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PROMATCH

AACHENER VERFAHRENSTECHNIK

A Grey-Box Modeling Approach for the Reduction of Nonlinear Systems

Reinout Romijn[1], Leyla Özkan[2],
Siep Weiland[3], Jobert Ludlage[2],
Wolfgang Marquardt[1]

[1] Process Systems Engineering, RWTH Aachen University, Germany
[2] IPCOS, Boxtel, The Netherlands
[3] Eindhoven University of Technology, Eindhoven, The Netherlands

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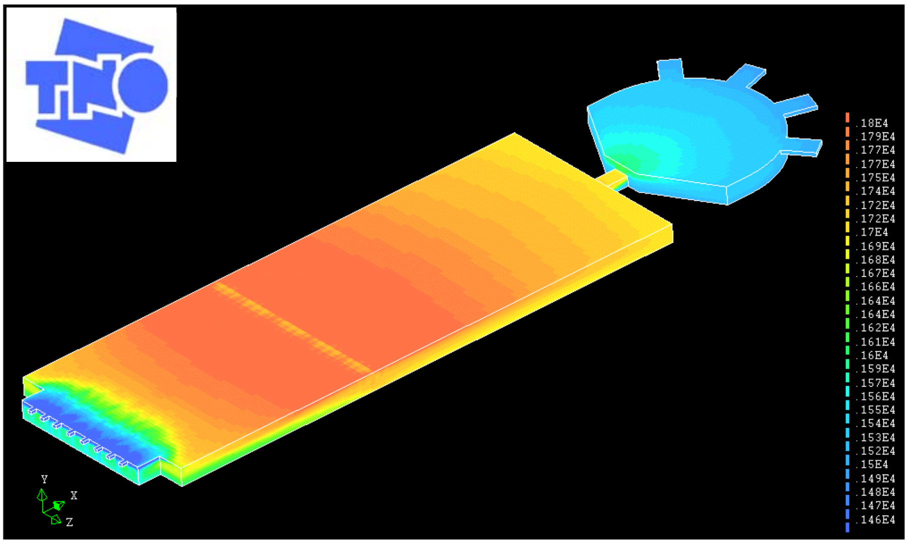
Motivating Example

Industrial Model of a Glass Melting Tank



- > 360.000 grid cells
- > 5 balance equations per cell
- > n constitutive equations per cell

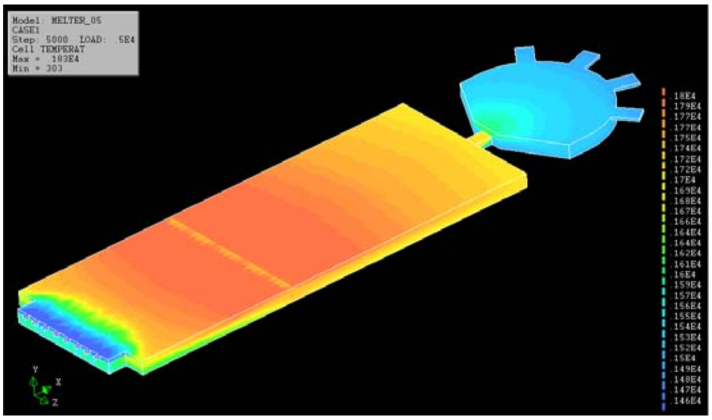
→ extremely high dimensional model

- simulation time: high
- online applications or parameter estimation: not feasible

Problem Setting

How to identify unknown relations *or* replace highly complex ones?

