MODELING AND CONTROL OF COMPLEX SYSTEMS

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Abstract: In this paper, an effective technique of hybrid modeling and control are presented. Simple thermal model of Air-handling unit (AHU) is defined in hybrid description language (HYSDEL). AHU model includes continuous dynamics together with discrete rules (if-then-else conditions) and discrete components (on/off switches) represented in unique model structure. Receding horizon optimal control is extended to solve mixed-integer programming problems and is presented as a suitable control scheme for hybrid models.

Keywords: hybrid systems, dynamic models, ventilation and air-conditionig system, predictive control

1 INTRODUCTION AND PRELIMINARIES

The mathematical model of system is traditionally associated with differential or difference equations. Consequently, most of control law and schemes have been designed for such systems, whose are characterized by the smooth linear or non-linear state transitions. On the other hand, many controlled processes consist of continuous dynamics governed by logic rules, such as for instance on/off switches or valves, gears or speed selectors. Often, the control of these systems is left to schemes based on heuristic rules inferred from practical plant operation. In this paper, as an example of hybrid control, is studied simple thermal process of AHU (fig 1).

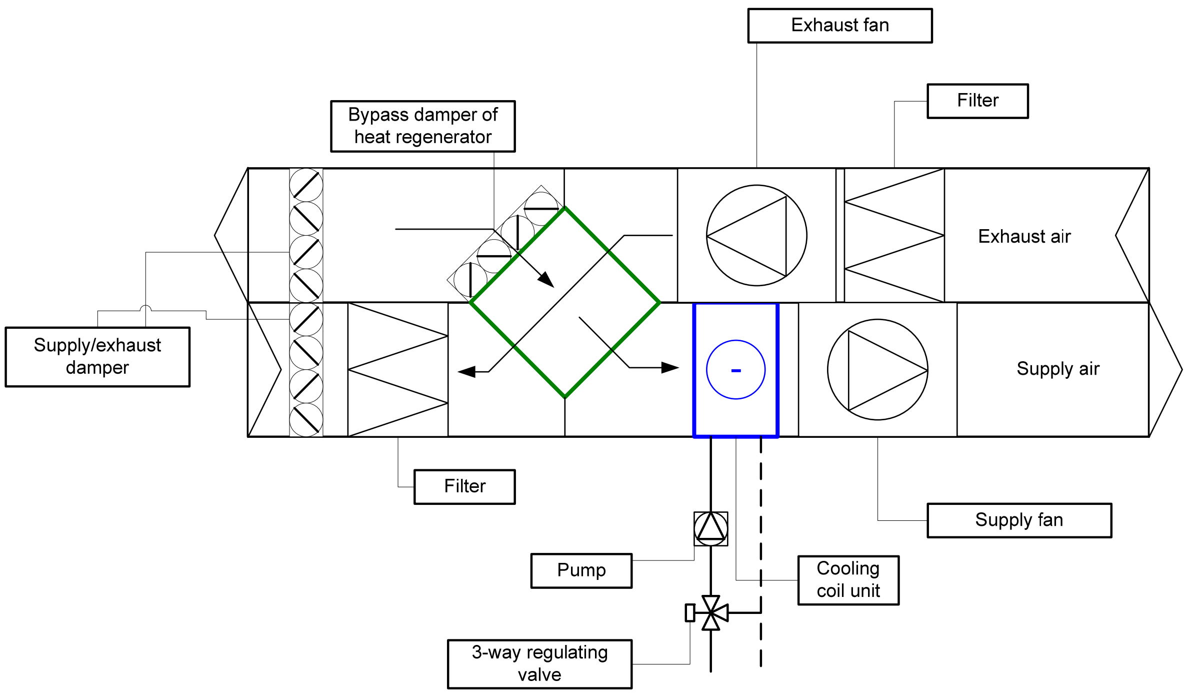

Fig. 1: Air-handling unit scheme

A simple dynamic model of Cooling coil unit (CCU) was developed in (Guang-Yu Jin, Wen-Jian Cai, 2006). The model relatively simple and exactly captures nonlinear characteristics over a wide operating range of the CCU without requiring geometric specification. The same approach was used to develop mathematical model of air-to-air heat exchanger (AAE). Moreover, AHU model contains discrete parts like bypass damper, cooling water pump and fan speed switcher. Objective of control problem is to design a control law that optimally selects discrete inputs (damper, fan speed) and continuous inputs (cooling coil valve openning). For this purpose we need hybrid model that includes continuous dynamics and discrete logic.

Hybrid system switches among many operating modes, where each mode is governed by its own laws. Mode transitions are triggered by variables crossing specific thresholds (state invents), by the elapse of certain time periods (time events), or by external inputs (input events). A particular case of hybrid systems is popular class of *piecewise affine* (PWA) systems. PWA is switched affine system whose mode only depends on the current location of the state vector. More precisely, the state space is partitioned into polyhedral regions, as depicted in Fig. 2, and each region is associated with a different affine state-update equation (Bemporad, 2008).

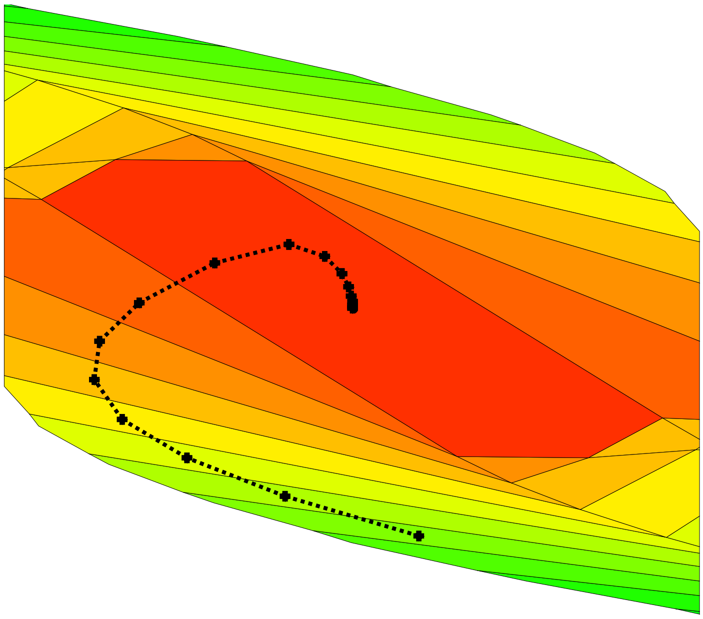

Fig. 2: Piecewise affine systems. Mode switches are triggered by threshold events

