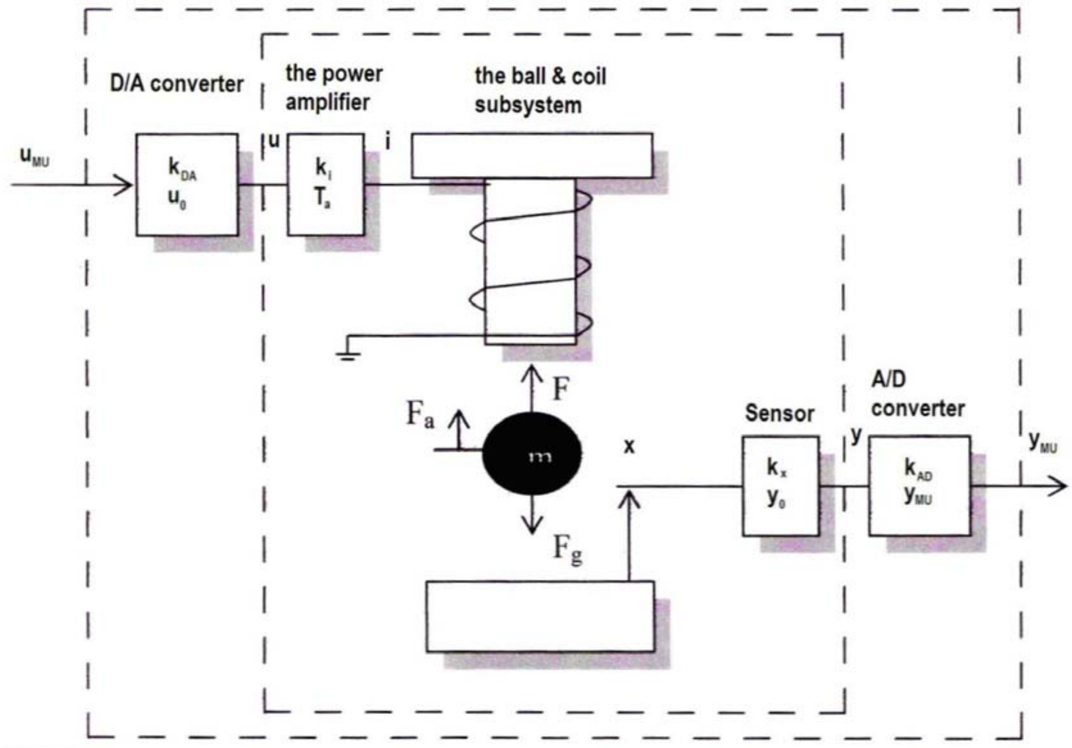

Figure 1: Scheme of the magnetic levitation model

converter. The model is connected to the PC via an universal data acquisition card, like the HUMUSOFT AD512 or MF614. The scheme shows that the model interface can be considered at two different levels:

- physical level – input and output voltage to the coil power amplifier and from the ball position sensors
- logical level – voltage converted by the data acquisition card and scaled to +-1 machine unit [MU].

The basic control task is to control the position of the ball freely levitation in the magnetic field of the coil.

3 PID CONTROLLER DESIGN WITH DESIRED PHASE MARGIN

Consider the closed-loop feedback system shown in Figure 2, where $G_R(s)$ is transfer function of a PID controller; $G_P(s)$ is a transfer function of the real plant; w, e, u and y are the reference, control error, manipulated variable and output of the plant signals, respectively.

