2nd IFAC Workshop on Automatic Control in Offshore Oil and Gas Production,
May 27-29, 2015, Florianópolis, Brazil

A Generalized Formulation for Oilfield Development Optimization

Shiva Navabi, Reza Khaninezhad, Behnam Jafarpour

(navabiso@usc.edu, m.khaninezhad@usc.edu, jafarpou@usc.edu)
Viterbi School of Engineering, University of Southern California, USA

Abstract: Oilfield development involves several key decisions, including the number, type, location, drilling schedule, and operating controls of the wells. If optimized independently, i.e., without considering the coupling between the decision variables, only suboptimal solutions are obtained. A generalized oilfield development optimization approach is proposed to simultaneously optimize these decision variables. It is shown that the source/sink term in the governing multiphase flow equations includes all the decision variables for optimization. By incorporating the drilling cost in the net present value objective function, the proposed formulation transforms the problem into a sparsity-promoting optimization, which is efficiently solved using sparse reconstruction algorithms. Numerical experiments are presented to show the performance of the method for simultaneous optimization of the number, location, type (injection/production), control trajectories, and drilling schedule of the wells.

Keywords: generalized field development optimization, well placement, production optimization, sparse reconstruction.

1. INTRODUCTION

Several methods have been proposed for solving field development optimization problems. The majority of the existing algorithms consider separate optimization problems for each individual decision variable without accounting for the interplay between these variables. Examples of these approaches are well placement optimization to identify the optimal well locations given predetermined and fixed well controls (Bangerth et al., 2006; Zandvliet et al., 2008), and well control optimization in which the operational settings of the wells (rates and pressures) are optimized for a fixed well configuration (Brouwer, 2004). Recent studies in the literature have attempted to include multiple variables in the field development optimization problem. As an initial step towards such integrative solution approaches, Li and Jafarpour (2012) proposed a sequential scheme for solving the joint well placement and control optimization problem. Li et al. (2013) applied a generalized version of the SPSA algorithm (Spall, 1992; Spall et al., 2006) to simultaneously optimize well locations and controls and reported significant improvement in production performance. Other methods have also been proposed to solve joint field development optimization problems (Forouzanfar and Reynolds, 2013; Humphries et al., 2013; Isebor et al., 2013). Both gradient-based local search methods and gradient-free global search algorithms have been applied to field development optimization problems. Gradient-based methods are computationally efficient and monotonically improve the objective function. However, they easily get trapped in local solutions and are sensitive to initialization. Global search methods, on the other hand, do not need gradient information, but are computationally more demanding and can be prohibitive for large-scale problems.

We propose a novel approach for formulating and solving generalized field development optimization problems by exploiting the inherently sparse nature of the vector of decision variables in field development optimization. We show that the decision variables are collectively captured by the source/sink term of the discretized governing flow equations, which are sparse since only a very small number (often far less than 1%) of grid cells in the model contain source/sink terms. By including the cost of drilling in the net present value (NPV) objective function, the problem is posed as a sparsity-promoting optimization. We adopt algorithmic developments in sparse reconstruction and compressive sensing (Donoho, 2006; Baraniuk, 2007; Candès and Wakin, 2008) to solve the resulting optimization problem.

2. PROBLEM STATEMENT

We motivate the problem formulation by presenting the governing equations of the subsurface flow from a systems view (Jansen, 2013). For a two-phase (oil-water) incompressible flow system, after ignoring capillary pressure, the governing equations for phase n (n=oil,water) can be expressed as (Jansen, 2013)

$$\underbrace{-\frac{h}{\mu_n}\left[\frac{\partial}{\partial x}\left(kk_{rn}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(kk_{rn}\frac{\partial p}{\partial y}\right)\right]}_{\text{flux term}}+ \underbrace{h\left[\phi S_n(c_n+c_r)\frac{\partial p}{\partial t}+\phi\frac{\partial S_n}{\partial t}\right]}_{\text{accumulation term}}-\underbrace{hq_n'''}_{\text{source term}}=0 \tag{1}$$

Note that, from physical saturation constraint, we have $S_w + S_o = 1$. The finite difference discretized version of the equations can be written as

$$\begin{aligned} &V\left[\phi S_n(c_n+c_r)\frac{\partial p}{\partial t}+\phi\frac{\partial S_n}{\partial t}\right]_{i,j}-(T_n)_{i-\frac{1}{2},j}\,p_{i-1,j}\\ &-(T_n)_{i,j-\frac{1}{2}}\,p_{i,j-1}+\Big[(T_n)_{i-\frac{1}{2},j}+(T_n)_{i,j-\frac{1}{2}}\\ &+(T_n)_{i,j+\frac{1}{2}}+(T_n)_{i+\frac{1}{2},j}\Big]p_{i,j}-(T_n)_{i,j+\frac{1}{2}}p_{i,j+1}\\ &-(T_n)_{i+\frac{1}{2},j}p_{i+1,j}=V(q_n''')_{i,j} \end{aligned} \tag{2}$$

where, the discretized transmissibilities are defined as

$$(T_n)_{i,j} \triangleq \frac{\triangle y}{\triangle x}\frac{h}{\mu_n}(kk_{rn})_{i,j} \quad (3)$$