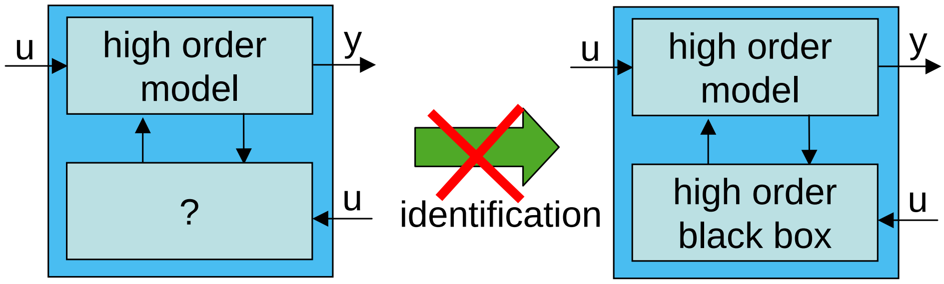
- Radiative heat sources
- Reaction kinetics
- Physical properties at extreme conditions



Conventional identification: not feasible

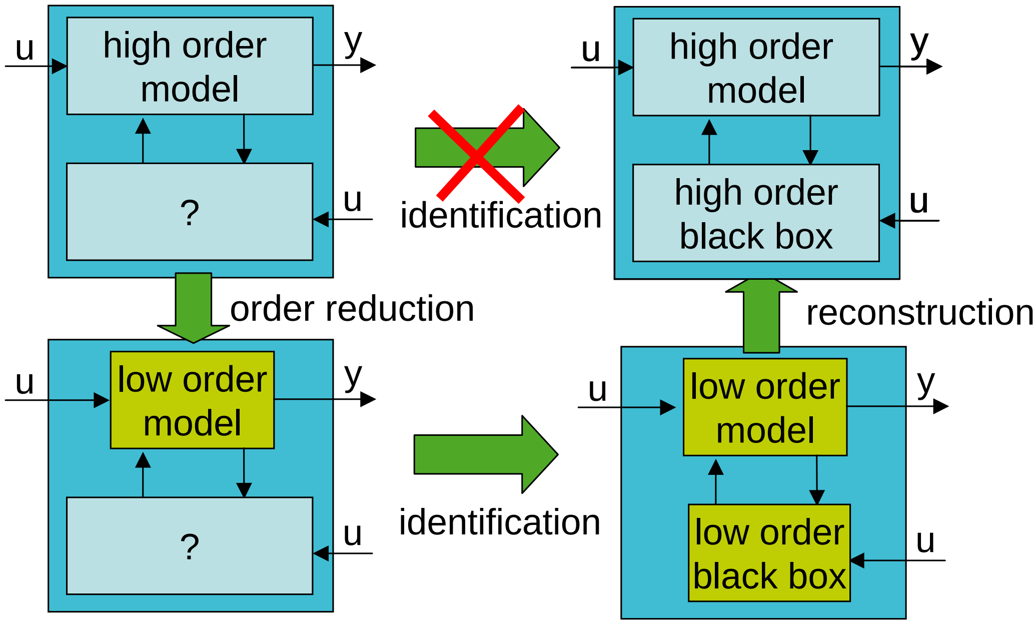
→ grey-box model reduction strategy is proposed