PWA systems are model structures for describing hybrid systems. Dynamical behavior of such systems

is captured by relations of the following form:

$$x(k+1) = A_i x(k) + B_i u(k) + f_i$$
$$y(k) = C_i x(k) + D_i u(k) + g_i \tag{1.1}$$
$$s.t.$$

$$y_{min} \leq y(k) \leq y_{max}$$
$$u_{min} \leq u(k) \leq u_{max} \tag{1.1a}$$
$$\Delta u_{min} \leq u(k) - u(k-1) \leq \Delta u_{max}$$

Each dynamics i is active in a polyhedral partition bounded by the so-called guard-lines:

$$guardX_i x(k) + guardU_i u(k) \leq guardC_i \,, \tag{1.2}$$

which means dynamics i will be applied if the above inequality is satisfied (Kvasnica, Grieder, Baotić, Christophersen, 2006).

In (Bemporad, Morari 2009) was presented compact structure of hybrid model so-called MLD structure. MLD structure is defined in the following form:

$$x(t+1) = A_t x(t) + B_{1t} u(t) + B_{2t} \delta(t) + B_{3t} z(t) \tag{1.3a}$$

$$y(t) = C_t x(t) + D_{1t} u(t) + D_{2t} \delta(t) + D_{3t} z(t) \tag{1.3b}$$

$$E_{2t} \delta(t) + E_{3t} z(t) \leq E_{1t} u(t) + E_{4t} x(t) + E_{5t} \tag{1.3c}$$

where $t \in Z$, $x = \begin{bmatrix} x_c \\ x_l \end{bmatrix},\ x_c \in \mathbb{R}^{n_c}, x_l \in \{0,1\}^{n_l}, n = n_c + n_l$ is system state, whose components are devided into continuous x_c and $0-1\ \ x_l$; $y = \begin{bmatrix} y_c \\ y_l \end{bmatrix},\ y_c \in \mathbb{R}^{p_c}, y_l \in \{0,1\}^{p_l}, p = p_c + p_l$ is output vector, $u = \begin{bmatrix} u_c \\ u_l \end{bmatrix},\ u_c \in \mathbb{R}^{m_c}, u_l \in \{0,1\}^{m_l}, m = m_c + m_l$ is control input which includes analog u_c and binary (On/Off) inputs u_l; $\delta \in \{0,1\}^n$ and $z \in \mathbb{R}^{r_c}$ represents auxiliary variables.

The class of MLD systems includes the following important classes of systems:

- Linear hybrid systems.
- Sequential logical systems (Finite State Machines, Automata) ($n_c = m_c = p_c = 0$).
- Nonlinear dynamic systems, where the nonlinearity can be expressed through combinational logic ($n_l = 0$).
- Some classes of discrete event systems ($n_c = p_c = 0$).
- Constrained linear systems ($n_l = m_l = p_l = r_l = r_c = 0$).
- Linear systems ($n_l = m_l = p_l = r_l = r_c = 0,\ E_{it} = 0, i = 1,4,5$).

In (Bemporad, Morari 1999) is showed that MLD and PWA equivalent model classes, and hence, in particular, MLD systems can be converted to equivalent PWA systems.

This paper is organized as follows. In section 2 are specified equations of CCU and AAE. In section 3 is formed model of AHU in MLD fashion and defined in Hybrid Description Language (Hysdel). Section 4 deals with optimal control of defined model.

2 MATHEMATICAL MODEL OF COOLING COIL UNIT AND REGENERATOR

A water CCU uses chilled water as the coolant inside the tubes. The chilled water cools and dehumidifies the moist air that flows over the external surface of the tubes and fins, as shown in Fig. 3. During heat exchange with the air outside the CCU tubes, the chilled water flows from the inlet to the outlet of the CCU forced by the chilled water pump with inlet temperature $T_{chw,i}$ and mass flow rate $m_{chw,i}$, and the outlet temperature of the chilled water rises to $T_{chw,o}$. The air flows from the inlet to the outlet of the CCU forced by the supply air fan with the on coil, dry bulb temperature $T_{a,i}$ and mass flow rate m_a, and likewise, the off-coil, dry bulb temperatures descend to $T_{a,o}$.

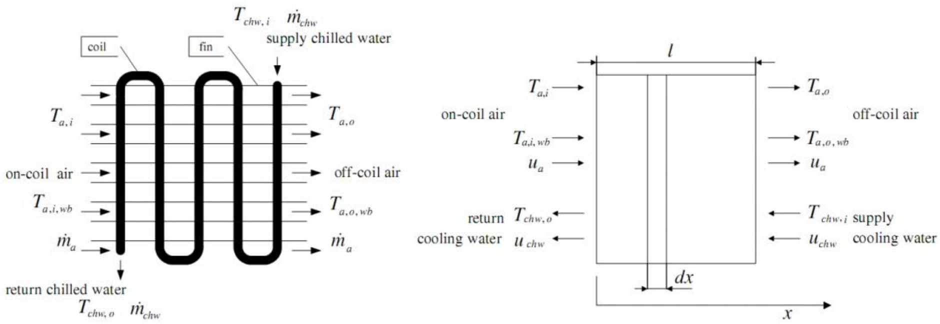

Fig. 3. Schematic diagram of finned tube CCU

Final differential equations of CCU by (Guang-Yu Jin, Wen-Jian Cai, 2006) are:

