PARAMETRIZATION AND CONVEX APPROXIMATION APPROACH TO STABILIZATION VIA OUTPUT FEEDBACK

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Abstract: A parametric description of all static output feedback stabililizing controllers for switching diffusion systems is presented. This description is expressed in terms of coupled linear matrix equations and non-convex quadratic matrix inequalities which depend on parameter matrices similar to weight matrices in LQR theory. A convexifying approximation technique is proposed to obtain the LMI-based.algorithms for computing of the gain matrix. These are non-iterative and used computationally efficient SDP solvers.The results are then applied to simultaneous stabilization of a set of diffusion systems, robust stabilization and stochastic passification problems.

Keywords: Swithching diffusion, parametrization, output feedback, simultaneous stabilization, robust stabilization, passification, linear quadratic regulator, linear matrix inequalities, convex approximation.

1 INTRODUCTION AND PRELIMINARIES

The problem of stabilization via static output feedback is a tricky problem in the modern control theory. On the one hand there exists a lot of necessary and sufficient conditions of stabilization on the other hand these conditions are hard to implement and can impose very difficult numerical problems. A survey of static output-feedback control is given in (Syrmos et al, 1997). A lot of work has been pursued after publication of this survey (Crusius and Trofino, 1999, Gadewadikar et al, 2007, Rosinova et al, 2003, Yu, 2004) and references therein, however several problems are still open. Moreover there exist few results concerning these problems for the class of stochastic systems (Pakshin and Soliviev, 2009) and references therein. Main objective of this paper is to present a LQR type parameterization of static output feedback controllers for continuous-time diffusion systems with Markovian switching. That parameterization is derived from classical LQR parameterizations of state-feedback controllers with restrictions to have an output-feedback structure (Gadewadikar et al, 2007). Based on this parametrization a new approach to design of static output feedback stabilizing control is developed. This approach leads to algorithms for computation of stabilizing gain which may be implemented with existing semi-definite solvers such as SeDuMi (Sturm, 1999) and easily coded in Matlab environment using YALMIP (Lofberg, 2004). It turns out that particular cases of obtained result give effective solution for simultaneous stabilization, robust stabilization and passification via static output feedback.control.

2 PROBLEM STATEMENT

Consider switching diffusion system (Kats and Martynyuk, 2002; Yin and Zhu, 2009) described by the following equations

$$dx(t) = [A(r(t))x(t) + B(r(t))u(t)] + \sum_{l=1}^{m}\gamma_l(r(t))[A_l(r(t))x(t) + B_l(r(t))u(t)]dw_l(t),$$
$$y(t) = C(r(t))x(t), t \geq 0, \quad (1)$$

where $x(t) \in \mathbb{R}^{n_x}$ is the continuous component of the state, $u(t) \in \mathbb{R}^{n_u}$ is the control input vector; $y(t) \in \mathbb{R}^{n_y}$ is the output vector; $r(t)\,(t \geq 0)$ is the discrete component of the state, taking values in a finite set $\mathbb{N} = \{1, \ldots, N\}$; this component is modeled by homogeneous Markov chain with the mode transition probabilities

$$\text{Prob}(r(t+h) = j \mid r(t) = i) = \begin{cases} \pi_{ij}h + o(h), & \text{if} \quad j \neq i, \\ 1 + \pi_{ii}h + o(h), & \text{if} \quad j = i, \end{cases} \quad (2)$$

$i, j \in \mathbb{N}$, $\pi_{ij} > 0\ (i \neq j)$ denotes the switching rate from mode i at time t to mode j at time $t+h$ for $h > 0$ and $\pi_{ii} = -\sum_{i \neq j}^{N}\pi_{ij}$; the Markov chain transition rates matrix is defined by $\Pi = [\pi_{ij}]_1^N$; $\gamma_l(\cdot)\ (l = 1, \ldots, m)$ are positive scalars; $w(t) = [w_1(t) \ldots w_m(t)]^{\mathrm{T}}$ is the $\mathbb{R}^m$-valued standard Wiener process defined on the probability space $(\Omega, \mathcal{F}, \mathbf{P})$ with natural filtration $\{\mathcal{F}_t\}$; for $r(t) \in \mathbb{N}$ the system matrices and scalar parameters of the i-th mode are denoted by $A_i, B_i, A_{li}, B_{li}, C_i$ and γ_{li} which are real known with appropriate dimensions; the initial conditions $x(0) = x_0, r(0) = r_0$ are deterministic.

