HYBRID PREDICTIVE CONTROL OF NONLINEAR PROCESS

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Abstract: In this paper hybrid fuzzy model based predictive control (HFMBPC) is addressed, proposed and tested. The proposed hybrid fuzzy convolution model consists of a steady-state fuzzy model and a gain independent impulse response model. The proposed model is tested in model based predictive control of the concentration control in the chemical reactor, manipulating its flow rate. The paper deals with theoretical and practical methodology, offering approach for intelligent fuzzy robust control design and its successful application.

Keywords: fuzzy models, hybrid models, predictive control

1 INTRODUCTION AND PRELIMINARIES

Predictive control has become popular over the past twenty years as a powerful tool in feedback control for solving many problems for which other control approaches have been proved to be ineffective. Predictive control is a control strategy that is based on the prediction of the plant output over the extended horizon in the future, which enables the controller to predict future changes of the measurement signal and to base control actions on the prediction. The proposed HFMBPC has been received well in process industry.

2 THE HYBRID FUZZY CONVOLUTION MODEL

The output of the model can be formulated as (Abonyi *et al.* 2000)

$$y_m(k+1) = y_s + K(u_s, x_2, \ldots, x_n).\sum_{i=1}^{N} g_i(x_2, \ldots, x_n).(u(k-i+1) - u_s) \qquad (1)$$

where $y_s + K(u_s, x_2, \ldots, x_n)$ is steady-state part, which is described by Takagi-Sugeno fuzzy model and $\sum_{i=1}^{N} g_i(x_2, \ldots, x_n).(u(k-i+1) - u_s)$ is dynamic part of model (the impulse response model). The gain independent impulse response model is $g_i(x_2,\ldots,x_n)$, the previous input values are $u(k\text{-}i\text{-}1)$ over N horizon, K is steady-state gain, u_s and y_s are steady-state input and output, $x_2,\ldots,x_n$ are other operating parameters having effects on the steady-state output.

The convolution is multiplied by steady-state gain

$$K = \frac{\partial f(u_s, x_2, \ldots, x_n)}{\partial u_s} \qquad (2)$$

2.1 The steady-state part of Hybrid Fuzzy- Neuro Convolution Model (HFNCM)

The steady state part is described by fuzzy-neuro model. A nonlinear discrete system can be expressed by fuzzy-neuro model (ANFIS) with n rules. The i-th rule of the model is described as follows (Paulusová, *et al.* 2008):

$$R^i : if\ x_1\ is\ A_{1,i}\ and \ldots and\ x_n\ is\ A_{n,i}\ then\ y_s = d_i \tag{3}$$

where n is the number of inputs, $x=[x_1,...,x_n]^T$ is a vector of inputs of the model, $A_{j,i}(x_j)$ is the i=1,2,...,M_j –th antecedent fuzzy set referring to the j-th input, where M_j is the number of the fuzzy set on the j-th input domain.

The first element of the input vector is the steady-state input $x_1=u_s$.

The output is computed as weighted average of the individual rules' consequents

$$y_s = \frac{\sum_{i=1}^{m} \mu_i d_i}{\sum_{i=1}^{m} \mu_i} \tag{4}$$

where the weights $0 \le \mu_i \le 1$ are computed as $\mu_i = \prod_{j=1}^{m} A_{ji}(x_j)$, where $\prod$ is fuzzy operator, usually been applied as the *min* or the *product* operator and m is number of rules.

Various types of membership functions were examined in our research. Different membership functions were employed for each fuzzy model: triangular, Gaussian, trapezoidal and bell-type as shown in Fig. 1.

The triangular membership functions are defined as follows:

$$\begin{aligned} A_{j,i}(x_j) &= \frac{x_j - a_{j,i-1}}{a_{j,i} - a_{j,i-1}}, \qquad a_{j,i-1} \le x_j < a_{j,i} \\ A_{j,i}(x_j) &= \frac{a_{j,i+1} - x_j}{a_{j,i+1} - a_{j,i}}, \qquad a_{j,i} \le x_j < a_{j,i+1} \end{aligned} \tag{5}$$

where $x_j \in (a_{j,m_j}, a_{j,m_j+1}\rangle$.