The matrix form of the resulting discretized equations (for water and oil, $n = w, o$) can be expressed as

$$\underbrace{\begin{bmatrix}\mathbf{V}_{wp} & \mathbf{V}_{ws}\\ \mathbf{V}_{op} & \mathbf{V}_{os}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{p}}\\ \dot{\mathbf{s}}\end{bmatrix}}_{\text{accumulation term}} + \underbrace{\begin{bmatrix}\mathbf{T}_w & 0\\ \mathbf{T}_o & 0\end{bmatrix}\begin{bmatrix}\mathbf{p}\\ \mathbf{s}\end{bmatrix}}_{\text{flux term}} = \underbrace{\begin{bmatrix}\mathbf{q}_w\\ \mathbf{q}_o\end{bmatrix}}_{\text{source term}} \quad (4)$$

In (4) the saturation and pressure variables in all cells are denoted by the vectors $\mathbf{p}$ and $\mathbf{s}$. Vectors $\mathbf{q}_w$ and $\mathbf{q}_o$ in the source/sink term of Equation (4) denote, respectively, the flow rates of water and oil phases in all the cells with their entries arranged in the following format

$$\mathbf{q}_w^T \triangleq [\cdots\ (q_w)_{i,j}\ \cdots], \mathbf{q}_o^T \triangleq [\cdots\ (q_o)_{i,j}\ \cdots] \quad (5)$$

From Equation (5), the entries of the source/sink vector are zero for cells that do not contain a well. Hence, the source/sink vectors in Equation (4) contain the information about well locations, types, and control settings. Therefore, these vectors constitute the decision variables of the generalized field development optimization problem as discussed below. Since in a large oilfield only a small number of grid blocks are intersected by wells, the vast majority of entries in the source/sink vector ($q^T = [\mathbf{q}_w^T\ \mathbf{q}_o^T]$) are 0s; that is, q is sparse. This interpretation highlights the similarity between the field development optimization problem and sparse reconstruction problem, where sparse solution to an optimization problem is sought. The optimal solution of the source/sink vector should minimize the number of wells while maximizing the production or NPV. Solution sparsity is promoted by the drilling cost. Note that, the entries in the source/sink vector ($[\mathbf{q}_w^T\ \mathbf{q}_o^T]^T$) in Equation (4) are expressed for a single timestep. In order to incorporate dynamic well controls, we expand the source/sink *vector* into a source/sink *matrix* $\mathbf{Q}$ as follows (note that, we use a matrix form to facilitate the discussion, but for optimization purpose this matrix is vectorized)

$$\mathbf{Q} = \begin{bmatrix} q_1^{t_1} & q_1^{t_2} & \cdots & q_1^{t_T}\\ q_2^{t_1} & q_2^{t_2} & \cdots & q_2^{t_T}\\ \vdots & \vdots & \ddots & \vdots\\ q_N^{t_1} & q_N^{t_2} & \cdots & q_N^{t_T}\end{bmatrix} \quad (6)$$

The rows of $\mathbf{Q}$ correspond to the indices of all grid blocks (that can contain wells) and its columns consist of well control trajectories (in time). In (6), T denotes the total number of control steps and N denotes the total number of grid blocks in the discretized domain. In the current matrix representation, the entry $q_i^{t_j}$ indicates the fluid flow rate of the well located in grid block i at control time step t_j. The well type is determined by the *sign* of each entry in $\mathbf{Q}$, by convention, injector (+) and producer (−). It is important to note that the sparse nature of the solution does not depend on the type of well controls used (rates or bottom-hole pressure). While we have used rate-controlled injection and production wells, the formulation can be generalized to the case where a mixture of BHP-controlled and rate-controlled wells are used.

A sparsity constraint can be applied to the rows of $\mathbf{Q}$ in (6) to minimize the number of wells to drill. On the other hand, since the columns of $\mathbf{Q}$ consist of control trajectories, there's no need to promote sparsity along the columns. Other constraints may be imposed along the columns of $\mathbf{Q}$ to impart a desirable behavior. Such constraints may be enforced through minimization of some norms (Bach et al., 2012). This formulation of the generalized field development focuses on finding $\mathbf{Q}$ with a sparse structure along its rows and a non-sparse structure along its columns. Note that $\mathbf{Q}$ contains all the decision variables for field development, including the number, location, type, and drilling schedule of the wells. The main challenge after formulating such an optimization problem is solving it with an efficient and reliable algorithm.