Conventional grey-box identification

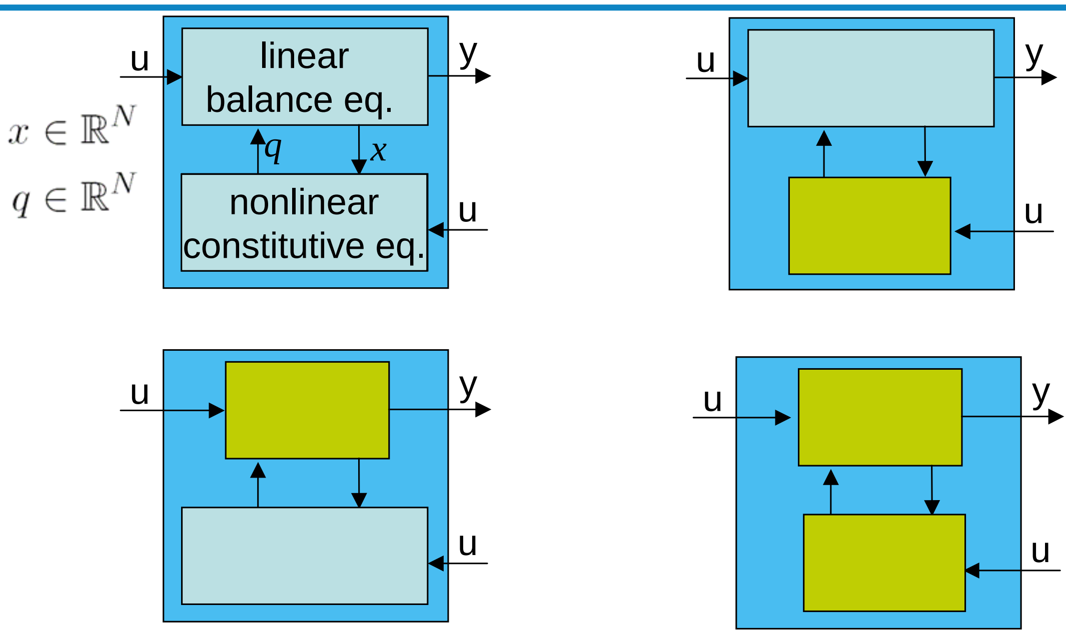


Psichogios and Ungar (1992), Thompson and Kramer (1994)

Grey-box model reduction strategy



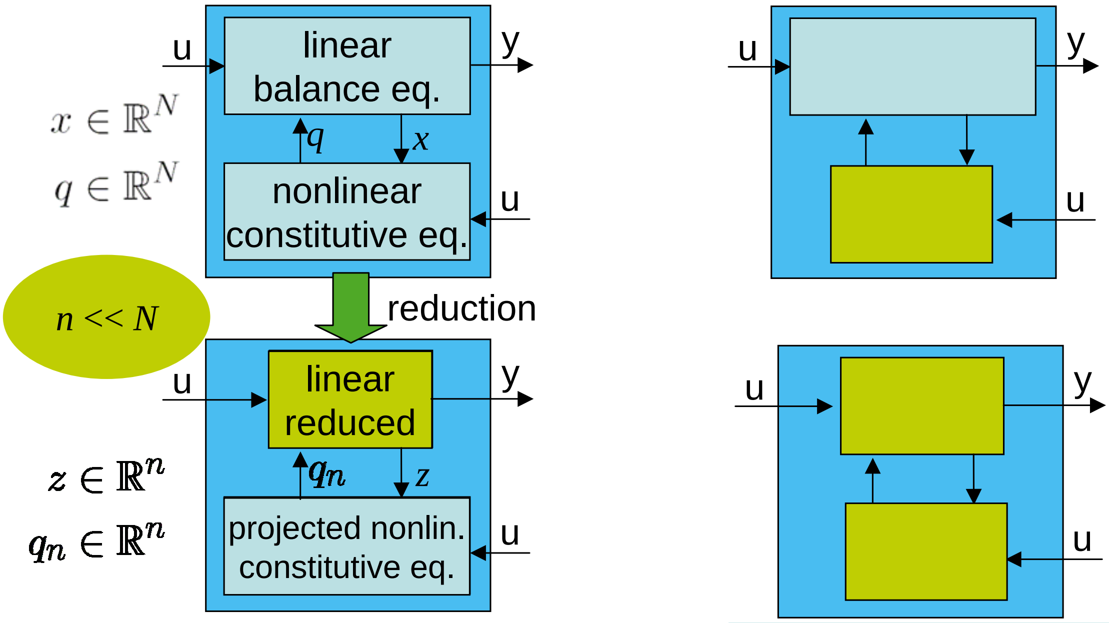
Reduction Strategy – Separation



- balance equations: linear
- constitutive equations: nonlinear

$$\frac{dx}{dt} = f(x,u) \rightarrow \begin{cases} \frac{dx}{dt} = Ax + Bu + q \\ q = \mathcal{F}(x,u) \end{cases}$$