The two-component process $[x(\cdot)\ r(\cdot)]^{\mathrm{T}}$ in the hybrid space $\mathbb{R}^{n_x} \times \mathbb{N}$ satisfying (1), (2) is termed a switching diffusion or a mode(regime)-switching diffusion. The components $x(t)$ and $r(t)$ are called continuous and discrete ones corresponding to their sample path properties.

Assume that the switching static output feedback control has the form

$$u(t) = -F_i y(t), \text{if } r(t) = i, i \in \mathbb{N}. \quad (3)$$

The purpose of the paper is to describe in terms of the LQR-type parameters all the gain matrices in (3), such that the system (1), is exponentially stable in the mean square (ESMS) (Kats and Martynyuk, 2002; Yin and Zhu, 2009) and to derive LMI based algorithms (Boyd et al, 1994; Lofberg, 2004; Sturm, 1999;) for computing these gain matrices. The paper extends to the class of switching diffusion systems the results obtained by the authors in (Pakshin and Peaucelle, 2009, a, b; Pakshin, Peaucelle and Zhilina, 2009).

3 LQR PARAMETRIZATION OF ALL STABILIZING GAINS

The following theorem gives a parametric description of all stabilizing static output feedback gains in LQR terms.

Theorem 1 *There exists a gain matrix* F_i *such that the system* (1), (2) *is exponentially stable in the mean square if and only if there exist parameter matrices* $Q_i = Q_i^{\mathrm{T}} \geq 0, R_i = R_i^{\mathrm{T}} > 0$ *such that*

$$F_i C_i = [R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}}P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i], i \in \mathbb{N}, \quad (4)$$

where $P_i = P_i^{\mathrm{T}} > 0$ *and* $L_i\ (i \in \mathbb{N})$ *is a solution to the system of coupled matrix inequalities*

$$A_i^{\mathrm{T}} P_i + P_i A_i - [P_i B_i + \Theta_i(P_i)][R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}}] + Q_i + \Delta_i(P_i)$$
$$+\sum_{j=1}^{N} \pi_{ij} P_j + L_i^{\mathrm{T}}[R_i + \Gamma_i(P_i)]^{-1} L_i < 0, i \in \mathbb{N}, \qquad (5)$$

$$\Gamma_i(P_i) = \sum_{l=1}^{m} \gamma_{li}^2 B_{li}^{\mathrm{T}} P_i B_{li}, \Delta_i(P_i) = \sum_{l=1}^{m} \gamma_{li}^2 A_{li}^{\mathrm{T}} P_i A_{li}, \Theta_i(P_i) = \sum_{l=1}^{m} \gamma_{li}^2 A_{li}^{\mathrm{T}} P_i B_{li}, i \in \mathbb{N}.$$

There is no known methods for solving the nonstandard system of coupled matrix inequalities (4), (5). We propose some convex sufficient conditions which allow to obtain LMI-based algorithms for computing of the gain matrix

This theorem is continuous-time counterpart of the result by (Pakshin and Soloviev,2009) and its proof is omitted.

4 CONVEX SUFFICIENT CONDITIONS AND ALGORITHMS

As for the general static output feedback design, there is no known exact convex methodology for the design of the gain matrix $F_i\,(i \in \mathbb{N})$. Based on existing convexifying techniques, we provide now two conservative LMI based results for the problem. Each of these techniques may possibly fail even if stabilizing gains exist, yet in practice, one or the other, happens to be successful on examples.

4.1 Convex approximation I

Assume given matrices Q_i, R_i and $L_i\,(i \in \mathbb{N})$ and let a scalar $\mu_i\,(i \in \mathbb{N})$ sufficiently large for the following inequality to hold

$$\begin{bmatrix} \mu_i Q_i & L_i^{\mathrm{T}} \\ L_i & R_i + \Gamma_i(P_i) \end{bmatrix} > 0, i \in \mathbb{N}. \qquad (6)$$

Assume as well $P_i = P_i^{\mathrm{T}} > 0\,(i \in \mathbb{N})$ solution to the coupled Riccati equations

$$A_i^{\mathrm{T}} P_i + P_i A_i - [P_i B_i + \Theta_i(P_i)][R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}}] + \sum_{j=1}^{N} \pi_{ij} P_j + (1+\mu_i) Q_i = 0. \qquad (7)$$