The Gaussian membership functions are defined as follows:

$$\begin{aligned} A_{j,i-1}(x_j) &= e^{-\frac{(x_j - a_{j,i-1})^2}{2\delta_{j,i-1}^2}} \\ A_{j,i}(x_j) &= e^{-\frac{(x_j - a_{j,i})^2}{2\delta_{j,i}^2}} \\ A_{j,i+1}(x_j) &= e^{-\frac{(x_j - a_{j,i+1})^2}{2\delta_{j,i+1}^2}} \end{aligned} \tag{6}$$

where $a_{j,i-1}$, $a_{j,i}$, $a_{j,i+1}$ are the centre, $\delta_{j,i-1}$, $\delta_{j,i}$, $\delta_{j,i+1}$ the width of the fuzzy sets and $x_j \in < a_{j,m_j-1}, a_{j,m_j+1} >$.

The trapezoidal membership functions are defined as follows:

$$A_{j,i}(x_j) = \begin{cases} 0 & x_j < a_{j,i-2}, \quad x_j \ge a_{j,i+2} \\ \frac{x_j - a_{j,i-2}}{a_{j,i-1} - a_{j,i-2}} & a_{j,i-2} \le x_j < a_{j,i-1} \\ 1 & a_{j,i-1} \le x_j < a_{j,i+1} \\ \frac{a_{j,i+2} - x_j}{a_{j,i+2} - a_{j,i+1}} & a_{j,i+1} \le x_j < a_{j,i+2} \end{cases} \tag{7}$$

