Comparison of Different Methods for Pressure Drop Calculation in 90° and 180° Elbows

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The resistance to flow through the various “piping” components, such as fittings, valves and connections significantly contributes to pressure drop of a whole piping system. Moreover, some of fittings, like elbows, are frequently used also as important part of heat transfer equipment and significantly influences its thermal-hydraulic characteristics.

Heat exchangers designed as tube bundles for boilers, fired heaters, heat recovery steam generators (HRSG) or waste-to energy incinerators prefers the 90° and/or 180° elbows for connection of individual straight circular tubes in the tube bundle. When such heat exchanger would be designed for minimum pressure drop of heated liquid or gas fluid, the accuracy of pressure drop calculation of the elbow plays very important role.

Elbow as a bend or curve in a pipe always induces a larger energy loss than a straight pipe of equivalent arc length. Paper presents comparison of the most frequently cited calculation methods (for 90° and 180° elbow) based on evaluation of results of series calculations with different elbow curvature (short or long elbow type) and with varying linear velocity of water.

Comparison of tested calculation methods results in the identification and recommendation of reliable calculation method(s). This analysis wants to provide engineers with overview of calculation methods for pressure drops in elbows and with recommendation for opting of the most suitable and most accurate method.

1. Introduction

Calculation of pressure drop in straight pipe is not a difficult engineering task to do. However, calculation of pressure drop in pipeline components (elbows, tube fittings and various valves, etc.) may become more complicated. This paper deals with calculation of pressure drop in 90° and 180° elbows (presented calculations may be applied for pressure drop in other types of pipe components, but with other coefficient); paper thus gives an overview of most used calculation methods and tries to give the recommendation for the best one for calculation of pressure drop. There are several methods for calculation of pressure drop in elbows. However, not all of them provide accurate results.

Paper describes various procedures for calculation of this type of pressure drops. Elbows are categorized according to the ratio of their radius (R) and inner diameter (D), thus we may talk about long radius elbow (Fig. 1a), i.e. R/D >= 1.5, and short radius elbow (Figure 1b), i.e. R/D < 1.5 (several sources give ratio R/D = 2 as a decisive value). It is valid that short radius gives higher pressure drops that long radius.

a) *long radius elbow*

b) *short radius elbow*

Figure 1: Schematic categorization of elbow depending on R/D ratio

2. Calculation of Pressure Drop in General

Calculation of pressure drop of fluid flowing in straight pipe is conducted by applying probably the most known hydraulic equation, the so called Darcy-Weisbach equation. This equation can be found in technical literature, for example in (Holland and Bragg, 1995; Coker, 2007; Verein Deutscher Ingenieure, 2006; Walas, 1990; Kakac and Liu, 2002).

Calculation of pressure drop for various pipe components is more complicated. We may apply either Darcy-Weisbach equation, where pipe length L is substituted by the so called equivalent length L_{ekv} (to be described further on), or a method using loss coefficient K (Holland and Bragg, 1995; Coker 2007; Verein Deutscher Ingenieure, 2006; Walas, 1990; Kakac and Liu, 2002). K loss coefficient may be expressed by a Greek letter ξ.

There are many possibilities to calculate pressure drop by application of loss coefficient. These possibilities differ in their level of elaborateness and of course in their accuracy of K coefficient. These methods are briefly outlined in this paper.

3. Briefly Description of Calculation Methods

As mentioned above, two basic approaches are employed for calculation of pressure drop of fluid in pipe components. First of the approaches consists of a substitution of pipe length by the so called equivalent length. The other approach applies K loss coefficient.

Seven calculation methods were selected for comparison of calculation of pressure drop in 90° and 180° elbows. They are the most used ones and most known too.

3.1 Equivalent Length Method

Method of equivalent length is among the most known and the oldest methods (Holland and Bragg, 1995; Coker, 2007; Walas, 1990; Kakac and Liu, 2002). This method is based on an assumption that elbow (or other type of pipe component) may be substituted by a pipe of a given diameter with a fictive length, the so called equivalent length with pressure drop identical to a given pipe component with a given Reynolds number in a pipeline. Equivalent length is independent on the Reynolds number and pipe diameter.

3.2 Crane's Method

This method is described in detail in (Crane, 1991) or in (Holland and Bragg, 1995). It is a modification of a previously mentioned method; however, it takes into consideration the fact that there is higher degree of turbulence in valves and fittings than in pipe with a given Reynolds number. That is the reason why fully developed turbulent area is considered in calculation of friction factor in pipe components regardless of current Reynolds number in the pipeline (Darby, 1999; Darby, 2001).

3.3 Loss Coefficient Method

This method represents a totally different approach to calculations of pressure drop. Calculation considers the so called *K* loss coefficient whose values are tabulated in various textbooks and handbooks. Approximation that these values are constant for given types of valves and fittings is not really true since the values change with respect to pipe components geometry (diameter, elbow radius, type of pipe connection, etc.) and depending on Reynolds number.

3.4 2-K Method (Hooper's Method)

The so called 2-K method (Coker, 2007; Hooper, 1981; Darby, 2001) published by Hooper is based on experimental data of many valves and fittings and elbows acquired for a wide spectrum of Reynolds numbers. *K* coefficient does not depend on roughness but is a function of Reynolds number, geometry of a given component and even of type of pipe connection for elbows. Compared to other methods, this one is valid for broad radius of Reynolds numbers but impact of pipe dimensions (1/D term) is not considered exactly. In case of elbows with larger diameters, values of Hooper's *K* coefficient are not compared with measured values (Darby, 2001). This method considers a type of pipe connection.