Taking into account (5) we easily obtain from (6):

$$A_i^{\mathrm{T}} P_i + P_i A_i - [P_i B_i + \Theta_i(P_i)][R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}}] + Q_i + \Delta_i(P_i)$$
$$+\sum_{j=1}^{N} \pi_{ij} P_j + L_i^{\mathrm{T}}[R_i + \Gamma_i(P_i)]^{-1} L_i < A_i^{\mathrm{T}} P_i + P_i A_i - [P_i B_i + \Theta_i(P_i)][R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}} P_i$$
$$+\Theta_i(P_i)^{\mathrm{T}}] + \Delta_i(P_i) + \sum_{j=1}^{N} \pi_{ij} P_j + (1+\mu_i) Q_i = 0.$$

The equation (4) has exact solution with respect to gain matrix only for special form of the right hand side. According to (Skelton et al., 1997) this equation is solvable with respect to if and only if

$$[B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i](I - C_i^{+} C_i) = 0, \qquad (8)$$

where C^{+} stands for the Moore-Penrose inverse of C. Moreover the solution of (4) is then given by

$$F_i = [R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i]C^{+}. \tag{9}$$

These conditions can be also formulated in terms of singular value decomposition of the output matrix C_i (Gadewadikar et al.,2007; Yu, 2004; Pakshin and Soloviev, 2009).

So we have the following result

Corollary 1 *Let for some scalar* $\mu_i > 0$ *and parameter matrices* $Q_i = Q_i^{\mathrm{T}} \geq 0$, $R_i = R_i^{\mathrm{T}} > 0$ $(i \in \mathbb{N})$ *the system of coupled Riccati equations* (7) *has positive definite solution* $P_i = P_i^{\mathrm{T}} > 0$ *satisfying* (6), (8) *for some matrix* L_i $(i \in \mathbb{N})$. *Then the control law* (3) *with the gain matrix given by* (9) *provides ESMS of the system* (1).

Based on convex sufficient conditions of Corollary 1 and LMI based method to solution of Riccati equation (Ait Rami and El Ghaoui, 1996) we can formulate the following algorithm for the design of stabilizing gains F_i.

Algorithm 1

Step 1 *Assign matrices* $Q_i = Q_i^{\mathrm{T}} \geq 0$, $R_i = R_i^{\mathrm{T}} > 0$ $(i \in \mathbb{N})$, *scalar* $\mu_i > 0$ *based on LQR reasons and solve the following LMI optimization problem with respect to variables* $P_i = P_i^{\mathrm{T}} > 0$ *and* L_i:

$$\mathrm{Tr}\sum_{j=1}^{N} P_j \to \max,$$

$$\begin{bmatrix} A_i^{\mathrm{T}} P_i + P_i A_i + \Delta_i(P_i) + \sum_{j=1}^{N}\pi_{ij} P_j + (1+\mu_i)Q_i & P_i B_i + \Theta_i(P_i) \\ B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} & \Gamma_i(P_i) + R_i \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} \mu_i Q_i & L_i^{\mathrm{T}} \\ L_i & \Gamma_i(P_i) + R_i \end{bmatrix} > 0,\ [B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i][I - C_i^{+} C_i] = 0,\ i \in \mathbb{N}.$$

Step 2 *If the LMI problem on the previous step is feasible then compute a static output feedback gain by the formula* (9)

Step 3 *If the LMI with respect to variables* S_i $(i \in \mathbb{N})$

$$(A_i - B_i F_i C_i)^{\mathrm{T}} S_i + S_i(A_i - B_i F_i C_i) + \sum_{j=1}^{N}\pi_{ij} S_j + \sum_{l=1}^{m}\gamma_{li}^2 (A_{li} - B_{li} F_i C_i)^{\mathrm{T}} S_i (A_{li} - B_{li} F_i C_i) < 0$$

is feasible, then F_i $(i \in \mathbb{N})$ given by (9)is a ESMS gain.