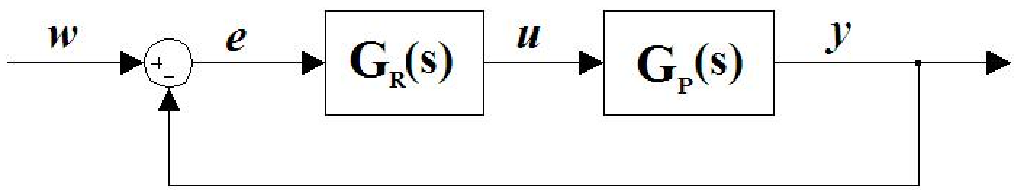

Figure 2: Classical feedback system

Consider the transfer functions in the form

$$G_P(s) = \frac{B(s)}{A(s)} \tag{1}$$

and

$$G_R(s) = k + \frac{k_i}{s} + k_d s \tag{2}$$

The problem studied in this paper can be formulated as follows: For system $G_P(s)$ described by (1) a PID controller G_R(s) is to be designed using Neimark D-partition so that not only stability will be ensured but performance in term of phase margin too.

Then the characteristic equation is

$$1 + G_R(s)G_P(s) = 0 \tag{3}$$

A small modification of (3) yields

$$k + \frac{k_i}{s} + k_d s = -\frac{A(s)}{B(s)} \tag{4}$$

Using substitution $s = j\omega$ is possible to obtain real and imaginary part:

$$RE: k = -\frac{A(j\omega)}{B(j\omega)} \qquad IM: -\frac{k_i}{\omega} j + k_d j\omega = -\frac{A(\omega)}{B(\omega)} \tag{5}$$

If ω is changing step by step in the interval $\omega \in (0,\infty)$ from real part of (5) is possible to calculate frequency dependent vector of complex numbers which plotted in complex plane create D-curve for parameter k. Similar is it with imaginary part of (5) from which is possible to obtain k_i or k_d but not both at once. In one step is possible to plot D-curve for parameters k and k_i (PI controller) or k and k_d (PD controller).

A small modification of characteristic equation yields

$$1 + G_R(s)G_P(s)e^{-j\varphi} = 0 \tag{6}$$

It is possible to rotate the frequency characteristics of a system, where φ is the angle of desired rotation in radians. Then the D-curves calculated with (6) are

$$RE: k = -\frac{A(j\omega)}{B(j\omega)e^{-j\varphi}} \qquad IM: -\frac{k_i j}{\omega} + k_d j\omega = -\frac{A(j\omega)}{B(j\omega)e^{-j\varphi}} \tag{7}$$

and controller parameters are chosen directly from the D-curves. The designed controller will ensure phase margin equal to the angle φ.

The PID controller design consists of two steps: in the first step, PD controller can be designed and in the second step, PI controller design can by applied for the plant with the PD controller. The final PID controller is than calculated as follows

$$G_R(s) = (k_1 + k_d s)(k_2 + \frac{k_i}{s}) = k_1 k_2 + k_d k_i + \frac{k_i k_1}{s} + k_d k_i s = K_P + \frac{K_I}{s} + K_D s \tag{8}$$

In this way controller for unstable plant can be designed, if this plant stabilizable with a PD controller. Hence in the first step, a PD controller is used for stabilization and a PI controller ensures desired phase margin and eliminates steady state offset.

4 DESIGN OF PID CONTROLLER FOR MAGNETIC LEVITATION MODEL

The chosen working point is given by the manipulated variable $u_{MU} = 0.17$ and the regulated variable $y_{MU} = 0.47$ what correspond the position $x = 2.85[mm]$. Consider the transfer function of a position of the Magnetic Levitation Model obtained by linearisation

$$G_P(s) = \frac{18400}{s^2 - 2.418s - 3998} \tag{9}$$

The system has two real roots: 64.45 and -62.03. The required phase margin is $\varphi_z = 65^\circ$.

The D-curves for k_1 and k_d are calculated and depicted in Figure 3.