$$\frac{dT_{chw,o}}{dt} + c_1 m_{chw}\left(T_{chw,o} - T_{chw,i}\right) = \frac{c_2 m_a^{\ell}}{1 + c_3\left(\frac{m_a}{m_{chw}}\right)^{\ell}}\left(T_{a,o} - T_{chw,o}\right) \tag{2.1}$$

$$\frac{dT_{a,o}}{dt} + c_4 m_a\left(T_{a,o} - T_{a,i}\right) = -\frac{c_5 m_a^{\ell}}{1 + c_3\left(\frac{m_a}{m_{chw}}\right)^{\ell}}\left(T_{a,o} - T_{chw,o}\right) \tag{2.2}$$

where $c_1 = \frac{\ell}{\xi}\frac{1}{\rho_{chw}A_{tube}}$, $c_2 = \frac{b_a A_a}{\rho_{chw}V_{chw}c_{chw}}$, $c_3 = \frac{b_a A_a}{b_{chw}A_{chw}}$, $c_4 = \frac{1}{\rho_a A_a}$, $c_5 = \frac{b_a A_a}{\rho_a V_a c_{ma}}$ a ℓ are constants that can be determined from the manufacturer´s data or by real time experimental data. In Tab. 1 are taken values of identified parameters.

Tab.1 Identifikované parametre							
Model	**Load range (kW)**	ℓ	c_1	c_2	c_3	c_4	c_5
Six-parameter	0.60-1.20	0.6078	6.7142	0.7412	0.7021	8.9936	3.9722

Time responses of input signals are in Fig. 4 and Fig. 5. Mass flow rate of air m_a and mass flow rate of cooling water m_{chw} are modeled by rectangular signals Fig. 6. Output temperatures are capured in Fig. 7.

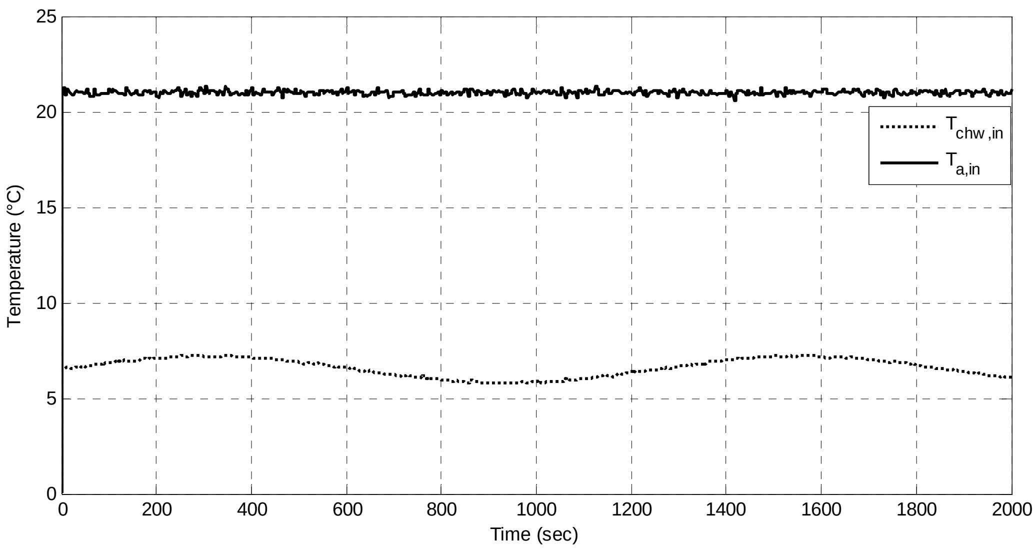

Fig. 4 Time response of in-coil air temperature and in-coil cooling water temperature