4.2 Convex approximation-II

Let for some parameter matrices $Q_i = Q_i^{\mathrm{T}} \geq 0$, $R_i = R_i^{\mathrm{T}} > 0$ $(i \in \mathbb{N})$ the following system of linear matrix inequalities with respect to variables $X_i = X_i^{\mathrm{T}} > 0$ and Y_i holds:

$$\begin{bmatrix} \Lambda_{11i} & \Lambda_{12i} & \Lambda_{13i} & \Lambda_{14i} \\ \Lambda_{12i}^{\mathrm{T}} & \Lambda_{22i} & 0 & 0 \\ \Lambda_{13i}^{\mathrm{T}} & 0 & \Lambda_{33i} & 0 \\ \Lambda_{14i}^{\mathrm{T}} & 0 & 0 & \Lambda_{44i} \end{bmatrix} < 0,\ i \in \mathbb{N}, \tag{10}$$

where

$$\Lambda_{11i} = [(A_i X_i - B_i Y_i C_i)^{\mathrm{T}} + (A_i X_i - B_i Y_i C_i) + \pi_{ii} X_i],$$

$$\Lambda_{12i} = [\pi_{i1}^{\frac{1}{2}} X_i \dots \pi_{ii-1}^{\frac{1}{2}} X_i \; \pi_{ii+1}^{\frac{1}{2}} X_i \dots \pi_{iN}^{\frac{1}{2}} X_i],$$

$$\Lambda_{13i} = [\gamma_{1i}(A_{1i} X_i - B_{1i} Y_i C_i)^{\mathrm{T}} \dots \gamma_{mi}(A_{mi} X_i - B_{mi} Y_i C_i)^{\mathrm{T}}],$$

$$\Lambda_{14i} = [X_i Q_i^{\frac{1}{2}} \;\; C_i^{\mathrm{T}} Y_i^{\mathrm{T}}],\ \Lambda_{22i} = \mathrm{diag}[-X_1 \dots - X_{i-1} - X_{i+1} \dots - X_N],$$

$$\Lambda_{33i} = \mathrm{diag}[-X_i \dots - X_i],\ \Lambda_{44i} = [-I_{n_x} - R_i^{-1}].$$

Following to (Crusius and Trofino, 1999) assume that there exists a decision variables $Z_i \ (i \in \mathbb{N})$ such that

$$C_i X_i = Z_i C_i \tag{11}$$

and suppose

$$F_i = Y_i Z_i^{-1}. \tag{12}$$

Denote $P_i = X_i^{-1} \ (i \in \mathbb{N})$. Then taking into account (11), (12) and using Schur complement arguments rewrite (10) in the following form

$$\begin{aligned}&(A_i - B_i F_i C_i)^{\mathrm{T}} P_i + P_i (A_i - B_i F_i C_i) + \sum_{j=1}^{N} \pi_{ij} P_j \\ &+ \sum_{l=1}^{m} \gamma_{li}^2 (A_{li} - B_{li} F_i C_i)^{\mathrm{T}} P_i (A_{li} - B_{li} F_i C_i) + Q_i + (F_i C_i)^T R_i F_i C_i < 0,\ i \in \mathbb{N}.\end{aligned} \tag{13}$$

Because $P_i = P_i^{\mathrm{T}} > 0$ it follows from (13) that system (1)-(3) is ESMS and stabilizing gain is given by (12)

So we have the following result

Corollary 2 *Let for some parameter matrices* $Q_i = Q_i^{\mathrm{T}} \geq 0,\ \ R_i = R_i^{\mathrm{T}} > 0 \ (i \in \mathbb{N})$ *the system of coupled linear matrix equations and inequalities* (10), (11) *with respect to variables* X_i, Y_i *and* $Z_i \ (i \in \mathbb{N})$ *is feasible. Then the control law* (3) *with the gain matrix* F_i *given by* (12) *provides ESMS of the system* (1).

Based on these sufficient conditions it is easy to formulate the algorithm for obtaining of the stabilizing gain.

Algorithm 2

Step 1. *Assign matrices* $Q_i = Q_i^{\mathrm{T}} \geq 0,\ \ R_i = R_i^{\mathrm{T}} > 0 \ (i \in \mathbb{N})$, *based on LQR reasons and solve the LMI/LME problem* (10), (11) *with respect to variables* X_i, Y_i *and* $Z_i \ (i \in \mathbb{N})$.

Step 2. *If the LMI/LME problem on the previous previous step is feasible then compute the static output feedback stabilizing gain matrix* F_i *by the formula* (12).