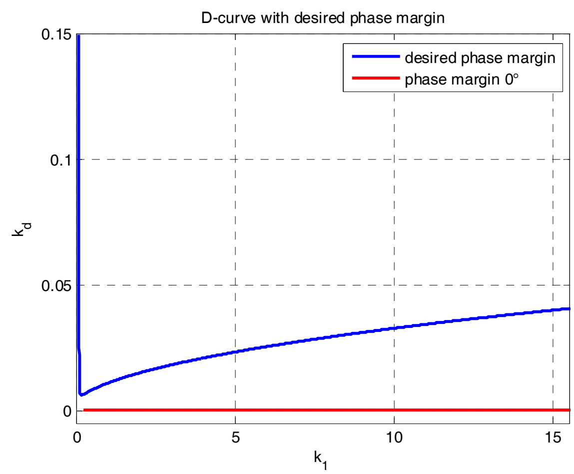

Figure 3: D-curve for parameters k_1 and k_d

In the first step it is not necessary to choose parameters from blue line because we just want to stabilize the system. System will be stable if the controller parameters will be chosen from region above red line which represents stability bound. From figure 3 we chose PD controller parameters $k_1 = 10$ and $k_d = 0.05$. The poles of characteristic equations with this PD controller are -633.39 and -284.18.

In the second step PI controller is designed for system consisting from plant and PD controller. The D-curves for k_2 and k_i are calculated and together depicted in Figure 4.

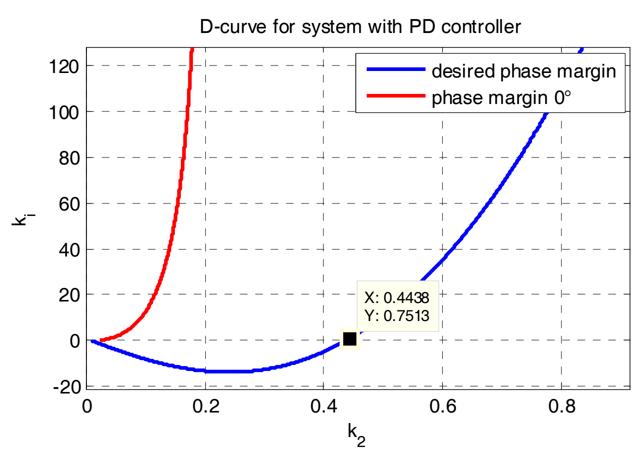

Figure 4: D-curve for parameters k_2 and k_i

From Figure 4 we chose parameters from blue line which ensure desired phase margin $\varphi = 65°$. Designed PI controller has following parameters: $k_2 = 0.4438$ and $k_i = 0.7513$. The final controller calculated according (8) is:

$$G_R(s) = 4.476 + \frac{7.513}{s} + 0.0222s \qquad (10)$$

In Figure 5 are depicted bode characteristics where the desired phase margin was achieved.

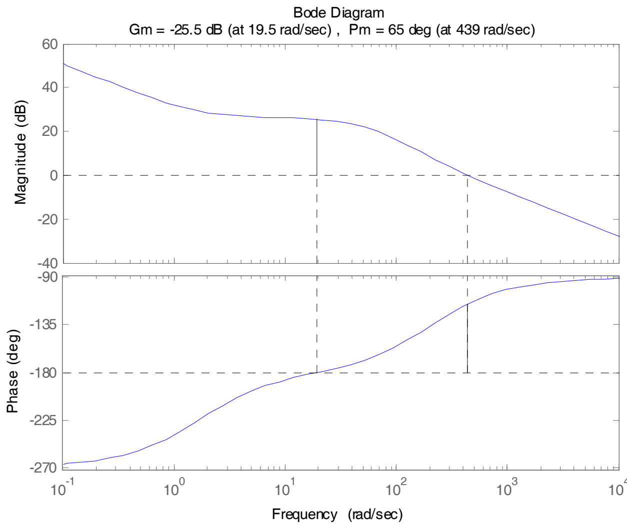

Figure 5 Bode characteristics for plant of Magnetic Levitation with designed controller

The poles of characteristic equations with designed PID controller are:

-202.05 ± 191.86i and -1.79

Figure 6 shows the closed loop step response under the designed controller in the working point.