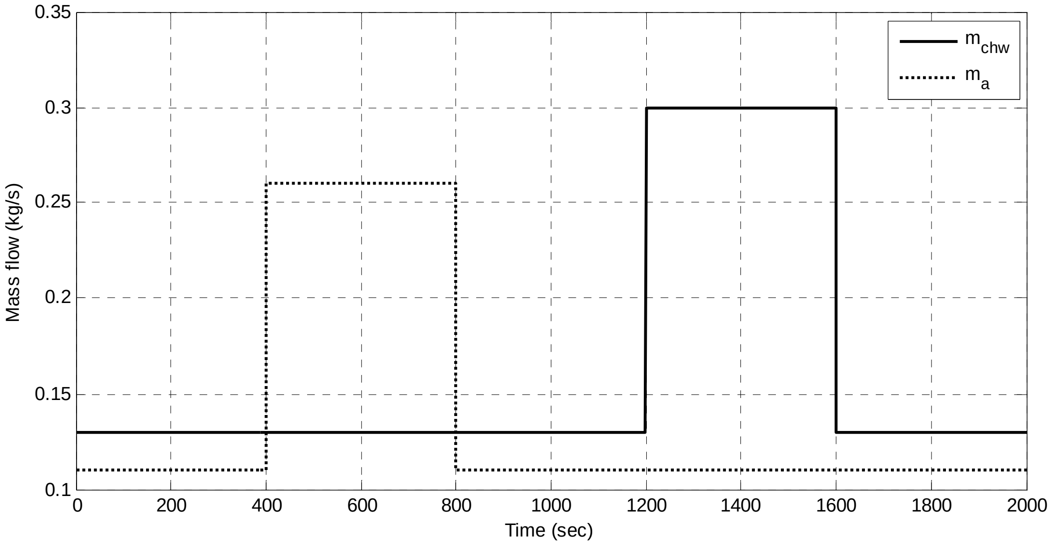

Fig. 5 Time response of air mass flow rate and cooling water mass flow rate

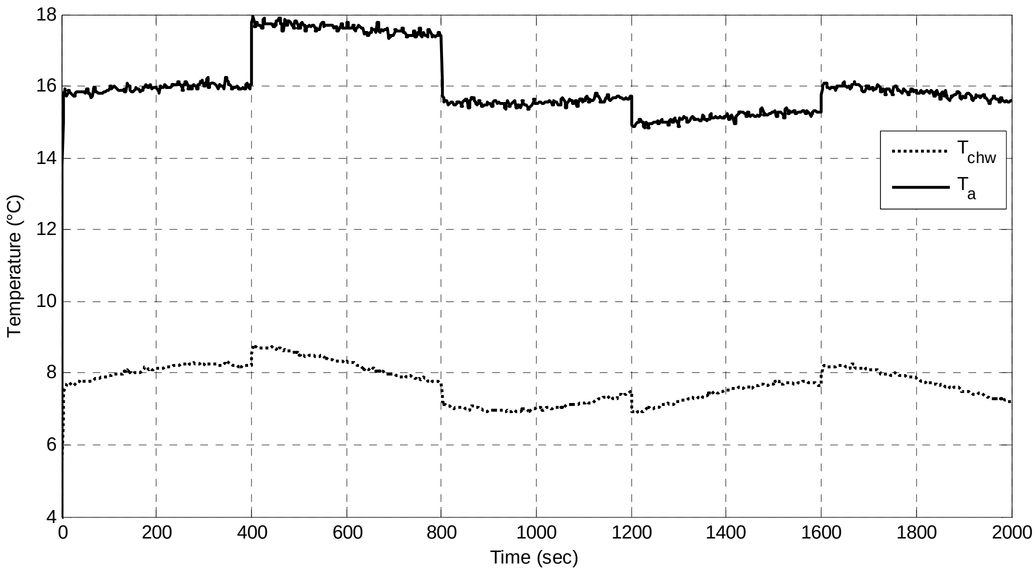

Fig. 6 Time response of off-coil air temperature and off-coil water temperature

For developing model of air-to-air heat exchanger (AAE) we use the same technique as in CCU case. In AAE exhaust air (from building) gives (or takes) heat to supply outdoor air. For example if temperature of outdoor air is 33°C and temperature of exhaust air is 24°C, the exhaust air can pre-cool supplied outdoor air, consequently, the requirements for capacity (size) of CCU decrease.

Relation which describes the quantity of transfered heat from aspect of geometry (Yao-Weng Wang, Wen-Jian Cai 2004) is:

$$q = \frac{b_{chw}A_{chw}m_{chw}^{\ell}b_aA_am_a^{\ell}}{b_{chw}A_{chw}m_{chw}^{\ell}+b_aA_am_a^{\ell}}(T_a - T_{chw}) \tag{2.3}$$

where A_{chw} and A_a is area of heat transfer from side of cooling water and air, respectively. b_{chw} and b_a are identified constants. ℓ is depth of CCU. If we use this relation and parameters of cooling medium replace with partameters of exhaust air, which are the same as for supply air we get:

$$q = \frac{1}{2} b_a A_a m_a^f (T_{ao} - T_{s,i}) \tag{2.4}$$

In this relation we can adjust the quantity of heat transfer by two parameters: b_a and f.

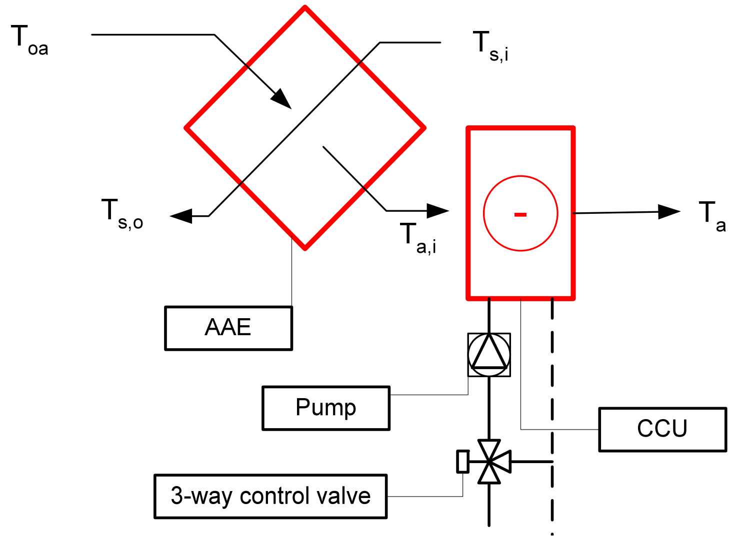

Fig. 7 Temperature labelling in AHU

Mathematical model of regenerator is in form:

$$\frac{dT_{a,i}}{dt} + c_1 m_a (T_{a,i} - T_{oa}) = c_2 m_a^f (T_{s,o} - T_{a,i}) \tag{2.5}$$

$$\frac{dT_{s,o}}{dt} + c_1 m_a (T_{s,o} - T_{s,i}) = c_2 m_a^f (T_{a,i} - T_{s,o}) \tag{2.6}$$

where $T_{a,i}$ is temperature behind the AAE (supply side), T_{oa} is temperature of outdoor air, $T_{s,i}$ is temperature of exhaust air from building, $T_{s,o}$ is exhaust temperature of AAE (Fig. 7), m_a is mass flow rate of air, , $c_1 = \frac{1}{\rho_a A_a l}$ and $c_2 = \frac{1}{2} \frac{(b_a A_a)}{\rho_a A_a C_{pa}}$ are constants. Time responses of inlet and outlet temperatures are captured in Fig. 8. In $T = 400\,sec$ we made step change of air flow. Increase of air flow is reflected in weaker heat transfer.