The multivariate optimization problem for generalized field development can now be expressed as

$$\begin{aligned}\mathbf{Q}^* = & \operatorname*{arg\,min}_{\mathbf{Q}\in[0,1]^{N\times T}} \quad J(\mathbf{Q})\\ \textit{subject to} \quad & \mathbf{Q} \in \Theta_{\mathbf{Q}}\\ & g_i(\mathbf{Q}) = 0 \quad , \quad i = 1, 2, \cdots, n\\ & f_j(\mathbf{Q}) \leq 0 \quad , \quad j = 1, 2, \cdots, m\end{aligned} \quad (7)$$

The dimension of this problem is $N \times T$ where $\Theta_{\mathbf{Q}}$ denotes the feasible set for the $N \times T$ dimensional matrix $\mathbf{Q}$. The general problem can be formulated by incorporating equality, $g_i(.)$, and inequality, $f_j(.)$, constraints. Example of equality and inequality constraints are mass balance equations and economic water-cut constraints, respectively.

2.1 Objective Function

In this study, NPV is used as the production performance metric for optimization. It incorporates the cost of drilling wells, operating costs (water injection, recycling, and disposal), as well as the revenue from produced hydrocarbon, and is formulated as

$$NPV = \int_0^T \left[\underbrace{\sum_{i=1}^{N_{\text{prod}}} r_o(t)\, q_{o,i}(t)}_{\text{oil revenue}} - \underbrace{\sum_{i=1}^{N_{\text{prod}}} r_{w,disp}(t)\, q_{w,disp,i}(t)}_{\text{water disposal cost}} \right. \\ \left. - \underbrace{\sum_{i=1}^{N_{\text{inj}}} r_{w,inj}(t)\, q_{w,inj,i}(t)}_{\text{water injection cost}} \right] \underbrace{\frac{1}{(1+d(t))^t}}_{\text{discount factor}} dt - \underbrace{C(\mathbf{Q})}_{\text{capital costs}} \quad (8)$$

In (8) the integral term represents the operating cost (with negative sign) and $C(\mathbf{Q})$ is the drilling cost; N_{prod} and N_{inj} are the number of production and injection wells, respectively; $r_o(t)$, $r_{w,disp}(t)$ and $r_{w,inj}(t)$ denote the time dependent oil production, water disposal, and water injection costs (\$/stb), respectively. The annual discount factor $d(t)$ is particularly important for drilling schedule, and is applied to the drilling cost ($C(\mathbf{Q})$) as well as the operating cost. Hence, the objective function is defined as:

$$J(\mathbf{Q}) = -NPV = O(\mathbf{Q}) + C(\mathbf{Q}) \quad (9)$$

Mathematically, the drilling cost in the objective function serves as a sparsity-promoting term; that is, it promotes solutions with minimum number of wells (drilling cost) to maximize the NPV. We rewrite $J(\mathbf{Q})$ as:

$$J(\mathbf{Q}) = O(\mathbf{Q}) + C(\lambda, \mathbf{Q}) \tag{10}$$

where,

$$C(\lambda, \mathbf{Q}) = \Big[\sum_{i=1}^{N} \lambda_i \Big(\epsilon + \sum_{t=1}^{T} (q_i^t)^p \Big)^{\frac{q}{p}} \Big]^{\frac{1}{q}} \simeq \| \mathbf{Q} \|_{\Lambda;p,q} \tag{11}$$

$$\lambda_i = \frac{\gamma}{(1+d)^{b_i}} \tag{12}$$

The term $(C(\lambda, \mathbf{Q}))$ in (11) is a *weighted mixed norm,* $L_{W;p,q}$ of $\mathbf{Q}$ that accounts for the drilling cost and serves as a sparsity constraint along the rows of $\mathbf{Q}$. Since the entries of the nonzero rows are control allocations of the corresponding wells in time, p affects the behavior of the control trajectory. The q-norm, on the other hand, is applied to the rows to promotes sparsity (minimize the number of wells). The constant ϵ is a small constant introduced to avoid matrix singularity during the iterations. The parameter λ_i is applied to the mixed-norm expression in (11) to properly account for the discounting factor (based on the drilling time). The cost of drilling a single well is denoted as γ, which is adjusted by the discount factor $\frac{1}{(1+d)^{b_i}}$, where d and b_i represent the annual interest rate and number of years passed since the i^{th} well is drilled. Note that the discounting factor in the objective function controls the drilling schedule by giving preference to wells that are drilled later.

The sparsity-based formulation in this work is inspired by the concept of group sparsity and group LASSO problem introduced in (Yuan and Lin, 2006; Turlach et al., 2005; Zhao et al., 2009). Since the ℓ_0-norm, used for counting the number of wells to quantify well costs, is not well-behaved, approximate methods are typically used to solve the problem. For sparsity along the rows of $\mathbf{Q}$, a sufficiently small q, $0 < q < 1, q \ll 1$, has been shown to have a similar sparsity-promoting behavior as ℓ_0-norm. However, given that the well cost will be affected for any norm other than ℓ_0, it is imperative to ensure that at the solution the well cost is properly reflected by using a very small q value (e.g.,$q < .01$). The ℓ_p-norm functions with different values of p are displayed in Figure 1.a.