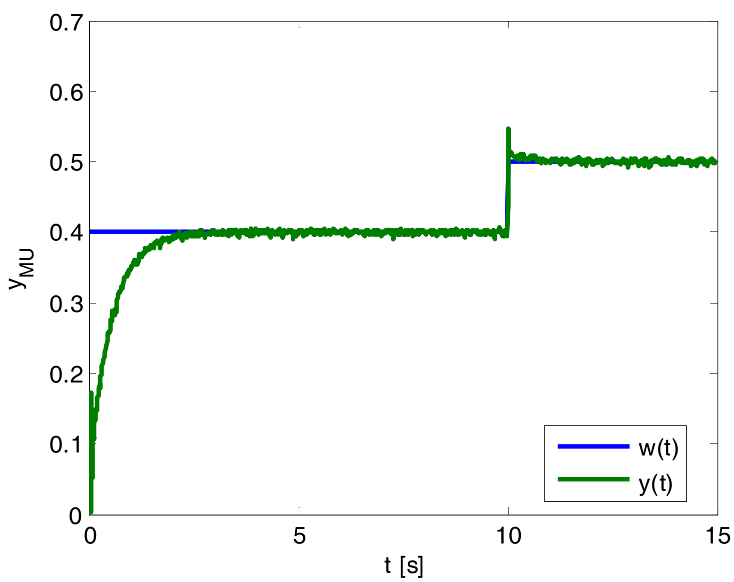

Figure 6 Closed-loop step response in the working point

The constraints of controller parameters (8) on the real model are:

$$K_P \in \langle 0;20 \rangle,\ K_I \in \langle 0;10 \rangle \text{ and } K_D \in \langle 0;5 \rangle.$$

Some of the calculated controller parameters for various choices of the point from figure 3 are listed in the Table 1.

Table 1: Parameter values for the required phase margin $\varphi_z = 65^\circ$

	k_1	k_d	k_2	k_i	$G_R(s)$
1	10	0.05	0.4438	0.7513	$4.476 + \frac{7.513}{s} + 0.0222s$
2	12	0.06	0.369	0.4966	$4.458 + \frac{5.950}{s} + 0.02214s$
3	5	0.03	0.6226	0.7457	$3.135 + \frac{3.729}{s} + 0.01868s$
4	8	0.04	0.552	0.5304	$4.437 + \frac{4.243}{s} + 0.02208s$
5	15	0.1	0.1736	0.6664	$2.671 + \frac{9.996}{s} + 0.01736s$

Figure 7 shows the closed loop step responses under some the designed controllers in the working point.