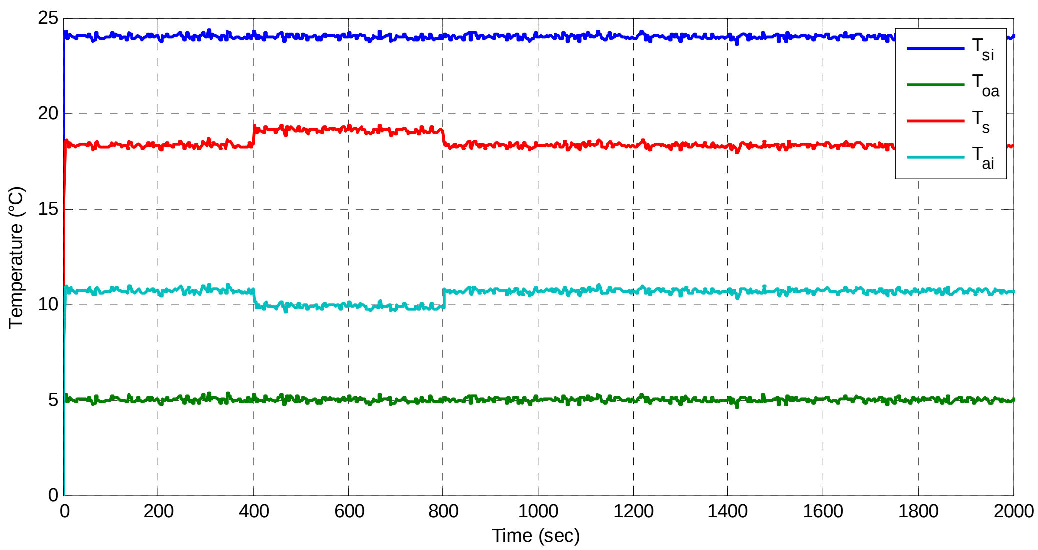

Fig. 8 Time response of AAE temperatures

3 HYBRID MODEL OF AIR-HANDLING UNIT

In Hysdel we defined hybrid model that contained AAE and CCU together with cooling water pump, fan speed swither and bypass damper. Continuous input signals of the model are $T_{chw,i}$ – inlet temperature of cooling water, T_{oa} - outside air temperature, $T_{s,i}$ - building exhaust temperature. Binary input signals of the model are s - fan speed swither, Cr - cooling water pump and $KLPK$ – regenerator bypass dumper.

States of the model are T_{chw} - outlet temperature of cooling water, T_a - supply air temperature (controlled variable), $T_{a,i}$ – CCU inlet air temperature and T_s - AAE outlet air temperature (Fig. 7).

Hybrid model definition in Hysdel (Torrisi et al. 2002):

```
SYSTEM ahu{
    INTERFACE{
        STATE{
            REAL T_chw [0, 15] ;
            REAL T_a [0, 100] ;
            REAL T_ai [0, 100] ;
            REAL T_s [0, 35] ;
        }
        INPUT{
            REAL T_chwi [0, 15] ;
            REAL T_oa [0, 35] ;
            REAL T_si [0, 35] ;
            BOOL s ;
            BOOL Cr ;
            BOOL KLPK ;
        }
        OUTPUT{
            REAL y1, y2 ;
        }
        PARAMETER{
            REAL ccu_a1_I, ccu_a2_I, ccu_a3_I, ccu_a4_I, ccu_b1_I,
                    ccu_b2_I, ccu_b3_I, ccu_b4_I ;
            REAL ccu_a1_II, ccu_a2_II, ccu_a3_II, ccu_a4_II, ccu_b1_II,
                    ccu_b2_II, ccu_b3_II, ccu_b4_II ;
            REAL aae_a1_I, aae_a2_I, aae_a3_I, aae_a4_I, aae_b1_I,
                    aae_b2_I, aae_b3_I, aae_b4_I ;
            REAL aae_a1_II, aae_a2_II, aae_a3_II, aae_a4_II, aae_b1_II,
                    aae_b2_II, aae_b3_II, aae_b4_II ;
         }
    }
    IMPLEMENTATION {
        AUX {
            REAL ccu_a1, ccu_a2, ccu_a3, ccu_a4, ccu_b1, ccu_b2, ccu_b3, ccu_b4 ;
            REAL aae_a1, aae_a2, aae_a3, aae_a4, aae_b1, aae_b2, aae_b3, aae_b4 ;
            REAL aae_a1_i, aae_a2_i, aae_a3_i, aae_a4_i, aae_b1_i, aae_b2_i,
                    aae_b3_i, aae_b4_i ;
        }
        DA {
            aae_a1_i = { IF s THEN aae_a1_II*T_ai ELSE aae_a1_I*T_ai} ;
            aae_a2_i = { IF s THEN aae_a2_II*T_s ELSE aae_a2_I*T_s} ;
            aae_a3_i = { IF s THEN aae_a3_II*T_ai ELSE aae_a3_I*T_ai} ;
            aae_a4_i = { IF s THEN aae_a4_II*T_s ELSE aae_a4_I*T_s} ;
            aae_b1_i = { IF s THEN aae_b1_II*T_oa ELSE aae_b1_I*T_oa} ;
            aae_b2_i = { IF s THEN aae_b2_II*T_si ELSE aae_b2_I*T_si} ;
            aae_b3_i = { IF s THEN aae_b3_II*T_oa ELSE aae_b3_I*T_oa} ;
            aae_b4_i = { IF s THEN aae_b4_II*T_si ELSE aae_b4_I*T_si} ;

            aae_a1 = { IF KLPK THEN 0 ELSE aae_a1_i} ;
            aae_a2 = { IF KLPK THEN 0 ELSE aae_a2_i} ;
            aae_a3 = { IF KLPK THEN 0 ELSE aae_a3_i} ;
            aae_a4 = { IF KLPK THEN 0 ELSE aae_a4_i} ;
            aae_b1 = { IF KLPK THEN T_oa ELSE aae_b1_i} ;
            aae_b2 = { IF KLPK THEN 0 ELSE aae_b2_i} ;
            aae_b3 = { IF KLPK THEN 0 ELSE aae_b3_i} ;
```

```
            aae_b4 = { IF KLPK THEN T_si ELSE aae_b4_i} ;

            ccu_a1 = { IF s THEN ccu_a1_II*T_chw ELSE ccu_a1_I*T_chw} ;
            ccu_a2 = { IF s THEN ccu_a2_II*T_a ELSE ccu_a2_I*T_a} ;
            ccu_a3 = { IF s THEN ccu_a3_II*T_chw ELSE ccu_a3_I*T_chw} ;
            ccu_a4 = { IF s THEN ccu_a4_II*T_a ELSE ccu_a4_I*T_a} ;
            ccu_b1 = { IF s THEN ccu_b1_II*T_chwi ELSE ccu_b1_I*T_chwi} ;
            ccu_b2 = { IF s THEN ccu_b2_II*T_ai ELSE ccu_b2_I*T_ai} ;
            ccu_b3 = { IF s THEN ccu_b3_II*T_chwi ELSE ccu_b3_I*T_chwi} ;
            ccu_b4 = { IF s THEN ccu_b4_II*T_ai ELSE ccu_b4_I*T_ai} ;

        }
        CONTINUOUS {
            T_ai = aae_a1 + aae_a2 + aae_b1 + aae_b2 ;
            T_s = aae_a3 + aae_a4 + aae_b3 + aae_b4 ;
            T_chw = ccu_a1 + ccu_a2 + ccu_b1 + ccu_b2 ;
            T_a = ccu_a3 + ccu_a4 + ccu_b3 + ccu_b4 ;
        }
        OUTPUT {
            y1 = T_chw ;
            y2 = T_a ;
        }
    }
}
```