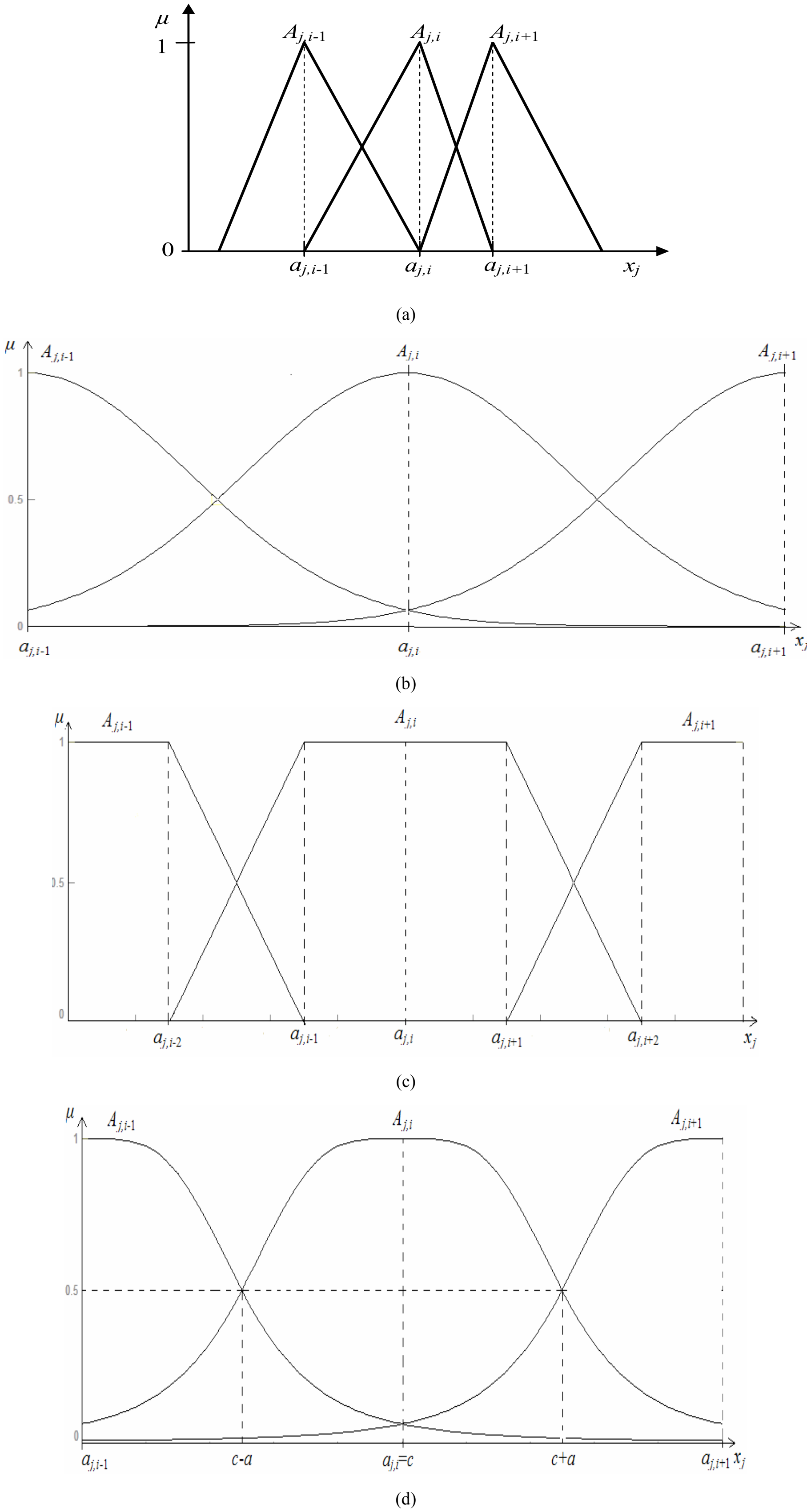

Figure 1: Membership functions used for the fuzzy model: triangular (a), Gaussian (b), trapezoidal (c) and bell-type (d)

The bell membership functions are defined as follows:

$$A_{j,i}\left(x_j\right)=\frac{1}{1+\left|\frac{x_j-c}{a}\right|^{2b}} \qquad a_{j,i-1}\le x_j < a_{j,i+1} \tag{8}$$

where $x_j \in (a_{j,m_j-1}, a_{j,m_j+1}\rangle$, and b is usually positive. The importance of each parameter can be seen in Fig. 1d. The slope function is equal $-\frac{b}{2a}$.

The gain of the steady-state fuzzy model can be computed as

$$K_j=\frac{\partial y_s}{\partial u_s}=\sum_{i=m_j}^{m_j+1}\left[\left(\frac{\Gamma_{i-1}(u_s)}{a_{1,i}-a_{1,i-1}}-\frac{\Gamma_i(u_s)}{a_{1,i+1}-a_{1,i}}\right)\prod_{j=2}^{n}A_{j,i}\left(x_j\right)d_i\right], \qquad K=\sum_{j=1}^{n}K_j \tag{9}$$

where

$$\Gamma_i=1 \qquad if \quad u_s\in(a_{1,i},a_{1,i+1}\rangle$$
$$\Gamma_i=0 \qquad if \quad u_s\notin(a_{1,i},a_{1,i+1}\rangle$$

2.2 Dynamic part of the HFNCM

The dynamic state part is described by the impulse response model (IRM). Parameters of the discrete IRM g_i (i=0,…, N, where N is the model horizon) can be easily calculated from the input-output data (u_i and y_i) of the process.

$$y(k)=\sum_{i=0}^{k}g_i u(k-i) \tag{10}$$

In matrix form

$$\begin{bmatrix} y_0\\ y_1\\ \vdots\\ y_N\end{bmatrix}=\begin{bmatrix} u_0 & 0 & \cdots & 0 & \cdots\\ u_1 & u_0 & \cdots & 0 & \cdots\\ \vdots & \vdots & & & \vdots\\ u_N & u_{N-1} & \cdots & u_0 & \cdots\end{bmatrix}\begin{bmatrix} g_0\\ g_1\\ \vdots\\ g_N\end{bmatrix}$$

The parameters are given as follows

$$g=\left(U^T U\right)^{-1}U^T y \tag{11}$$

3 HYBRID FUZZY-NEURO MODEL BASED PREDICTIVE CONTROLLER

The nonlinear HFNCM can be easily applied in model based predictive control scheme.