5 APPLICATION TO SIMULTANEOUS STABILIZATION PROBLEM

The particular case $F_i = F$, $\pi_{ij} \equiv 0$, $i, j \in \mathbb{N}$ corresponds to the problem of simultaneous stabilization of the set of linear diffusion systems

$$dx(t) = [A_i x(t) + B_i u(t)] + \sum_{l=1}^{m} \gamma_{li} [A_{li} x(t) + B_{li} u(t)] dw_l(t), \quad y(t) = C_i x(t), t \geq 0, \; i \in \mathbb{N} \tag{14}$$

via output feedback with constant gain

$$u(t) = -Fy(t). \tag{15}$$

Corollary 3 *Let for some scalar* $\mu_i > 0$ *and parameter matrices* $Q_i = Q_i^{\mathrm{T}} \geq 0$, $R_i = R_i^{\mathrm{T}} > 0$ $(i \in \mathbb{N})$ *the system of coupled Riccati equations*

$$A_i^{\mathrm{T}} P_i + P_i A_i - [P_i B_i + \Theta_i(P_i)][R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}}] + (1 + \mu_i) Q_i = 0. \tag{16}$$

has positive definite solution $P_i = P_i^{\mathrm{T}} > 0$ *satisfying*

$$\begin{bmatrix} \mu_i Q_i & L_i^{\mathrm{T}} \\ L_i & R_i + \Gamma_i(P_i) \end{bmatrix} > 0, \; B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i](I - C_i^{+} C_i) = 0,$$
$$[R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i]C_i^{+} \tag{17}$$
$$= [R_{i+1} + \Gamma_{i+1}(P_{i+1})]^{-1}[B_{i+1}^{\mathrm{T}} P_{i+1} + \Theta_{i+1}(P_{i+1})^{\mathrm{T}} + L_{i+1}]C_{i+1}^{+}, \; i \in \mathbb{N}$$

for some matrix L_i $(i \in \mathbb{N})$. *Then the control law* (15) *with the gain matrix given by*

$$F = [R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i]C_i^{+} \tag{18}$$

for some $i \in \mathbb{N}$ *provides ESMS of all the systems* (14).

Based on that result and with the same methodology one gets the following algorithm to produce simultaneously stabilizing gains. Note that as the previous algorithm it is used the same conservative assumptions. Moreover, in order to have a unique feedback gain for all systems two additional assumptions in the form of the equality constraints are added.

Algorithm 3

Step 1 *Assign matrix* $Q_i = Q_i^{\mathrm{T}} \geq 0$, *and, scalar* $\mu_i > 0$ $(i \in \mathbb{N})$ *based on LQR reasons and solve the following LMI optimization problem with respect to variables* $P_i = P_i^{\mathrm{T}} > 0, R_i = R_i^{\mathrm{T}} > 0$ *and* L_i:

$$\mathrm{Tr} \sum_{j=1}^{N} P_j \to \max,$$

$$\begin{bmatrix} A_i^{\mathrm{T}} P_i + P_i A_i + \Delta_i(P_i) + (1 + \mu_i) Q_i & P_i B_i + \Theta_i(P_i) \\ B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} & \Gamma_i(P_i) + R_i \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} \mu_i Q_i & L_i^{\mathrm{T}} \\ L_i & \Gamma_i(P_i) + R_i \end{bmatrix} > 0, \; [B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i][I - C_i^{+} C_i] = 0, \quad i \in \mathbb{N}.$$

$$[R_i + \Gamma_i(P_i)] = [R_{i+1} + \Gamma_{i+1}(P_{i+1})], \quad [B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i]C_i^{+}$$
$$= [B_{i+1}^{\mathrm{T}} P_{i+1} + \Theta_{i+1}(P_{i+1})^{\mathrm{T}} + L_{i+1}]C_{i+1}^{+}, \; i \in \mathbb{N}.$$

Step 2 *If the LMI optimization problem on the previous step is feasible then compute a static output feedback gain by the formula* (18)

Step 3 *If the LMI with respect to variables* $S_i\ (i \in \mathbb{N})$

$$(A_i - B_i F C_i)^{\mathrm{T}} S_i + S_i (A_i - B_i F C_i) + \sum_{l=1}^{m} \gamma_{li}^2 (A_{li} - B_{li} F C_i)^{\mathrm{T}} S_i (A_{li} - B_{li} F C_i) < 0$$

is feasible, then the control law with the gain matrix F given by formula (18)is simultaneously stabilizing one.