After transformation to PWA form we get model with eight dynamics. We connected measured values of outside air temperature T_{oa} and building exhaust temperature $T_{s,i}$ to the input of model (Fig. 9).

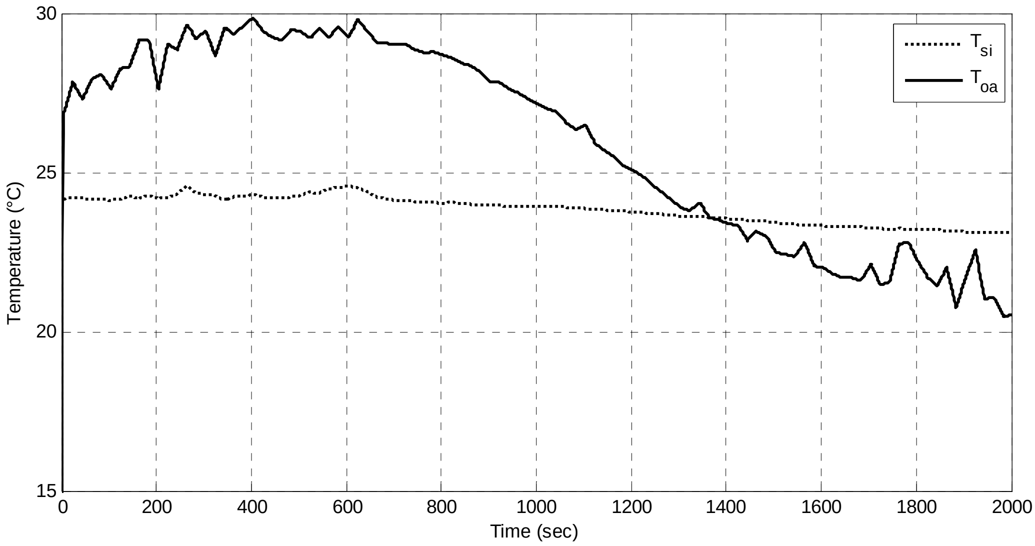

Fig. 9 Time response of outside air and building exhaust air temperature

4 OPTIMAL CONTROL OF HYBRID SYSTEM

Hybrid toolbox (Bemporad 2008) defines two approaches to control of hybrid systems based on Receding horizon optimal control RHC, also known as Model Predictive Control MPC (Goodwin, Seron, De Doná 2005). In on-line (implicit) version of RHC a open-loop optimal control problem is solved over a finite horizon. At the next time step the computation is repeated starting from the new state and over a shifted horizon, leading to a moving horizon policy. For systems where on-line computational effort may limit its aplicability have been developed off-line RHC (explicit) version (Bemporad, Morari, Pistikopoulos, Dua 2002). In explicit approach state space is considered as space of parameters of Multiparametric program. Output of explicit method is partitioning of state space to polytopes and for each polytop is designed optimal feedback controller . All the computational requirement is moved to off-line calculation.

Criterial function of MPC (Bemporad, Morari, Pistikopoulos, Dua 2002) – tracking problem:

$$\min_{U=\{\delta u_t,\dots,\delta u_{t+N_u-1}\}} \sum_{k=0}^{N_y-1} \left\{ \left[y_{(t+k|t)} - r(t)\right]' Q \left[y_{(t+k|t)} - r(t)\right] + \delta u'_{(t+k|t)} R \delta u_{(t+k|t)} \right\} \tag{4.1a}$$

$$s.t. \quad y_{min} \le y_{(t+k|t)} \le y_{min}, \quad k = 1, \dots, N_c \tag{4.1b}$$

$$u_{min} \le u_{t+k} \le u_{max}, \quad k = 0,1, \dots, N_c \tag{4.1c}$$

$$\delta u_{min} \le \delta u_{t+k} \le \delta u_{max}, \quad k = 0,1, \dots, N_u - 1 \tag{4.1d}$$

$$x_{(t+k|t)} = A x_{(t+k|t)} + B u_{t+k}, \quad k \ge 0, \tag{4.1e}$$

$$y_{(t+k|t)} = C x_{(t+k|t)}, \quad \mathrm{k} \ge 0, \tag{4.1f}$$

$$u_{t+k} = u_{t+k-1} + \delta u_{t+k}, \quad k \ge 0 \tag{4.1g}$$

$$\delta u_{t+k} = 0, \quad k \ge N_u. \tag{4.1h}$$

The constrained finite-time optimal control problems above can be converted into the multiparametric Quadratic Program (mpQP)

$$\min_U \frac{1}{2} U'HU + [x'(t)u'(t-1)r'(t)]FU$$

$$s.t. \qquad GU \le W + E \begin{bmatrix} x(t) \\ u(t-1) \\ r(t) \end{bmatrix}, \tag{4.2}$$