In most cases, the difference between system outputs and reference trajectory is used in combination with a cost function on the control effort. A general objective function is the following quadratic form (Paulusová, *et al.* 2005)

$$J=\sum_{i=1}^{p}[\hat{y}(k+i\,|\,k)-r(k+i)]^2\Gamma_y+\sum_{i=1}^{m}(\Delta u(k+i-1))^2\Gamma_u \tag{12}$$

Here, r is desired set point, Γ_u ($\Gamma_u=\gamma K^2$) and Γ_y are weight parameters, determining relative importance of different terms in the cost function, u and Δu are the control signal and its increment, respectively. Parameter p represents length of the prediction horizon, m is the length of the control horizon. Output predicted by the nonlinear fuzzy model is $\hat{y}(k)$.

$$\hat{y}(k) = K\sum_{i=1}^{\infty} s_i \Delta u(k-i) \tag{13}$$

where $s_i = \sum_{j=1}^{i} g_j$ are the step response coefficients and the change on the control variable is $\Delta u(k)=u(k)-u(k-1)$.
Model predictions along the prediction horizon p are

$$\hat{y}(k+j\,|\,k) = K\sum_{i=1}^{\infty} s_i \Delta u(k+j-i) + e(k+j\,|\,k) \tag{14}$$

Disturbances are considered to be constant between sample instants

$$e(k+j\,|\,k) = y(k\,|\,k) - K\sum_{i=1}^{\infty} s_i \Delta u(k+j-i) \tag{15}$$

where $y(k \mid k)$ represents the measured value of the process output at time k.
So

$$\hat{y}(k+j\,|\,k) = K\sum_{i=1}^{N} s_i \Delta u(k+j-i) + f(k+j\,|\,k) \tag{16}$$

Where

$$f(k+j\,|\,k) = y(k\,|\,k) + K\sum_{i=1}^{N} (s_{k+1} - s_i)\Delta u(k-i) \tag{17}$$

Prediction of the process output along the length of the prediction horizon, can be written compactly using matrix notation

$$\hat{y}(k) = KS\Delta u(k) + f(k) \tag{18}$$

Matrix S is called the system's dynamic matrix (19) (Morari, *et al.* 1989).

$$S = \begin{bmatrix} s_1 & 0 & \cdots & 0 \\ s_2 & s_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s_m & s_{m-1} & \cdots & s_1 \\ \vdots & \vdots & \ddots & \vdots \\ s_p & s_{p-1} & \cdots & s_{p-m+1} \end{bmatrix}_{p\times m} \tag{19}$$

By minimizing its objective function (12) the optimal solution is then given

$$\Delta u(k) = \frac{1}{K}(S^T\Gamma_y S + \gamma I)^{-1} S^T \Gamma_y e(k) \tag{20}$$

In many control applications the desired performance cannot be expressed solely as a trajectory following problem. Many practical requirements are more naturally expressed as constraints on process variables such as manipulated variable constraints, manipulated variable rate constraints or output variable constraints. The solution calls into existence of quadratic programming solution of the control problem.

3.1 Algorithm for the HFCM based control

The algorithm has the following steps (Abonyi, *et al*, 1999):

1. Calculation impulse response model g_i from (11),
2. Calculation of u_s from y_s=$y(k)$, considering the inversion of the fuzzy-neuro model,
3. Calculation of the value of the steady-state gain K by (9),
4. Calculation of S by (19) and e by (15),
5. Calculation of the controller output from the first element of the calculated Δu vector generated from (20).

4 CASE STUDY AND SIMULATION RESULTS

4.1 Case study

The application considered involves an isothermal reactor in which the Van Vusse reaction kinetic scheme is carried out. In the following analysis, A is the educt, B the desired product, C and D are unwanted byproducts (Paulusová, *et al.* 2006).

$$\begin{aligned} &A \xrightarrow{k_1} B \xrightarrow{k_2} C \\ &2A \xrightarrow{k_3} D \end{aligned} \qquad (21)$$

From a design perspective the objective is to make k_2 and k_3 small in comparison to k_1 by appropriate choice of catalyst and reaction conditions. The concentration of B in the product may be controlled by the manipulating the inlet flow rate and/or the reaction temperature.
The educt flow contains only cyclopentadiene in low concentration, C_{Af}. Assuming constant density and an ideal residence time distribution within the reactor, the mass balance equations for the relevant concentrations of cyclopentadiene and of the desired product cyclopentanol, C_A and C_B, are as follows:

$$\begin{aligned} &\dot{C}_A = -k_1 C_A - k_3 C_A^2 + \frac{F}{V}(C_{Af} - C_A) \\ &\dot{C}_B = k_1 C_A - k_2 C_B - \frac{F}{V} C_B \\ &y = C_B \end{aligned} \qquad (22)$$

