PREDICTIVE CONTROL OF NONLINEAR PROCESS

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Abstract: In this paper two predictive control approaches are addressed, proposed and tested. The first method described in this paper is Generalized Predictive Control (GPC). The second one is Dynamic Matrix Control (DMC). The proposed algorithms are tested in model based predictive control of the concentration control in the chemical reactor, manipulating its flow rate.

Keywords: DMC, GPC, model predictive control

1   INTRODUCTION AND PRELIMINARIES

Concept of model based predictive control (MBPC) has been heralded as one of the most significant control developments in recent ten years. Wide range of choice of model structures, prediction horizon, and optimization criteria allows the designer to easily tailor MBPC to its application in industry.

The main idea of Model Predictive Control (MPC) is the prediction of the output signal at each sampling instant. The prediction is implicit or explicit depending on the model of the process to be controlled. In the next step a control is selected to bring the predicted process output signal back to the reference signal in the way to minimize the area between the reference and the output signal.

Within basic methods, which are essentially referred as predictive control, DMC and GPC with the added capability of handling nonlinear systems are included.

2   MBPC FORMULATION

MBPC is a general methodology for solving control problems in the time domain. It is based on three main concepts:

1. Explicit use of a model to predict the process output.
2. Computation of a sequence of future control actions by minimizing a given objective function.
3. The use of the receding horizon strategy: only the first control action in the sequence is applied, and the horizons are moved one sample period towards the future, optimisation is repeated.

2.1 Dynamic Matrix Control (DMC)

One of the first proposed MBPC methods, and still commercially the most successful one, is DMC. Cutler introduced this method in 1980.

MPC is an optimization based control methodology that explicitly utilizes a dynamic mathematical model of a process to obtain a control signal by minimizing an objective function. The model must describe the system well. The future process outputs $y(k+i)$ for $i=1,\ldots,p$, are predicted over the prediction horizon (p) using a model of the process.

These values depend on the current process state, and on the future control signals $u(k+i)$ for $i=0,\ldots,m$-1, over the control horizon (m), where $m \leq p$. The control variable is manipulated only within the control horizon and remains constant afterwards, $u(k+i)=u(k+m$-1) for $i=m,\ldots,p$-1. The basic principle of MBPC is shown in Fig. 1.

Process interactions and deadtimes can be intrinsically handled with model predictive control schemes such as DMC. The block-scheme of MPC is shown in Fig. 1 and the principle of DMC in Fig. 2.

The sequence of future control signals is computed by optimizing a given (cost) function. Often, the system needs to follow a certain reference trajectory defined through set points. In most cases, the difference between system outputs and reference trajectory is used by combination with a cost function on the control effort.
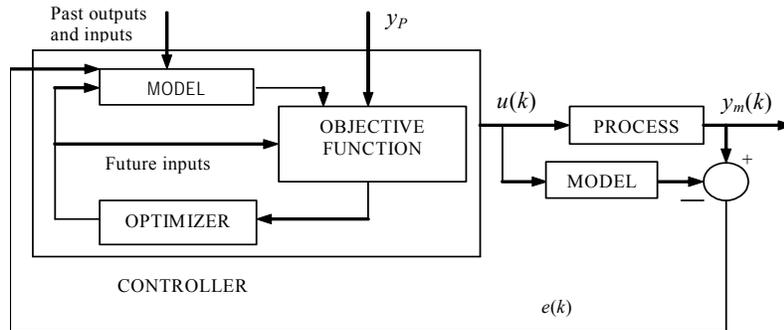

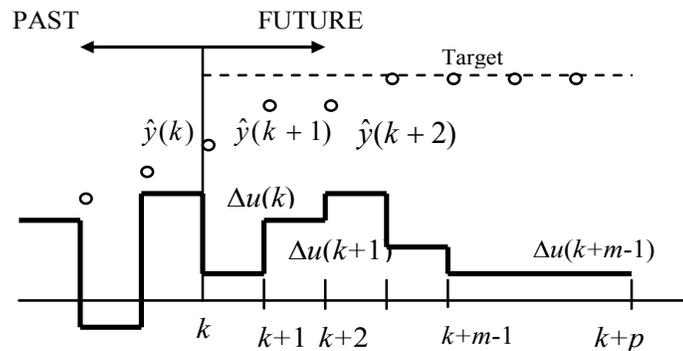Figure 1: Block scheme of model predictive control (MPC)

Figure 2: The principle of DMC

A general objective function is the following quadratic form

$$J = \sum_{i=1}^{p} [y_p(k+1\,|\,k) - \hat{y}(k+i\,|\,k)]^2 \Gamma_y + \sum_{i=1}^{m} \Delta u(k+i-1\,|\,k)^2 \Gamma_u \tag{1}$$

where: y_p - desired set point,

           Γ_u, Γ_y - weight matrix,

           $\Delta u(k-i)=u(k-i)-u(k-i-1)$ - the change in manipulation variable,

p — the length of the prediction horizon,

m — the length of the control horizon,

$\hat{y}(k)$ — the process output, at sample instant is given as

$$\hat{y}(k) = \sum_{i=1}^{\infty} g_i \Delta u(k-i) \tag{2}$$

where: g_i — the step response coefficients.