Where $r(t)$ lies in a given (possibly unbounded) polyhedral set. Piecewise affine solution is in form $\delta u(t) = F(x(t), u(t-1), r(t))$. In case the reference r(t) is known in advance, one can replace $r(t)$ by $r(t+k)$ in (4.1) and get a piecewise affine anticipative controller $\delta u(t) = F(x(t), u(t-1), r(t), \dots, r(t+N_y-1))$.

Efficiency of all methods for multi-parametric programs depends on dimension of the problem. As predictive horizon N grows, number of convex regions grows exponentially.

In following section the explicit RHC is designed for study-case of AHU. Algorithms for generating of explicit linear quadratic regulator are part of Hybrid Toolbox (Bemporad 2008) or MPT Toolboxu (Kvasnica, Grieder, Baotić, Christophersen, 2006). MPT Toolbox uses PWA as a default form of hybrid model. After transformation to PWA form we get model with eight dynamics.

Hybrid model definition in PWA form

```
sysStruct =
             A: {1x8 cell}
             B: {1x8 cell}
             f: {1x8 cell}
             C: {[0 1 0 0]  [0 1 0 0]  [0 1 0 0]  [0 1 0 0]  [0 1 0
                  0]  [0 1 0 0]  [0 1 0 0]  [0 1 0 0]}
             D: {1x8 cell}
             g: {[0]  [0]  [0]  [0]  [0]  [0]  [0]  [0]}
        guardX: {1x8 cell}
        guardU: {1x8 cell}
        guardC: {1x8 cell}
          Uset: {[-Inf Inf]  [-Inf Inf]  [-Inf Inf]  [0 1]  [0 1]  [0 1]}
          umax: [6x1 double]
          umin: [6x1 double]
          xmin: [4x1 double]
```

```
        xmax: [4x1 double]
        Pbnd: [1x1 polytope]
        data: [1x1 struct]
   StateName: {'T_chw'  'T_a'  'T_ai'  'T_s'}
   InputName: {'T_chwi'  'T_oa'  'T_si'  's'  'Cr'  'KLPK'}
  OutputName: {'T_a'}
        ymin: 0
        ymax: 35
       dumin: [6x1 double]
       dumax: [6x1 double]
```

We have choosed criteria function with quadratic norm and prediction horiznon $N = 2$. Reference value of targed temperature $T_a = 22°C$.

```
probStruct.Q = eye(4) ;
probStruct.R = eye(6) ;
probStruct.R(1,1) = 0.1 ;
probStruct.N = 2 ;
probStruct.norm = 2 ;
probStruct.subopt_lev = 0 ;
probStruct.y0bounds = 0 ;
probStruct.tracking = 0 ;
probStruct.Tconstraint = 1 ;
probStruct.useSymmetry = 0 ;
probStruct.feedback = 0 ;
probStruct.yref = 22 ;
probStruct.Qy = 100 ;
```

Control inputs are cooling water temperature $T_{chw,i}$ [6°C÷13°C], fan speed [0 – first speed, 1 – second speed], regenerator bypass damper [0 – regenerator opened, 1 – regenerator closed] and cooling water pump [0 – pump ON, 1 – pump OFF].