This example has been considered by a number of researchers as a benchmark problem for evaluating nonlinear process control algorithm.
By normalizing the process variables around the following operating point and substituting the values for the physical constants, the process model becomes:

$$\begin{aligned} &\dot{x}_1(t) = -50x_1(t) - 10x_1^2(t) + u(10 - x_1(t)) \\ &\dot{x}_2(t) = 50x_1(t) - 100x_2(t) + u(-x_2(t)) \\ &y(t) = x_2(t) \end{aligned} \qquad (23)$$

where the deviation variable for the concentration of component A is denoted by x_1, the concentration of component B by x_2, and the inlet flow rate by u.

4.2 Simulation results

The comparison of time responses of output of HFNCM with nonlinear plant is shown in Fig. 2. Time responses of the controlled and reference variables under HFNMBPC are shown in Fig. 3.

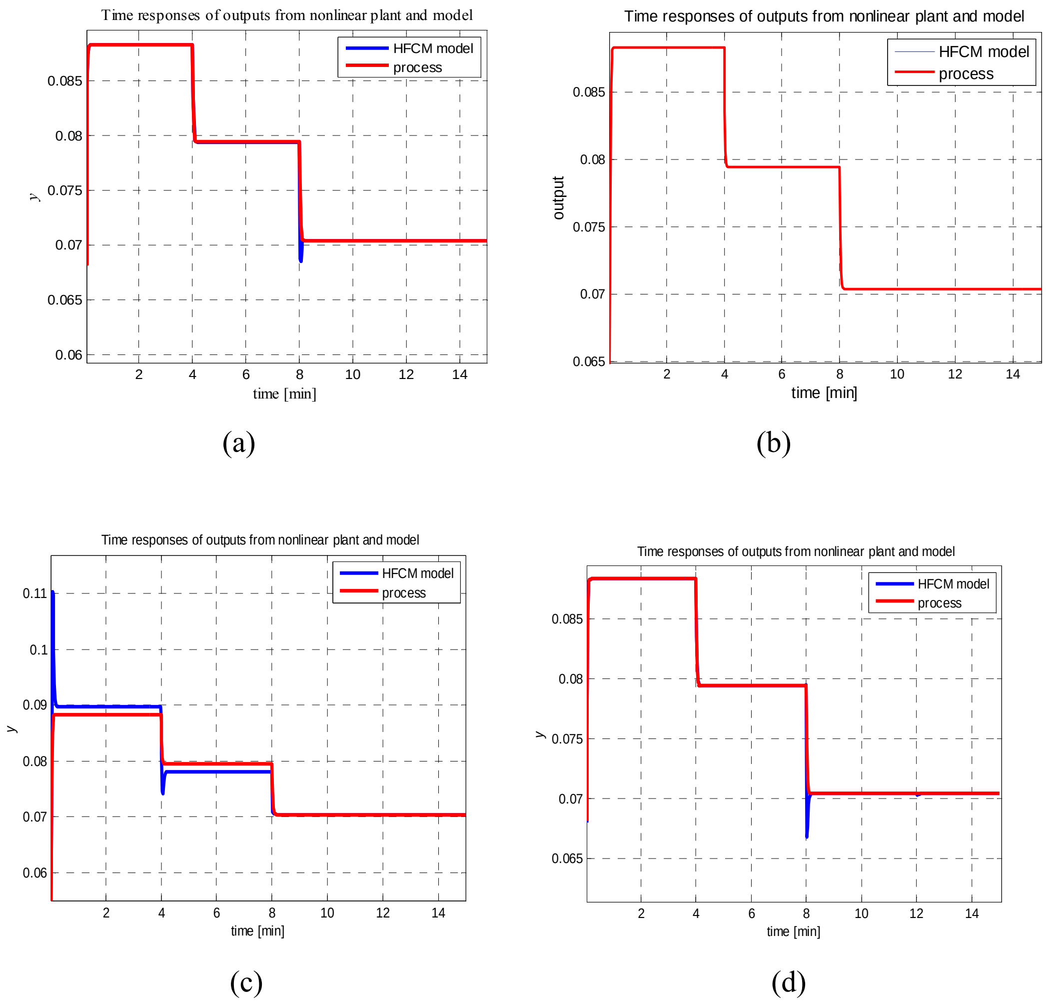

Figure 2: Time responses of output from the nonlinear plant and the HFNCM model with membership functions triangular (a), Gaussian (b), trapezoidal (c) and bell-type (d)