Reduction Strategy – Order Reduction

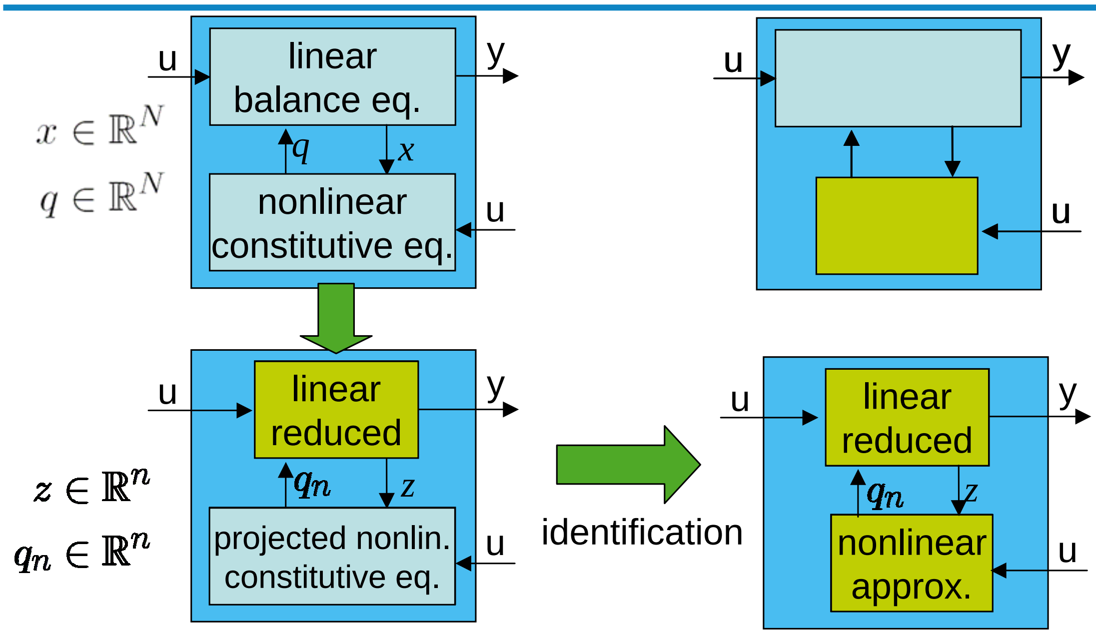


projection of equations

$$\begin{cases} \frac{dz}{dt} = A_n z + B_n u + q_n \\ q_n = \Phi_n^\top \mathcal{F}(\Phi_n z, u) \end{cases}$$

- projection matrix $\Phi \in \mathbb{R}^{NxN}$
- basis $\Phi_n \in \mathbb{R}^{Nxn}$
- new variable $x \approx \Phi_n z$