6 APPLICATION TO ROBUST STABILIZATION PROBLEM

Assume now that the pairs of matrices $(A_i\ B_i)$ are vertices of a polytope defining an uncertain model in which the matrix C defining measurements is uncertainty independent and assume one seeks for a unique Lyapunov matrix $P_i = P\ (i \in \mathbb{N})$. This case corresponds to the problem of quadratic stabilization via output feedback (15) of the linear system with polytopic uncertainty

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{N} \xi_i(t)[A_i x(t) + B_i u(t)], \\ y(t) &= Cx(t),\ i \in \mathbb{N}, \end{aligned} \tag{19}$$

where $\xi(t) = \left(\xi_1(t) \quad \dots \quad \xi_N(t)\right)$ belongs for all t to the simplex

$$\Xi = \left\{ \ \xi_i \geq 0\ ,\ \sum_{i=1}^{N} \xi_i = 1 \ \right\}.$$

Results of Theorem 1 apply and produce the following corollary.

Corollary 3 *There exists a gain matrix* F *such that the uncertain system* (19), (15) *is quadratically stable if and only if there exist parameter matrices* $Q_i = Q_i^{\mathrm{T}} \geq 0,\ R_i = R_i^{\mathrm{T}} > 0$ *such that*

$$FC = R_i^{-1}[B_i^{\mathrm{T}} P + L_i],\ i \in \mathbb{N},$$

where $P = P^T > 0$ *and* $L_i\ (i \in \mathbb{N})$ *is a solution to the system of matrix inequalities*

$$A_i^{\mathrm{T}} P + P A_i - P B_i R_i^{-1} B_i^{\mathrm{T}} P + Q_i + L_i^{\mathrm{T}} R_i^{-1} L_i < 0,\ i \in \mathbb{N}.$$

Based on this corollary and with the the same methodology as upper we can formulate the following robust stabilization algorithm. Its conservatism is analogous to the previous ones, only with the assumption that the C matrix is unique for all uncertainties.

Algorithm 4

Step 1 Assign matrices Q_i and R_i, based on LQR reasons on the vertices of the polytope and solve the LMI/LME problem with respect to variables X, Y_i, Z and K

$$\begin{bmatrix} XA_i^{\mathrm{T}} + A_i X - B_i^{\mathrm{T}} R_i^{-1} B_i & XQ_i^{\frac{1}{2}} & Y_i^{\mathrm{T}} \\ Q_i^{\frac{1}{2}} X & -I & 0 \\ Y_i & 0 & -R_i \end{bmatrix} < 0, \tag{20}$$

$$CX = ZC, \quad K = R_i^{-1}(B_i^{\mathrm{T}} + Y_i)C^{+}, \quad (B_i^{\mathrm{T}} + Y_i)(I - C^{+}C) = 0, i \in \mathbb{N}.$$

Step 2 If the problem (20) is feasible then compute the static output feedback stabilizing gain matrix by the formula

$$F = KZ^{-1} \tag{21}$$

It is easy to see that Algorithm 2 and Algorithm 3 can be also applied with corresponding conservative assumptions.

7 STOCHASTIC PASSIVITY AND PASSIFICATION

Stochastic passivity and dissipativity properties have been studied in (Florchinger, 1999; Pakshin, 2007; Yaesh and Shaked, 2009) and references therein. We consider here the following definition which is a particular case of stochastic exponential dissipativity, see (Pakshin, 2007) and it is stochastic counterpart of G-passivity, see (Andrievskii and Fradkov, 2006). Define for the system (1) some input w and the output z of the same dimensions by the formula

$$z(t) = G(r(t))y(t) + D(r(t)w(t), \tag{22}$$

where G and D are matrices of compatible dimension. System (1) is said to be stochastically G-passive with respect to input w and output z if there exists nonnegative scalar function $V(x,i)$ $(x \in \mathbb{R}^{n_x},\ i \in \mathbb{N})$ and scalar function $\mu(x,i) > 0$ for $x \neq 0$ $(x \in \mathbb{R}^{n_x},\ i \in \mathbb{N})$ such that

$$\mathbf{E}_{xi} V(x(t), r(t)) \le V(x,i) + \mathbf{E}_{xi} \int_0^t [w^{\mathrm{T}}(s) z(s) - \mu(x(s), r(s))] ds, \tag{23}$$

for each solution of the system (1) with deterministic initial conditions $x(0) = x,\ r(0) = i$, where $\mathbf{E}_{xi}$ is expectation operator with $x(0) = x,\ r(0) = i$.