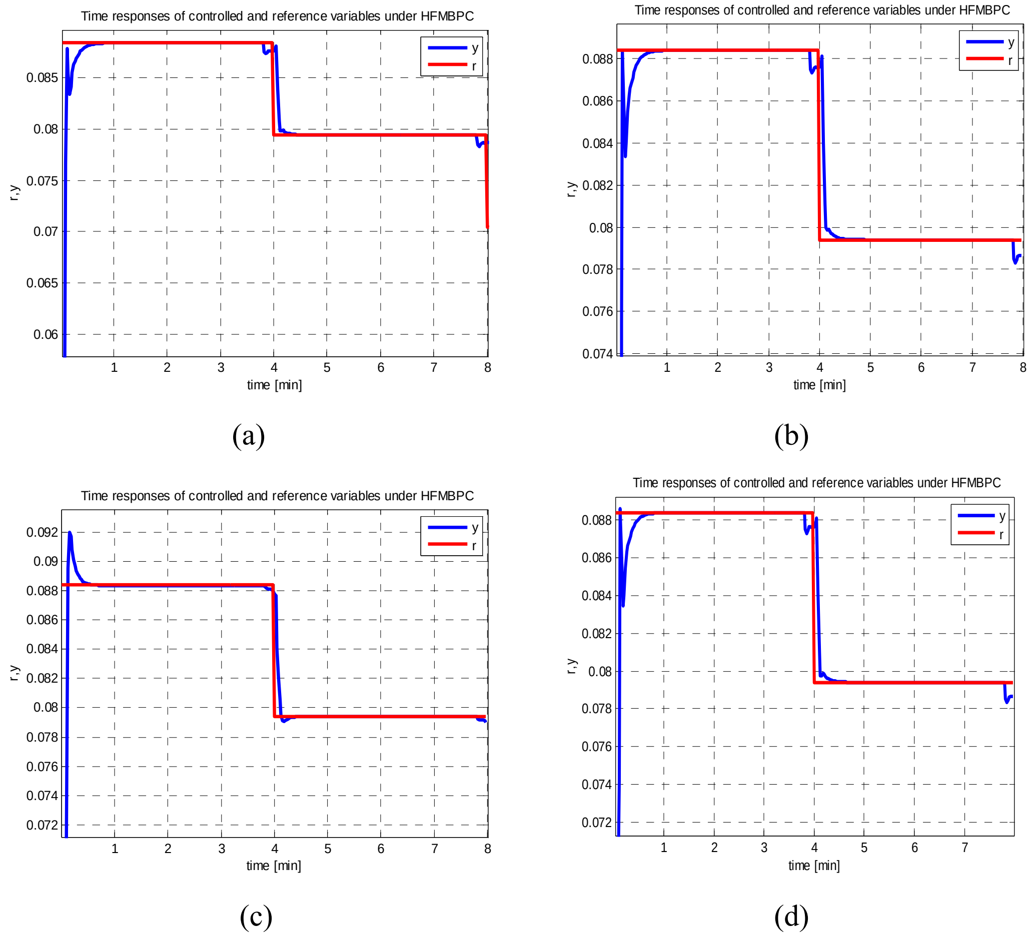

Figure 3: Time responses of the controlled and reference variables under HFNMBPC (m=5, p=10, Γ_y=K, Γ_u=γK^2, γ=1) with membership functions triangular (a), Gaussian (b), trapezoidal (c) and bell-type (d)

5 CONCLUSIONS

The HFMBPC uses the advantage of fuzzy systems in the representation of the steady-state behavior of the system. Other advantage is that it tries to combine knowledge about the system in form of a priori knowledge and measured data in the identification of a control relevant model.

Simulation example illustrates the potential offered by the HFNCMBPC.

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