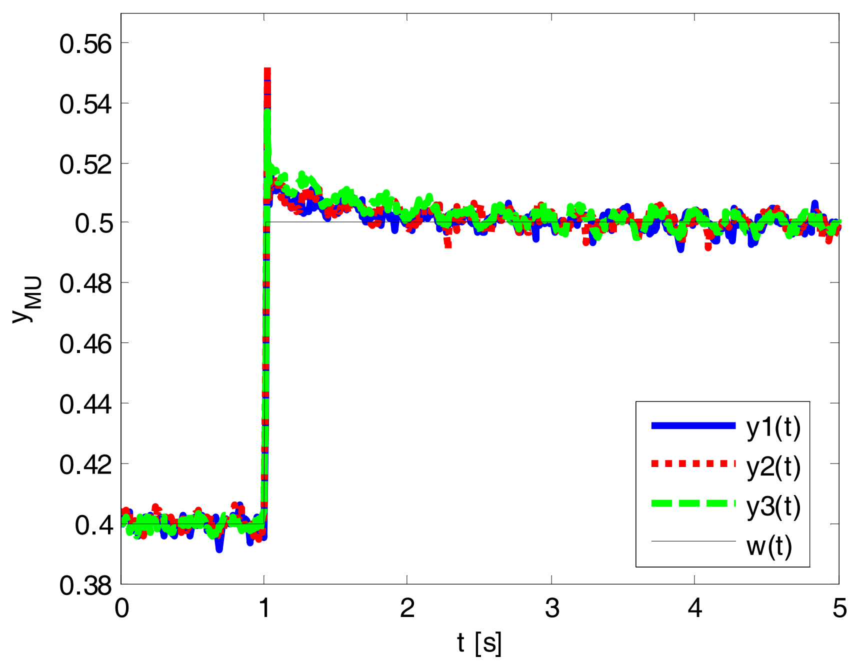

Figure 7 Closed-loop step responses in the working point

5 CONCLUSION

In this paper a modification of the Neimark D-partition method for the unstable system of Magnetic Levitation Model was presented. This controller design approach ensures not only stability but performance in terms of phase margin, too. The developed frequency domain design technique is graphical, interactive and insightful and it is useful for stable and unstable systems.

Acknowledgments

The work on this paper has been supported by the VEGA Grant No. 1/0544/09.

REFERENCES

ASTRÖM, K.J., T. HÄGGLUND (1995): PID Controllers: Theory, Design and Tuning. *2nd Edition. Research Triangle Park, NC: Instrument Sociaty of America*

BERKELMAN, P.J., R.L. HOLLIS (2000): Lorentz magnetic levitation for haptic interaction: Device design, performance, and integration with physical simulations, *International Journal of Robotics Research*, Vol. 19, No. 7, pp. 644-667

Fung, H.W., Q.G. Wang, T.H. Lee (1998): PI Tuning in terms of gain and phase margins, Automatica, 34, pp. 1145-1149

Ho, W.K., C.C. Hang, L. Cao (1995): Tuning of PID controllers based on gain and phase margin specifications, Automatica, 31, pp. 497-502

Holmer, P. (2003): Faster than a speeding bullet train, IEEE Spectrum, Vol. 40, No. 8, pp. 30-34

ISERMANN R. (1989): Digital Control Systems, Volume 1. Fundamentals Deterministic Control, *2nd revised Edition, Springer-Verlag*

KHAMESEE, M.B., N. KATO, Y. NOMURA, T. NAKAMURA (2002): Design and control of a microrobotic system using magnetic levitation, *EEE-ASME Transactions on Mechatronics*, Vol. 7, No. 1, pp. 1-14

KIM, W.J., D.L. TRUMPER (1998): High-precision magnetic levitation stage for photolithography, *Precision Engineering*, Vol. 22, No. 2, pp. 66-77

MASUZAWA, T., S. EZOE, T. KATO, Y. OKADA (2003): Magnetically suspended centrifugal blood pump with an axially levitated motor, *Artificial Organs*, Vol. 27, No. 7. pp. 631-638

NAGURKA, M., O. YANIV (2003): Robust PI Controller Design Satisfying Gain and Phase Margin Constraints. *Proceedings of the American Control Conference*, Denver, Colorado June 4-6

NEIMARK, Y.I. (1992): Robust stability and D-partition, *Automation and Remote Control 53* (7), pp. 957–965

PELIKÁN R. (2006): Magnetic Levitation - modeling, simulations and control, Dipl. thesis, UTB, Zlín

VARVELLA, M., E. CALLONI, L. DI FIORE, L. MILANO, N. ARNAUD (2004): Feasibility of magnetic suspension for second generation gravitational wave interferometers, Astroparticle Physics, Vol. 21, No. 3, pp. 325-335

VÍTEČKOVÁ, M., A. VÍTEČEK, L. SMUTNÝ (2000): Simple PI and PID Controllers Tuning for Monotone Self-Regulating Plants. In*: IFAC Workshop PID'00, 289-294, Terrasa*

ZIEGLER, J.G., N.B. NICHOLS (1942): Optimum settings for Automatic Controllers. *Trans ASME 64*, 759-768