3. SOLUTION METHOD

In this section, we develop an iterative gradient-based algorithm for solving the generalized field development optimization problem. We first describe the proximal gradient optimization techniques with properties that are suitable for optimization of non-smooth (black-box type) functions in large-scale problems.

3.1 Iterative Shrinkage Thresholding Algorithms

Proximal splitting optimization methods (Combettes and Pesquet, 2011; Parikh and Boyd, 2013) use the concept of proximal mapping to find the solution to a convex optimization problem more effectively than classical approaches. Proximity operator of a convex function is in fact a generalization of the notion of projection onto a convex set and can be defined as:

$$\text{prox}_f(\mathbf{x}) = \arg\min_{\mathbf{u}} \Big(f(\mathbf{u}) + \frac{1}{2} \| \mathbf{u} - \mathbf{x} \|_2^2 \Big) \tag{13}$$

For the constrained optimization problem

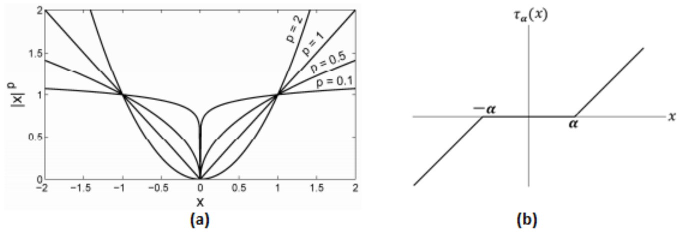

Fig. 1. (a) ℓ_p norm functions for different values of p; (b) Shrinkage function

$$\begin{aligned} \mathbf{x}^* &= \arg\min_{\mathbf{x}} \; O(\mathbf{x}) \\ O(\mathbf{x}) &= f(\mathbf{x}) + g(\mathbf{x}) \end{aligned} \tag{14}$$

proximal mapping can be used to reformulate the problem in (14) into an unconstrained problem that can be solved using the following iterative scheme

$$\begin{aligned} &\mathbf{x}^{k+1} = \text{prox}_{\alpha_k g}(\mathbf{x}^k - \alpha_k \nabla f(\mathbf{x}^k)) = \\ &\arg\min_{u} \Big(g(\mathbf{u}) + \frac{1}{2\alpha_k} \| \mathbf{u} - (\mathbf{x}^k - \alpha_k \nabla f(\mathbf{x}^k)) \|_2^2 \Big) \end{aligned} \tag{15}$$

where, α_k is the step size. This simple framework can be generalized to handle as many non-smooth constraint functions as desired, implying that a complicated objective function can be decomposed into the sum of its separate *simpler* components. The iterative shrinkage algorithms

Table 1. Iterative shrinkage thresholding pseudocode

initialization $\quad t_1 = 1 \quad , \quad \mathbf{X}_1 = \mathbf{Q}^0$
for k $\geq$ 1(while convergence criteria not satisfied)
$t_{k+1} = \frac{1+\sqrt{1+4t_k^2}}{2}$
$\mathbf{Q}^{k+1} = \mathbf{Q}^k - \frac{1}{L} P_k \nabla_{\mathbf{Q}} O(\mathbf{Q}^k)$
$\mathbf{X}^{k+1} = \arg\min_{\mathbf{X}} \left\{ \frac{L}{2} \| \mathbf{X} - \mathbf{Q}^{k+1} \|_2^2 + C(\lambda, \mathbf{Q}^k) \right\}$
$= \tau_{\frac{\lambda}{L}}(\mathbf{Q}^{k+1}) = \left(\mid \mathbf{Q}^{k+1} \mid - \frac{\lambda}{L} \right)_+ \text{sign}(\mathbf{Q}^{k+1})$
$\mathbf{Q}^{k+1} = \mathbf{X}^{k+1} + \frac{t_k - 1}{t_{k+1}} (\mathbf{X}^{k+1} - \mathbf{X}^k)$
$\mathbf{Q}^{k+1} = P_\Omega(\mathbf{Q}^{k+1})$
End

In Table 1, $\tau_\alpha(.)$ is a shrinkage operator for the updated matrix $(\mathbf{Q}^{k+1})$, according to the specified threshold $\alpha (= \frac{\lambda}{L})$. For soft thresholding, we have used the shrinkage function shown in Figure 1.b, which is defined as

$$\tau_\alpha(x) = \begin{cases} x - \alpha & x \geq \alpha \\ 0 & -\alpha \leq x \leq \alpha \\ x + \alpha & x \leq \alpha \end{cases} \tag{16}$$

The term $\nabla_{\mathbf{Q}} O(\mathbf{Q}^k)$ in Table 1 is the gradient of the operating cost with respect to the rate allocations at k^{th} iteration and is obtained using an adjoint model (Zandvliet et al., 2008; Sarma et al., 2008); and L, defined in (17), represents the gradient Lipschiz constant of $\nabla_{\mathbf{Q}} O(.)$ that sets an upper bound on the magnitude of the slope of $\nabla_{\mathbf{Q}} O(.)$ within its feasible domain.