Controller partition with 129 regions. Cut through x_4=0.00

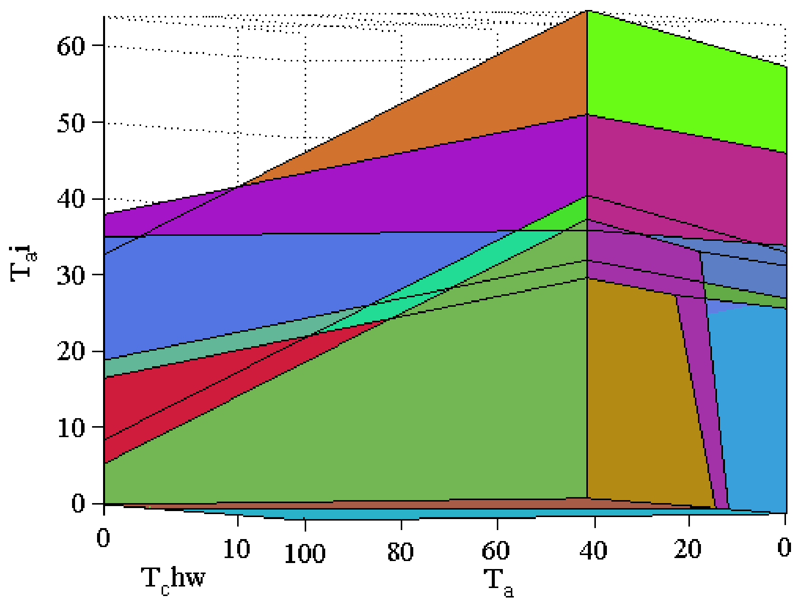

Fig. 10 State-space partition in linear optimal control problem of AHU

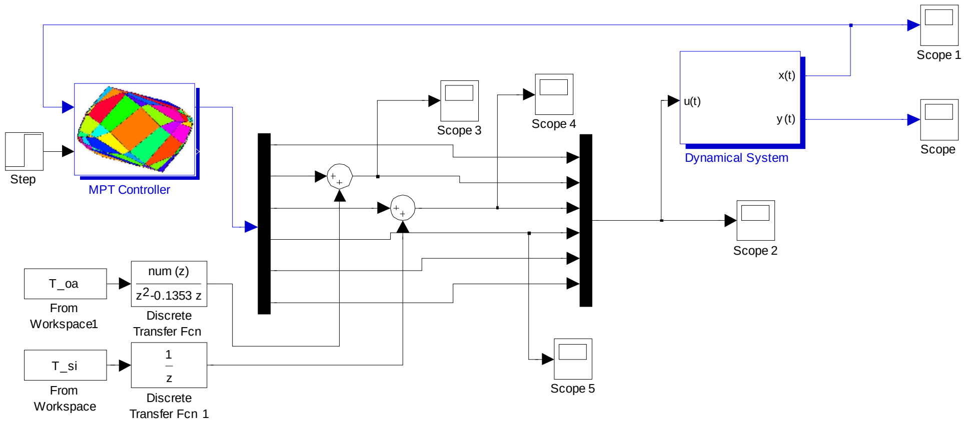

Fig. 11 Simulation scheme in Matlab-Simulink

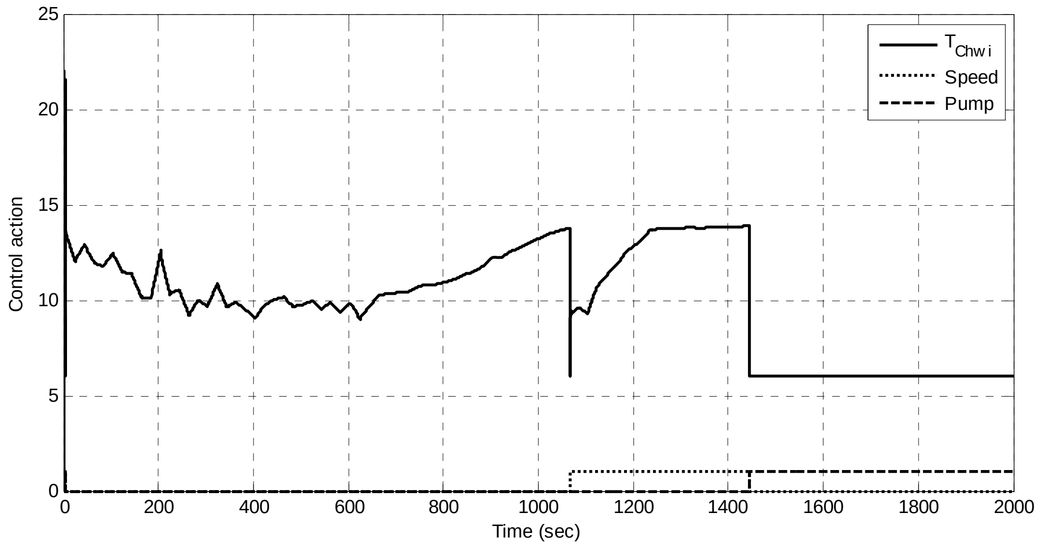

Fig. 12 Time response of control action in linear optimal control problem of AHU

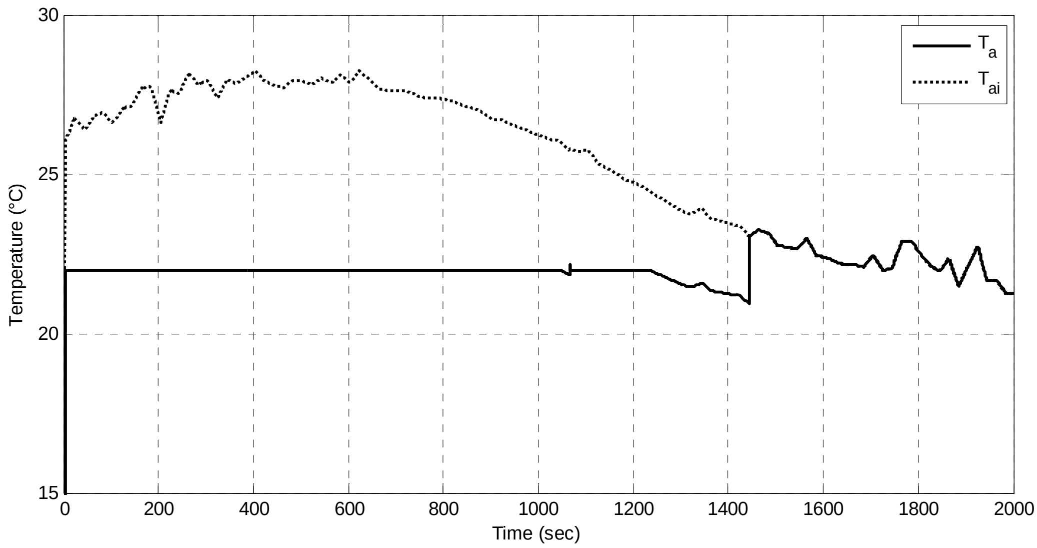

Fig. 13 Time response of controlled temperature in linear optimal control problem of AHU

5 CONCLUSION

Model predictive control belongs to advanced control methods with robust performance and stability. In every control step has to be solved optimization problem (linear or quadratic) which disqualifies this method in situations when computations requirements are prohibitive for technical or cost reasons. In this paper we introduced modeling method for system which consists of continuous dynamics and discrete parts in unique structure of hybrid model (MLD). It was showed that hybrid model can be included to constrained finite-time optimal control (CFTOC) problem. Solution of the CFTOC problem is control law in vector form (on-line/implicit) or as a set of feedback controllers (off-line/explicit). In explicit form the state space is partitioned into polyhedral sets (polytopes) and for each polytopes is defined simple feedback controller. Original computational complexity of MPC is moved to off-line calculation and controller is piecewise linear function.

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