The stochastic passification problem is to find the pair of matrices (F_i, G_i) $(i \in \mathbb{N})$ such that the system (1) with reference input w and with static output feedback

$$u(t) = w(t) - F(r(t))y(t) \tag{24}$$

is exponentially stable in the mean square and stochastically G-passive with respect to input w and output z .Consider $V(x,i) = x^{\mathrm{T}} H_i x,\ H_i = H_i^{\mathrm{T}} > 0\ (x \in \mathbb{R}^{n_x}, i \in \mathbb{N})$ as a candidate storage function and let $\mu(x,i) = x^{\mathrm{T}} W_i x,\ W_i^{\mathrm{T}} = W_i^{\mathrm{T}} > 0\ \ (x \in \mathbb{R}^{n_x}, i \in \mathbb{N})$, then according to (Pakshin, 2007) and (23) the stochastic G-passivity conditions with respect to input (24) and output (22) can be written as

$$\begin{bmatrix} (A_{ci}^{\mathrm{T}} H_i + H_i A_{ci} + W_i + \sum_{j=1}^{N} \pi_{ij} H_j + \sum_{l=1}^{m} A_{cli}^{\mathrm{T}} H_i A_{cli} & H_i B_i - (G_i C_i)^{\mathrm{T}} \\ B_i^{\mathrm{T}} H_i - G_i C_i & -D_i - D_i^{\mathrm{T}} \end{bmatrix} \le 0, i \in \mathbb{N}, \tag{25}$$

where $A_{ci} = A_i - B_i F_i C_i$, $A_{cli} = A_{li} - B_{li} F_i C_i$.

If matrix F_i $(i \in \mathbb{N})$ is known then these bilinear matrix inequalities will be LMIs with respect to matrix G and the passification problem can be solved in the following way. Find the matrix F_i $(i \in \mathbb{N})$ using Algorithm 1 or 2. Then for obtained matrix F_i $(i \in \mathbb{N})$ find matrices G_i $(i \in \mathbb{N})$ and D_i $(i \in \mathbb{N})$ as a solution of LMI (25).

Note that given a ESMS gain F_i $(i \in \mathbb{N})$, any matrix G_i $(i \in \mathbb{N})$ is solution to the problem if one takes D_i $(i \in \mathbb{N})$ positive definite with sufficiently large eigenvalues. But recall that D_i $(i \in \mathbb{N})$ is some parallel feedthrough gain. In practice one would expect it to be zero. Hence it is required to have D_i $(i \in \mathbb{N})$ as close to zero as possible and hence the LMIs should be solved along with some minimization of the norm of D_i $(i \in \mathbb{N})$ for example by performing

$$\min \sum_{i=1}^{N} \mathrm{Trace}(D_i), \quad D_i + D_i^{\mathrm{T}} \geq 0 \quad i \in \mathbb{N}.$$

It one gets at the optimum $D_i = 0\,(i \in \mathbb{N})$ then passivity is demonstrated with respect to the output $z(t) = G(r(t))y(t)$.

8 CONCLUSIONS

LQR type paramerization of static output feedback gains for linear diffusion systems with Markovian switching is proposed. This parametrization gives more complicated non-convex relations than original Lyapunov like inequalities, but it turns out that convex approximation technique can be effectively used to these relation to obtain LMI-based algorithm for computing of stabilizing gain. This result then applied to simultaneous and robust stabilization and robust passification problems. The results are conservative because additional convexifying restrictions. The evaluation of the degree of conservatism is an interesting open problem.

The reader is addressed to (Pakshin and Peaucelle, 2009, a,b; Pakshin, Peaucelle and Zhilina, 2009) for computation details connected with the example of angular longitudinal stabilization of aircraft with uncertain parameters via static output feedback. The passive output design procedure is also considered for this example.

ACKNOWLEDGEMENT

This work is supported in part by Russian Foundation for Basic Research under grants 07-01-92166, 08-01-97036 and 10-08-00843. It is supported as well by CNRS-RFBR research cooperation program PICS No. 4281

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