The model employed is a step response of the plant. The model predictions along the prediction horizon p are

$$\hat{y}(k+j\,|\,k) = \sum_{i=1}^{\infty} g_i \Delta u(k+j-i) + d(k+j\,|\,k) \tag{3}$$

Disturbances are considered to be constant between sample instants

$$d(k+j\,|\,k) = y_m(k\,|\,k) - \sum_{i=1}^{\infty} g_i \Delta u(k+j-i) \tag{4}$$

where: $y_m(k|k)$ — the measured value of the process output at time k.

$$\hat{y}(k+j\,|\,k) = \sum_{i=1}^{k} g_i \Delta u(k+j-i) + f(k+j\,|\,k) \tag{5}$$

where

$$f(k+j\,|\,k) = y_m(k\,|\,k) + \sum_{i=1}^{N} (g_{k+1} - g_i)\Delta u(k-i) \tag{6}$$

The prediction of the process output along the length of the prediction horizon, can be written compactly using matrix notation

$$y_P(k) = G\Delta u(k) + f(k) \tag{7}$$

where: G — dynamic matrix

$$G = \begin{bmatrix} g_1 & 0 & \cdots & 0 \\ g_2 & g_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_m & g_{m-1} & \cdots & g_1 \\ \vdots & \vdots & \ddots & \vdots \\ g_p & g_{p-1} & \cdots & g_{p-m+1} \end{bmatrix} \tag{8}$$

By minimizing objective function the optimal solution is then given in matrix form

$$\Delta u(k) = (G^T \Gamma_y G + \Gamma_u)^{-1} G^T \Gamma_y (y_m(k) - f(k)) \tag{9}$$

2.2 Generalized Predictive Control (GPC)

The CARIMA model, representing the plant model in GPC design, is defined as:

$$A(z^{-1})y(t) = B(z^{-1})u(k-1) + C(z^{-1})\xi(t)/\Delta \tag{10}$$

where Δ – is the differencing operator $1-z^{-1}$,

$\xi(t)$ – denotes white noise sequence,

A, B, C – are polynomials in the backward shift operator z^{-1}.

For simplicity here $C(z^{-1})=1$. To derive j-step ahead predictor of $y(k+j)$ based on (10) consider the identify (Clarce, *et al.* 1987):

$$1 = E_j(z^{-1})A(z^{-1})\Delta + z^{-j}F_j(z^{-1}) \tag{11}$$

Then predicted process output is

$$\hat{y}(k+j \mid k) = G_j(z^{-1})\Delta u(k+j-1 \mid k) + F_j(z^{-1})y(k) \tag{12}$$

Where $G_j(z^{-1}) = E_j(z^{-1})B(z^{-1})$.

Then (12) can be written in vector form

$$\hat{y} = G\Delta u + f \tag{13}$$

Where

$$\hat{y} = \begin{bmatrix} \hat{y}(k+1 \mid k) \\ \hat{y}(k+2 \mid k) \\ \dots \\ \hat{y}(k+p \mid k) \end{bmatrix}, \ \Delta u = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \dots \\ \Delta u(k+m-1) \end{bmatrix}, G = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{p-1} & g_{p-2} & \cdots & g_0 \end{bmatrix}, f = \begin{bmatrix} f(k) \\ f(k+1) \\ \dots \\ f(k+p) \end{bmatrix}$$

Where $f(k+1) = [G_1(z^{-1}) - g_{10}]\Delta u(k) + F_1 y(k)$,

$$f(k+2) = z[G_1(z^{-1}) - z^{-1}g_{21} - g_{20}]\Delta u(k) + F_2 y(k), \text{ etc.}$$

And $G_i(z^{-1}) = g_{i0} + g_{i1}z^{-1} + \dots$

Consider the minimalization of cost function (1) the optimal solution in vector form is

$$\Delta u = (G^T \Gamma_y G + \Gamma_u)^{-1} G^T \Gamma_y (w - f) \tag{14}$$

Where $w = \begin{bmatrix} w(k+1) \\ w(k+2) \\ \dots \\ w(k+p) \end{bmatrix}$

3   CASE STUDY AND SIMULATION RESULTS

The application considered involves an isothermal reactor in which the Van Vusse reaction kinetic scheme is carried out. In the following analysis, *A* is the educt, *B* the desired product, *C* and *D* are unwanted byproducts (Paulusová, *et al.* 2006).