$$L = \sup \left\{ \frac{\| \nabla f(\mathbf{x}_1) - \nabla f(\mathbf{x}_2) \|}{\| \mathbf{x}_1 - \mathbf{x}_2 \|} \right\} \tag{17}$$

Various methods have been proposed to numerically estimate the Lipschiz constant of a function (Wood and Zhang, 1996). For the numerical experiments in this study, at each iteration, we approximate the Lipschiz constant of $\nabla_{\mathbf{Q}} O(.)$ using

$$L^k \simeq \max_{\substack{m,n=1 \\ m \neq n}}^{k} \left\{ \frac{\| \nabla O(\mathbf{q}_m) - \nabla O(\mathbf{q}_n) \|}{\| \mathbf{q}_m - \mathbf{q}_n \|} \right\} \tag{18}$$

where $\mathbf{q}$ denotes the column-wise vectorized version of the allocations matrix $\mathbf{Q}$. The Lipschiz constant is approximated at each iteration as the maximum value among all computed Lipschiz constants using all pairs of the gradients calculated up to the current iteration. The gradient projection matrix $\mathbf{P}_k$ is applied to the gradient to modify the update direction to ensure that the updated solution lies within the feasible set defined by the constraints.

The field development optimization can be formulated as a constrained nonlinear programming problem with box constraints as well as a linear constraint (Luenberger and Ye, 2008, Chapter 12):

$$\begin{array}{rl} \min & O(\mathbf{Q}) \\ \text{subject to} & \mathbf{q} \leq \mathbf{1} \;\;,\;\; -\mathbf{q} \leq \mathbf{1} \\ & \mathbf{1}^T \mathbf{q}_i = 0 \;\;\;,\;\;\; i = 1, 2, \cdots, T \end{array} \tag{19}$$

where $\mathbf{q}$ denotes the column-wise vectorized version of $\mathbf{Q}$. Since entries of $\mathbf{Q}$ denote the control *allocations* for each well at the specified control step, they should neither exceed the normalized upper bound (the maximum control allocation for an injector, normalized to +1) nor the normalized lower bound (the maximum control allocation for a producer, normalized to -1); hence, the inequality constraints in (19) are designed to enforce the required box constraints on entries of $\mathbf{Q}$. The mass balance constraint is met through a set of equality constraints to ensure that the sum of injection/production allocations, with their respective signs, is zero. The gradient projection matrix can then be formed as (Luenberger and Ye, 2008, Chapter 12),

$$\mathbf{P}_k = [\mathbf{I} - \mathbf{A}_q^T (\mathbf{A}_q \, \mathbf{A}_q^T)^{-1} \mathbf{A}_q] \tag{20}$$

in which $\mathbf{A}_q$ contains concatenated 1's and 0's according to the indices of the updated matrix $\mathbf{Q}$ and the necessary modifications to the gradient direction. The projection operator $P_\Omega(.)$ (Goldstein, 1964; Levitin and Polyak, 1966; Bertsekas, 1982) acts on a nonempty closed convex set (Ω) through the mapping $P : \mathbb{R}^n \longrightarrow \Omega$, and is defined as

$$\mathbf{P}_\Omega(\mathbf{x}) = \arg\min \{ \; \| \mathbf{x} - \mathbf{u} \| : \mathbf{u} \in \Omega \; \} \tag{21}$$

The convex set Ω is defined based on the lower and upper bounds, $\mathbf{l}$ and $\mathbf{u}$ respectively, as follows

$$\Omega = \{ \mathbf{x} \in \mathbb{R}^n : \mathbf{l} \leq \mathbf{x} \leq \mathbf{u} \} \tag{22}$$

As outlined in Table 1, at the end of each iteration, mass balance is preserved by applying the projection operator $(P_\Omega(.))$ to the updated allocations matrix.

4. NUMERICAL EXPERIMENT

We examine the performance of the proposed algorithm using the PUNQ-S3 reservoir model and a two-phase (oil-water) flow simulation in Matlab Lie et al. (2012). The log-permeability maps of the 5 layers in the PUNQ-S3 model are shown in Figure 2. The field contains $28 \times 19 \times 5$ discretized grid blocks, many of which are not active as displayed in Figure 2. The simulation parameters are summarized in Table 2. The price of oil, and cost of water injection and disposal were set to \$80/stb, \$10/stb and \$10/stb, respectively. The annual discount rate is assumed to be 10% and the control trajectories are divided into 5 evenly spaced increments over the reservoir life cycle of 10 years. All wells are under fluid flow rate controls.