Reduction Strategy – Black-box Identification

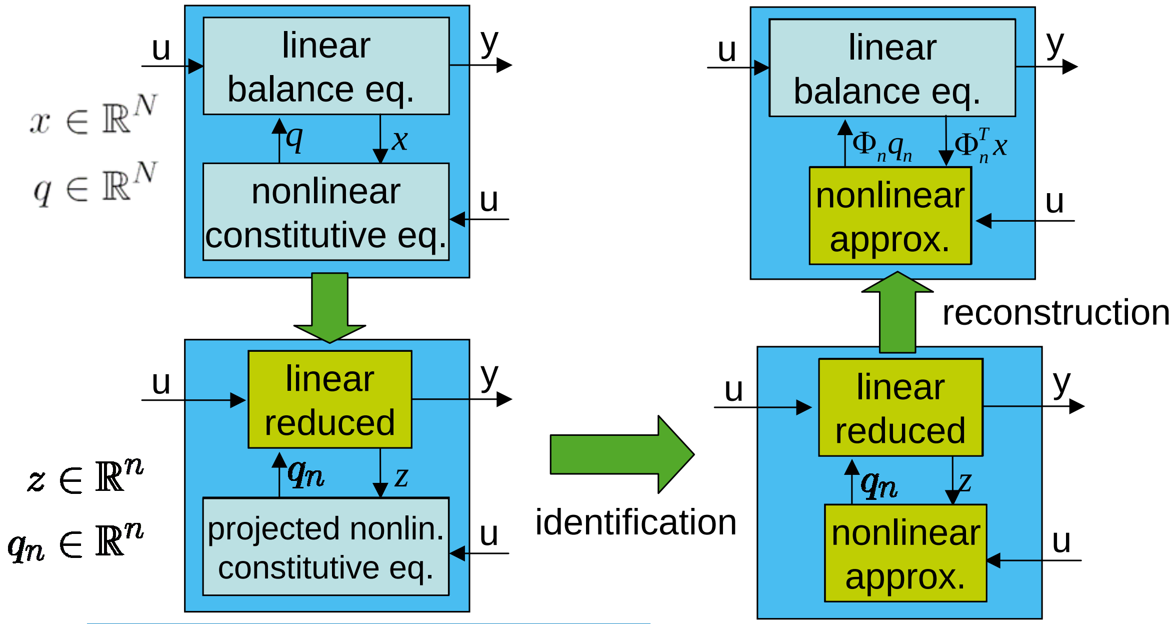


$$\begin{cases} \frac{dz}{dt} = A_n z + B_n u + q_n \\ q_n = \Phi_n^\top \mathcal{F}(\Phi_n z, u) \end{cases}$$

completely reduced model

$$\begin{cases} \frac{dz}{dt} = A_n z + B_n u + q_n \\ q_n = \mathcal{E}(z, u, \theta) \end{cases}$$

Reduction Strategy – Model Reconstruction



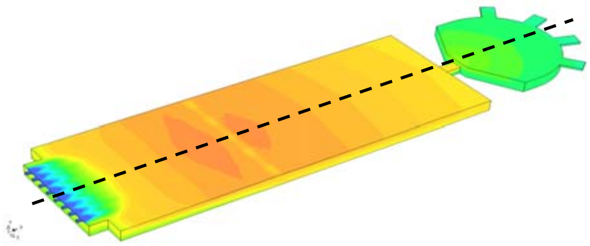
reconstructed by coupling

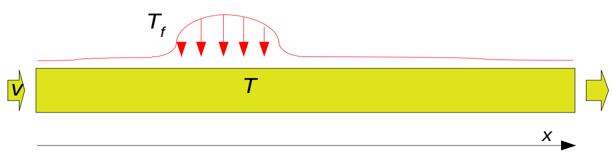
$$\begin{cases} \frac{dx}{dt} = Ax + Bu + \Phi_n q_n \\ q_n = \mathcal{E}\left(\Phi_n^\top x, u, \theta\right) \end{cases}$$

completely reduced model

$$\begin{cases} \frac{dz}{dt} = A_n z + B_n u + q_n \\ q_n = \mathcal{E}\left(z, u, \theta\right) \end{cases}$$

Case Study - Model





- balance equation

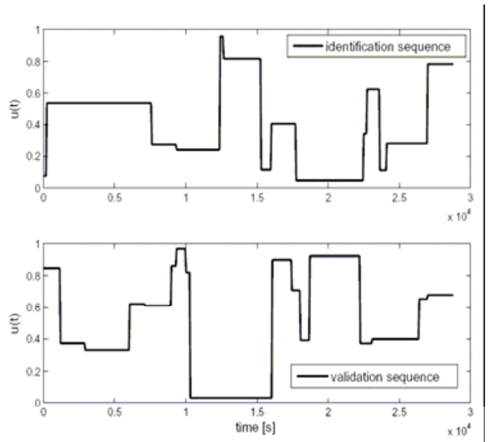
$$\frac{\partial T}{\partial t} = -v\frac{\partial T}{\partial x} + \alpha\frac{\partial^2 T}{\partial x^2} + q_{\text{ext}}$$