3.5 3-K Method (Darby's Method)

Darbys 3-K method (Coker, 1995; Darby, 1999; Darby, 2001) is similar to Hooper's 2-K method but with higher predicative value for broad radius of Reynolds numbers and fittings dimensions. Darby's 3-K method is also dependent on elbow inner diameter and values of Reynolds number. Darby's method provides good results even for laminar flow. This method also considered a type of pipe connection.

3.6 Blevins' Method

This sophisticated method is preferred for pressure drop calculation in elbows in some commercial software packages. The method depends on the velocity of process fluid, pipe diameter and is applicable for wide range of elbows radius (Blevins, 2003). The calculations are, however, independent on the roughness of pipe and type of connection elbow.

3.7 Idelchik’s Method

This is also very sophisticated method for calculation of pressure drop in elbows (Idelchik, 1975). This method depends also on the roughness of pipe in comparison with previous one. Discreteness of some coefficients is certain disadvantage.

4. Application of Calculation Methods

Calculation of pressure drop in 90° and 180° elbows of short and long radius depending on Reynolds number (and/or velocity of process medium) are performed for comparison of the above mentioned methods. Calculations are carried out in *Maple* software by Maplesoft, Inc. in version 14 (MapleSoft, 2011).

4.1 Example to Solution

Water is as a flowing fluid for comparison of individual methods. Its properties are specified in Table 1. Both short radius (R/D = 1) and long radius (R/D = 1.5) elbows are considered for pressure drop calculation. Because of paper size limitation the variations of pressure drop elbows are presented only for inner elbow diameter 26.5 mm.

Table 1: Water parameters of given pressure and temperature

	Water parameters	
Temperature [°C]	Dynamic viscosity [Pa.s]	Density [kg/m^3]
80	352.10^{-6}	971.8

4.2 Results Assessment

Comparison of obtained results is presented in graphical form in Figure 2.

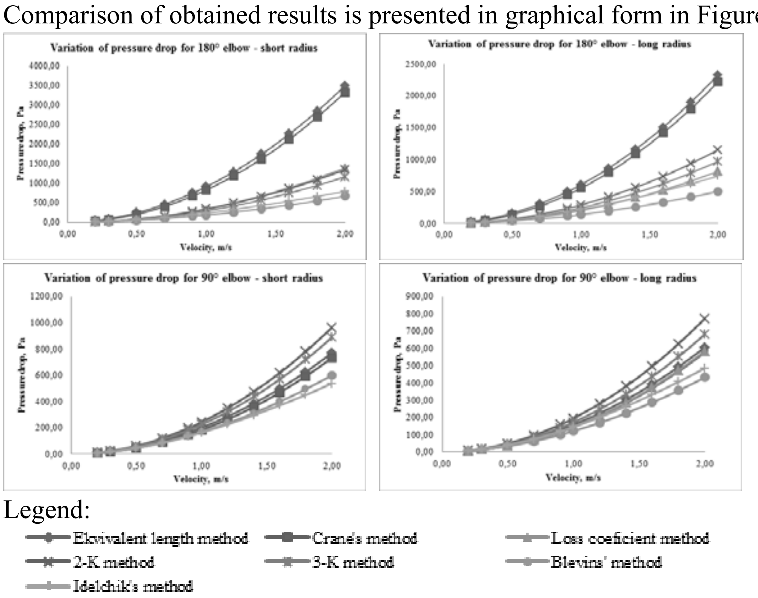

Figure 2: Variations of pressure drop for 90° and 180° elbows

Presented graphs in Figure 2 give clear idea about dependency of pressure drop upon velocity of process medium and elbow geometry. Method of equivalent length and Crane's method give quite different results for 180° elbow. It is because of problem with finding of an appropriate equivalent length values. Following methods give more or less similar results. Blevins' method and Idelchik's method give smallest value of pressure drop for 90° and 180° elbows. These methods are very sophisticated, but not take into account the type of connection to pipe compared to 2-K and 3-K method.
Equivalent length method, Crane's method and loss coefficient method are suitable for preliminary calculation. These methods do not faithfully represent dependence on elbow diameter, R/D ratio and dependence on the Reynolds number. 3-K method was opted as the best method for 90° and 180° elbows based on thorough analyses and assessment results. This method is considered as calculation standard by many engineers. Disadvantage of this method is small range of R/D ratio for 180° elbows. For these cases is possible to use the Blevins' method (Blevins, 2003).

5. Conclusion

Exact analytic determination of pressure drop of one-phase fluid in individual pipe components, i.e. valves and fittings and elbows etc., is still an engineering problem. Presence of elbows in pipe system results in significantly higher energy losses. Knowledge of more accurate results of fluid pressure drop in elbows allows to optimize final design of heat exchangers. However, proper calculation relations are necessary not only for calculation of pressure drops in elbows in heat exchangers but also in flue gas routes and wherever the medium streams in pipes with elbows and pressure drop has to be precisely determined. This paper wants to provide overview of calculation methods of pressure drops in elbows and present recommendations for most suitable methods.
As obvious from performed analyses of particular approaches, equivalent length method, Crane's method and loss coefficient method do not faithfully represent dependence of equivalent length and/or loss coefficient on elbow diameter, R/D ratio and Reynolds number. On the other hand, these dependences are well reflected in 2-K, 3-K, Blevins' and Idelchik's methods. The first two methods are based on measured data or this data correspond to the results of the calculation.
If the K-value characteristics of pipe components are given by manufacturer, it is obviously better to use these loss coefficients. If manufacturer does not present any dependence, one of the above mentioned methods has to be applied.
3-K method (Darby, 2001) is recommended for calculation of pressure drop of 90° and 180° elbows because it accounts directly for the effect of both Reynolds number and fitting size on the loss coefficient and reflects more accurately the scale effect of fitting size and connection type. Disadvantage of this method is small range of R/D ratio for 180° elbows. For these cases is possible to use the Blevins' method (Blevins, 2003).

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