Table 2. Simulation parameter setup used for the numerical experiment in example 2

Parameters	PUNQ-S3 Reservoir
Reservoir grid dimensions	$28{\times}19 \times 5 = 2660$
Number of active cells	1761
Physical cell dimension	$40{\times}40 \times 40ft^3$
Rock porosity	30 %
Simulated reservoir life cycle	10 yrs
Number of control steps	5
Fluid phases	Oil-water (2-phase)
Initial water saturation	0.05
Injection volume	1 PV
Perforation direction	Vertical
Injectors control mode	Total water injection rate
Producers control mode	Total fluid production rate
Well drilling cost	\$10 M

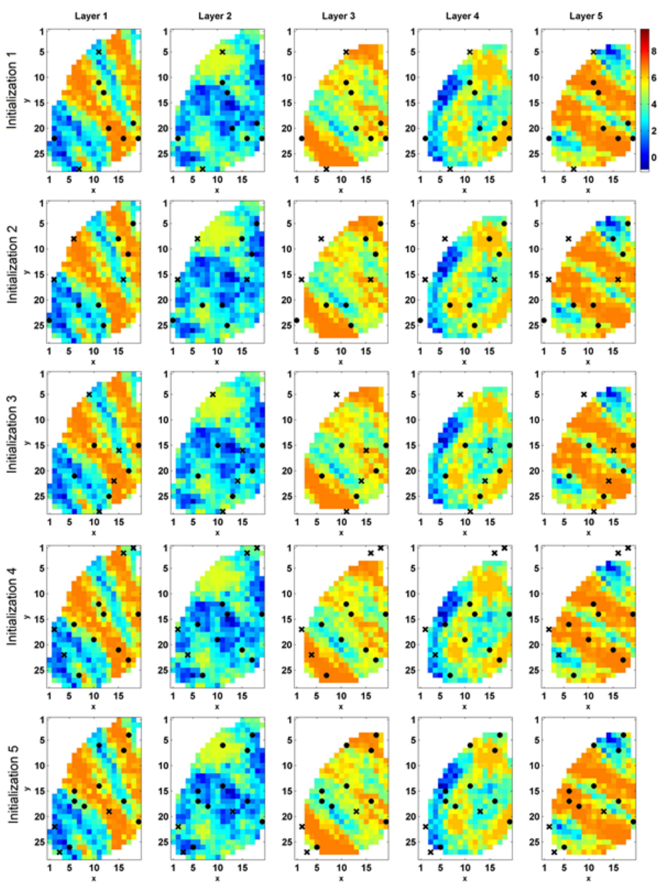

Fig. 2. Optimized well locations for five initializations; injectors and producers are marked with (×) and (•), respectively.

Table 3. Optimization results

Initialization no	N_{inj}	N_{prod}	NPV
1	2	7	\$94.6 M
2	3	7	\$93.8 M
3	4	5	\$93.8 M
4	4	8	\$87.8 M
5	3	10	\$76.0 M

The solution is initialized by assigning very small rates to a large number of randomly placed producers and injectors.

The wells are perforated in all 5 layers of the PUNQ reservoir except for the ones placed in the grid cells that are only active within the first 2 layers of the formation. Using the proposed sparsity-promoting gradient projection method and the adjoint sensitivities with respect to well controls, insignificant wells are successively removed to reduce drilling cost and increase the contribution of dominant wells, leading to the optimal configuration of the wells at convergence. In addition, the control trajectories are automatically estimated and used to infer the drilling schedule. Optimized well configurations for 5 different initializations are displayed in Figure 2. Details of the optimized parameters are summarized in Table 3. From Figure 2, it can be seen that the optimized configuration for different initializations vary in number and location of the wells, which is expected given the local nature of gradient-based algorithms, the number (and type) of variables, and the complexity of the objective function. From Table 3, it is also observed that the optimized NPVs are close to each other, and fluctuate around $\sim$ \$90 M, except for Case 5. To check for solution (local) optimality, we perturbed the optimal locations of the wells and verified that the NPV values actually decreased when local perturbations were applied.

The evolution of the objective function throughout the iterations is plotted in Figure 4 where it can be seen that on average, it has taken about 60 iterations for the algorithm to converge. Optimized well control trajectories for each of the five initializations are displayed in Figure 3. Overall, as a result of incorporating the discount factor, the algorithm tends to add some of the wells later in time to reduce the capital cost. For instance, from the control trajectories for wells in the optimized configuration for initialization 3, it can be seen that producers 2, 1 and 3 are added in the third, fourth and fifth control steps, respectively, while producers 4 and 5 start operating at the beginning of water injection.

An advantage of the developed formulation is that it can easily impose constraints on the control trajectories by adjusting the parameters p and q in the $\ell_{\Lambda;p,q}$ norm (Equation 11) or by supplying different constraints along the columns of the control matrix. In our experiments, the parameter p controls the variation in the control trajectories of active wells by imposing minimum ℓ_p norm constraint along the columns of $\mathbf{Q}$. In general, promoting sparsity on the control trajectories (columns) is not desirable, hence we set $p > 1$. Large values of p penalize high injection or production rates. In fact, very large values of p can transform the problem into $\ell_{\infty,1}$ minimization (Turlach et al., 2005), which is more sensitive to large entries in the active rows of $\mathbf{Q}$. Other user-specified constraints on the control trajectory (such as bound constraints or smoothness) may also be adopted with the proposed formulation.