- nonlinear source term

$$q_{\text{ext}} = \frac{\sigma}{\rho c_p}\left(c_1 T_f^4 - c_2 T^4\right)$$

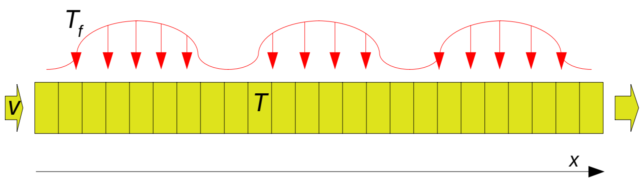
- input

$$T_f(x,t) = T_f(x)u(t)$$



Case Study - Reduction

- Method Of Lines
  - discretization of spatial domain
  - approximation of spatial derivatives by finite differences



→ finite set of 1st order ODE's

- Order reduction by Proper Orthogonal Decomposition
  - data obtained from simulation
  - Φ : singular vectors of data matrix
  - Basis $\Phi_n \in \mathbb{R}^{100x5}$

$$T \in \mathbb{R}^{100} \to z \in \mathbb{R}^5$$

order reduction by factor 20

Case Study - Identification

- replacement of

$$q_{\text{ext}} = \frac{\sigma}{\rho c_p}\left(c_1 T_{\text{f}}^4 - c_2 T^4\right)$$

- by neural network

$$q = \mathcal{E}\left(z, z_{\text{f}}, \theta\right)$$

with

$$\theta \in \mathbb{R}^{69}$$

parameter estimation problem

$$\min_{\theta} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(z_{i,j} - \hat{z}_{i,j}\right)^2$$

s.t.

$$\begin{cases} \dot{z} = A_r z + B_r T_b + q_r \\ q_r = \mathcal{E}\left(z, z_{\text{f}}, \theta\right) \\ \hat{z} = \Phi_n^\top \hat{T} \\ z_0 = \Phi_n^\top T_0 \end{cases}$$

	Function evaluations	Iterations	CPU time
High-order model (Eq. (12))	175	67	1972 s
Reduced order model (Eq. (21))	331	224	330 s

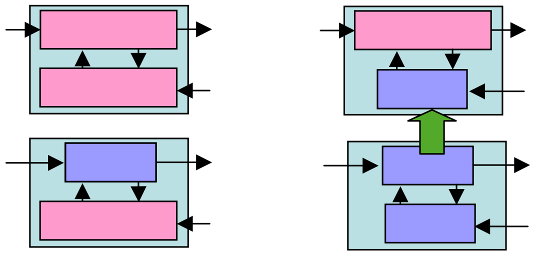
speed-up of identification by factor 6

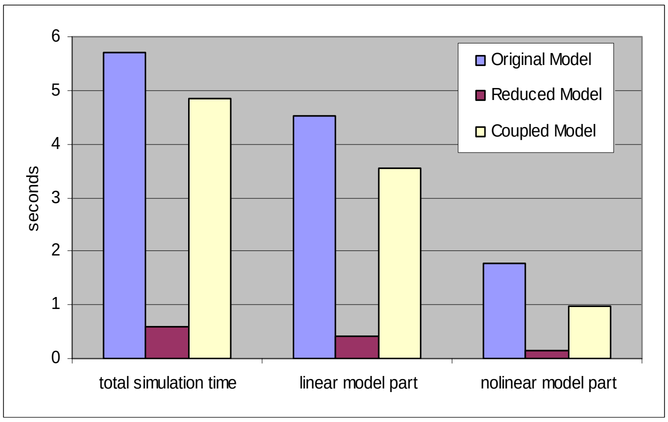
Computational load of the various models

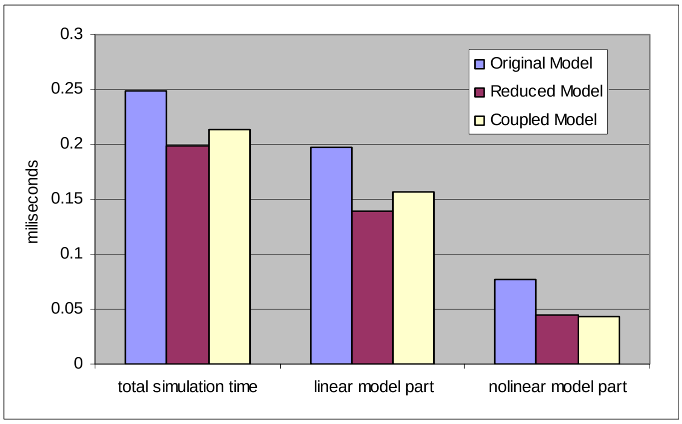
- simulation of reduced model
- reconstruct temperatures

$$\tilde{T} = \Phi_n z|$$

- coupling neural network to high order linear part
- simulation of reconstructed model