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$
$$2A \xrightarrow{k_3} D \tag{15}$$

From a design perspective the objective is to make k_2 and k_3 small in comparison to k_1 by appropriate choice of catalyst and reaction conditions. The concentration of *B* in the product may be controlled by the manipulating the inlet flow rate and/or the reaction temperature.

The educt flow contains only cyclopentadiene in low concentration, C_{Af}. Assuming constant density and an ideal residence time distribution within the reactor, the mass balance equations for the relevant concentrations of cyclopentadiene and of the desired product cyclopentanol, C_A and C_B, are as follows:

$$\dot{C}_A = -k_1 C_A - k_3 C_A^2 + \frac{F}{V}(C_{Af} - C_A)$$
$$\dot{C}_B = k_1 C_A - k_2 C_B - \frac{F}{V} C_B \tag{16}$$
$$y = C_B$$

This example has been considered by a number of researchers as a benchmark problem for evaluating nonlinear process control algorithm.

By normalizing the process variables around the following operating point and substituting the values for the physical constants, the process model becomes:

$$\dot{x}_1(t) = -50 x_1(t) - 10 x_1^2(t) + u(10 - x_1(t))$$
$$\dot{x}_2(t) = 50 x_1(t) - 100 x_2(t) + u(-x_2(t)) \tag{17}$$
$$y(t) = x_2(t)$$

where the deviation variable for the concentration of component A is denoted by x_1, the concentration of component B by x_2, and the inlet flow rate by u.

3.2 Simulation results

The comparison of time responses of controlled and referenced variables under DMC with GPC is shown in Fig. 3.

Parameters for DMC are $p=10$, $m=5$, $\Gamma_y=0.4$, $\Gamma_u=0.4$.

Parameters for GPC are $p=3$, $m=3$, $\Gamma_y=1$, $\Gamma_u=0.1$.

Quality of control is depended on the choice of parameters p, m, Γ_y, and Γ_u. For our selected parameters is quality of control under DMC better than GPC.
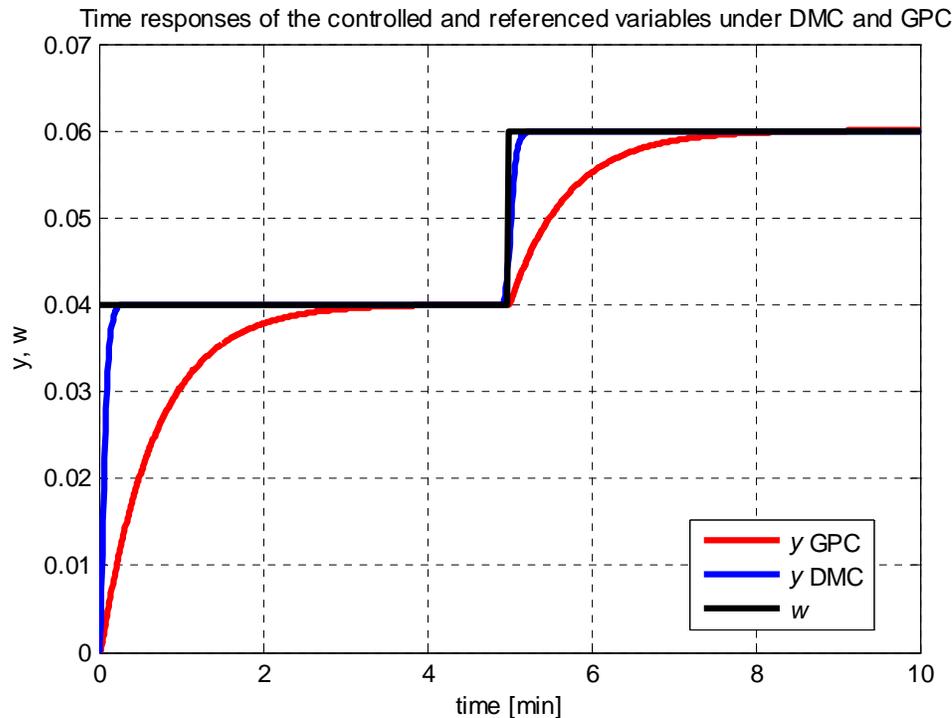
Figure 3: Time responses of the controlled and reference variables under DMC and GPC

4   CONCLUSION

This paper has described two predictive approaches GPC and DMC algorithms. These methods are based on model, which is the future behaviour of procees predicted. The advantages and disadvantages of both methods were presented here.

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