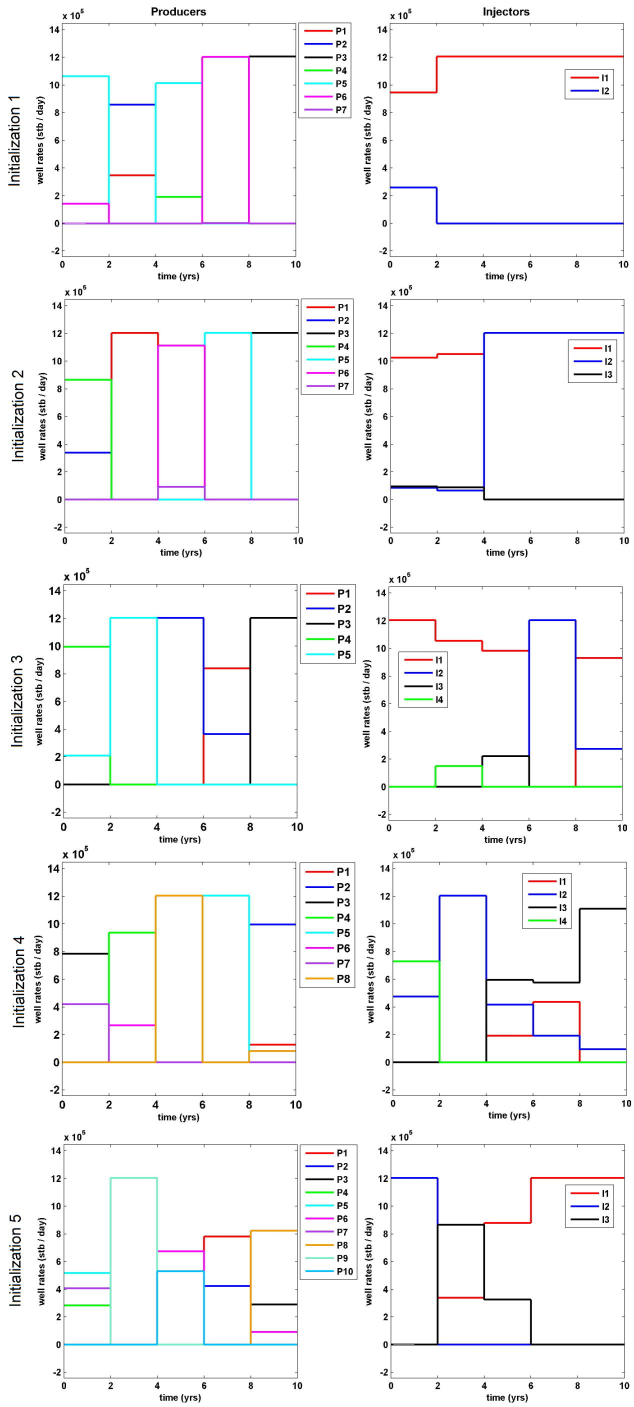

Fig. 3. Optimized well control trajectories for five initializations.

5. CONCLUSION

We have developed a novel generalized framework for oilfield development optimization. This framework can be used to optimize the number, location, type of wells, as well as their operating control trajectories in time, and their drilling schedule. The generalized problem is formulated to jointly optimize these design parameters by accounting for the coupling between them. In our formulation, the decision variables consist of the dynamic source/sink (well) terms for all grid cells in the reservoir (embedded in the wells total fluid flow rates) and their dynamic trajectories throughout the reservoir life cycle. By including the drilling and operating costs in the formulation of the NPV cost function, we show that the NPV objective function has the structure of a sparsity-promoting function. Hence, we solve the resulting optimization problem by adopting efficient optimization algorithms from the sparse reconstruction literature. Specifically, an iterative shrinkage thresholding scheme is used to eliminate insignificant wells from the reservoir model (or add new ones) to optimize the number of injectors and producers,

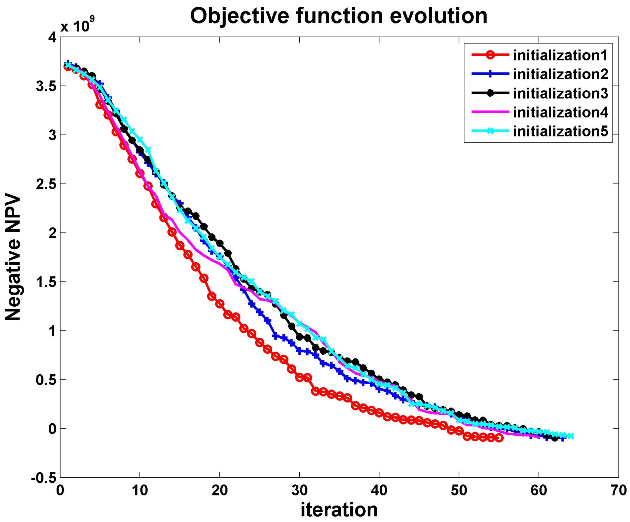

Fig. 4. Objective function evolution.

to identify promising well locations, control trajectories, and drilling schedule. The applicability and effectiveness of the developed method was demonstrated using a two-phase three-dimensional example based on the PUNQ-S3 geologic model.