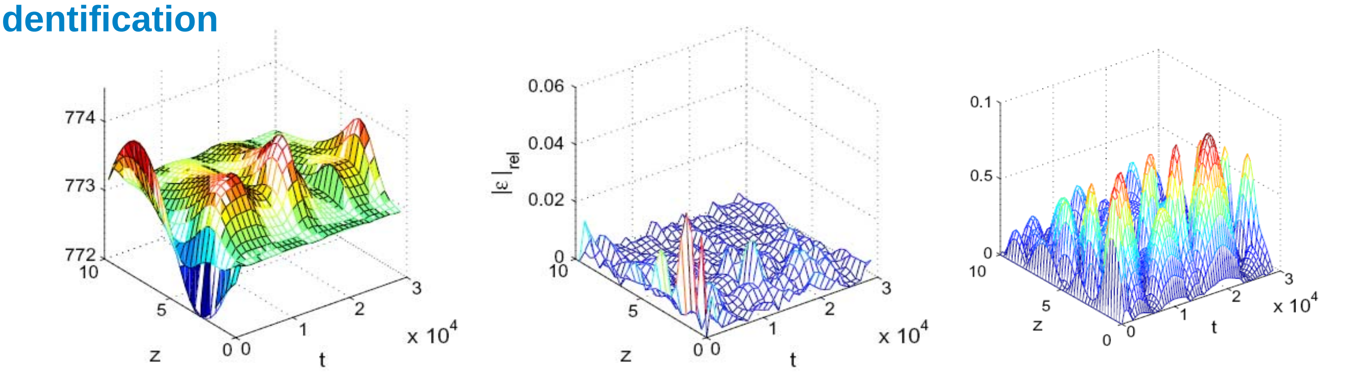




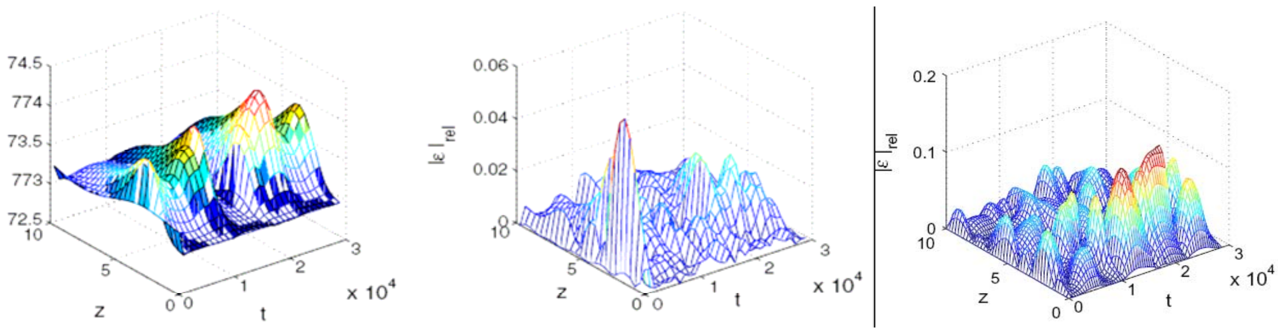
Identification and Validation errors

Temperature profiles	Error Reduced Model	Error Coupled Model

Identification



Validation



Conclusions

- proposed a novel reduction framework especially suited for structured PDE models, which reduces both linear (balances) and nonlinear (constitutive eq.) model parts

- case study exploration
  - proof of concept for model identification in reduction framework
  - identification of a black box in low dimension considerably faster by factor 6