REFERENCES

Bach, F., Jenatton, R., Mairal, J., and Obozinski, G. (2012). Optimization with sparsity-inducing penalties. *Foundations and Trends® in Machine Learning*, 4(1), 1–106.

Bangerth, W., Klie, H., Wheeler, M., Stoffa, P., and Sen, M. (2006). On optimization algorithms for the reservoir oil well placement problem. *Computational Geosciences*, 10(3), 303–319.

Baraniuk, R.G. (2007). Compressive sensing. *IEEE signal processing magazine*, 24(4).

Bertsekas, D.P. (1982). Projected newton methods for optimization problems with simple constraints. *SIAM Journal on control and Optimization*, 20(2), 221–246.

Brouwer, D.R. (2004). *Dynamic water flood optimization with smart wells using optimal control theory*. Delft University of Technology.

Candès, E.J. and Wakin, M.B. (2008). An introduction to compressive sampling. *Signal Processing Magazine, IEEE*, 25(2), 21–30.

Combettes, P.L. and Pesquet, J.C. (2011). Proximal splitting methods in signal processing. In *Fixed-point algorithms for inverse problems in science and engineering*, 185–212. Springer.

Donoho, D.L. (2006). Compressed sensing. *Information Theory, IEEE Transactions on*, 52(4), 1289–1306.

Forouzanfar, F. and Reynolds, A. (2013). Joint optimization of number of wells, well locations and controls using a gradient-based algorithm. *Chemical Engineering Research and Design*.

Goldstein, A.A. (1964). Convex programming in hilbert space. *Bulletin of the American Mathematical Society*, 70(5), 709–710.

Humphries, T.D., Haynes, R.D., and James, L.A. (2013). Simultaneous and sequential approaches to joint optimization of well placement and control. *Computational Geosciences*, 1–16.

Isebor, O.J., Durlofsky, L.J., and Ciaurri, D.E. (2013). A derivative-free methodology with local and global search for the constrained joint optimization of well locations and controls. *Computational Geosciences*, 1–20.

Jansen, J.D. (2013). *A systems description of flow through porous media*. Springer.

Levitin, E.S. and Polyak, B.T. (1966). Constrained minimization methods. *USSR Computational mathematics and mathematical physics*, 6(5), 1–50.

Li, L. and Jafarpour, B. (2012). A variable-control well placement optimization for improved reservoir development. *Computational Geosciences*, 16(4), 871–889.

Li, L., Jafarpour, B., and Mohammad-Khaninezhad, M.R. (2013). A simultaneous perturbation stochastic approximation algorithm for coupled well placement and control optimization under geologic uncertainty. *Computational Geosciences*, 17(1), 167–188.

Lie, K.A., Krogstad, S., Ligaarden, I.S., Natvig, J.R., Nilsen, H.M., and Skaflestad, B. (2012). Open-source matlab implementation of consistent discretisations on complex grids. *Computational Geosciences*, 16(2), 297–322.

Luenberger, D.G. and Ye, Y. (2008). *Linear and nonlinear programming*, volume 116. Springer.

Parikh, N. and Boyd, S. (2013). Proximal algorithms. *Foundations and Trends in Optimization*, 1(3), 123–231.

Sarma, P., Chen, W.H., et al. (2008). Efficient well placement optimization with gradient-based algorithms and adjoint models. In *Intelligent Energy Conference and Exhibition*. Society of Petroleum Engineers.

Spall, J.C. (1992). Multivariate stochastic approximation using a simultaneous perturbation gradient approximation. *Automatic Control, IEEE Transactions on*, 37(3), 332–341.

Spall, J.C., Hill, S.D., and Stark, D.R. (2006). Theoretical framework for comparing several stochastic optimization approaches. In *Probabilistic and Randomized Methods for Design under Uncertainty*, 99–117. Springer.

Turlach, B.A., Venables, W.N., and Wright, S.J. (2005). Simultaneous variable selection. *Technometrics*, 47(3), 349–363.

Wood, G. and Zhang, B. (1996). Estimation of the lipschitz constant of a function. *Journal of Global Optimization*, 8(1), 91–103.

Yuan, M. and Lin, Y. (2006). Model selection and estimation in regression with grouped variables. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 68(1), 49–67.

Zandvliet, M., Handels, M., van Essen, G., Brouwer, R., Jansen, J.D., et al. (2008). Adjoint-based well-placement optimization under production constraints. *SPE Journal*, 13(04), 392–399.

Zhao, P., Rocha, G., and Yu, B. (2009). The composite absolute penalties family for grouped and hierarchical variable selection. *The Annals of Statistics*, 3468–3497.

