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Modelling and Extremum Seeking Control of Gas Lifted Oil Wells

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Abstract This paper addresses the design of a perturbation-based extremum seeking control (ESC) method to maintain the oil production around the optimum point of the well-performance curve in gas lifted wells. The method uses periodic perturbation which is injected into the plant with intention to extract the information about the gradient of the well-performance curve. A simple nonlinear dynamic model is proposed and the essential dynamics of the Eikrem's model are captured, i.e., the transient behavior and the optimal steady state well-performance curve. Based on this simple model, a pre-compensation is developed which allows the application of the ESC scheme without reducing excessively the perturbation frequency. The control performance is evaluated via numerical simulations using an appropriate environment for modelling, simulation and optimization (EMSO) of process dynamic systems.

Keywords: extremum seeking control, oil production, process control, gas lifted wells.

1. INTRODUCTION

In oil production, when the pressure of the reservoir is not enough to maintain the oil flow reaching the wellhead or the well is at the end of its productive life, the use of artificial lift method is required (Thomas, 2001). Hence, artificial lift is a common technique to increase production from mature fields. One of the most used technique is the gas injection (Eikrem et al., 2005) (Skogestad and Storkaas, 2002), which consists of applying compressed gas into the annulus space of the well so that it enters in the bottom of the tubing reducing the average fluid density. As a result, the fluid becomes lighter by decreasing density and allows higher oil flow at the wellhead (Elgsaeter et al., 2010) (Ribeiro, 2012).

The curve representing the relationship between the gas injection flow and the oil flow at the wellhead is named *well-performance curve* (WPC) of gas lifted wells. By increasing excessively the gas flow, the friction increases while the oil production decreases. This results in a WPC with an extremum. The difficulty of estimating the WPC has motivated the search for a robust real-time method that leads to the oil production flow values near the optimum of the WPC (Garcia et al., 2010) (Garcia Irausquin et al., 2008) (Redden et al., 1974) (Aamo et al., 2005).

Optimal lift-gas allocation with constraints was formulated as a mixed-integer nonlinear programming problem in (Camponogara and de Conto, 2002). Optimal allocation of limited resources, such as the lift-gas flow, fluid handling capacities and water-treatment processing capacity was considered in (Camponogara et al., 2009) and (Teixeira, 2013) and a control strategy for the pressure of the gas lift was also proposed. Model predictive control was explored in (Plucenio, 2010) and (Ribeiro, 2012) to assure optimum oil production with quality constrains. Extremum-seeking control (ESC) is an alternative approach for online optimization that deals with uncertain situations when the plant model and/or the cost to optimize are not available to the designer. Using the available signals (plant input and output), the goal is to design a controller that dynamically searches for the optimizing inputs.

In (Ariyur and Krstic, 2003), the method was generalized for a class of dynamic plants stabilizable via state feedback. The general idea was to generate a closed-loop system with sufficiently fast dynamics in order to behave approximately as a static plant. A more general class of nonlinear plants were treated in (Krstić and Wang, 2000) by assuming again that the system (in closed loop via state feedback) can behave approximately as a static one or by assuming that the period of the periodic perturbation is large when compared to the time constant of the system (low excitation frequency). For the class of Hammerstein-Wienner (HW) systems, compensators can be added to the ESC scheme so that the period of the periodic perturbation can be reduced, leading to faster transients to reach the vicinity of the maximizer (Ariyur and Krstic, 2003), (Krstíc, 2000). It must be highlighted that, in all cases (Krstić and Wang, 2000), (Ariyur and Krstic, 2003) and (Krstíc, 2000), the mentioned phase difference is evident. In fact, this ESC method is deeply dependent on a good phase difference detection between input and output for average values of the input below and above the maximizer.

This paper considers the modified version of Eikrem's model by Ribeiro (2012) for gas lifted oil wells. Via numerical simulations it is apparent that phase difference is not observed between the input and output of the WPC mapping corresponding to average values of the input below and above the maximizer. A clear phase difference appears only for very low frequencies of the periodic perturbation, which impairs the directly applicability of the ESC method. Seeking to circumvent this problem, in this paper, the modified Eikrem's model is analyzed and a suitable model is proposed. We consider a simple stable first order linear system followed by a nonlinearity containing a product of the plant input and state. This model allows us to capture the main features of the Eikrem' model and clarify the main reason for the difficult to detect the phase difference. Moreover, a simple pre-compensation is proposed in order to approximate the nonlinear system to a HW system, allowing to reduce the period of the periodic perturbation. To the best of our knowledge, perturbation-based extremum seeking control via output feedback has remained unsolved for the class of uncertain nonlinear systems considered here, without reducing the frequency of operation. Hence, as a sub product, this paper also contributes with a preliminar solution to this problem.

Remark 1. (Notation and Terminology) The symbol "s" represents either the Laplace variable or the differential operator "d/dt", according to the context. As in (Ioannou and Sun, 1996), $H(s)u$ denotes the output of a LTI system with transfer function $H(s)$ and input u. Pure convolutions $h(t)*u(t)$, with $h(t)$ being the impulse response from $H(s)$, will be eventually written as $H(s)*u$. Classes of $\mathcal{K}, \mathcal{K}_\infty, \mathcal{L}$ functions are defined according to (Khalil, 2002, p. 144), in particular, a function $\beta : \mathbb{R}^+ \to \mathbb{R}^+$ belongs to class $\mathcal{L}$ if it is continuous, strictly decreasing and $\lim_{t\to\infty} \beta(t) = 0$. A function $\alpha : \mathbb{R}^+ \to \mathbb{R}^+$ belongs to class $\mathcal{K}$ if it is continuous, strictly increasing and $\alpha(0) = 0$.

2. MODIFIED EIKREM'S MODEL FOR WELL PRODUCTION

According to (Eikrem et al., 2002), the model to describe the well production (3 phases – water, oil and gas) is given in four parts: (i) mass balance model of the phases, (ii) the densities models, (iii) the pressures models and (iv) the flows models. It is assumed that the oil and water form one single phase (inside the well column) and only slow changes occurs in the quantity of gas in the mixture. The mass balance of the production process of a single well operating via gas lift can be described by:

$$\dot{x}_1 = u - w_{iv}\,, \tag{1}$$
$$\dot{x}_2 = w_{iv} - w_{pg}\,, \tag{2}$$
$$\dot{x}_3 = w_{ro} - w_{po}\,, \tag{3}$$
$$y = w_{po}\,, \tag{4}$$

where, x_1 is the mass of gas in the annulus, x_2 is the mass of gas in the tubing, x_3 is the mass of oil production in the column above the injection point, $u = w_{gc}$ is the flow gas injection in the annulus (control input), w_{iv} is the flow gas from the annulus to the tubing, w_{pg} is the gas flow though the production valve (choke), w_{ro} is the oil flow from the reservoir into the tubing, and $y = w_{po}$ is the oil flow though the production choke (plant output).

The oil density in the reservoir is given by $\rho_0 = \frac{1}{v_0}$, where v_0 is the specific volume of the oil in the reservoir (the oil is considered incompressible). The densities ρ_{ai} (gas density in the annulus at the injection point), ρ_m (density of the oil and gas mixture at the wellhead) are described by:

$$\rho_{ai} = \frac{M}{RT_a}\left(\frac{RT_a}{V_a M} + \frac{gL_a}{V_a}\right)x_1, \tag{5}$$
$$\rho_m = \frac{x_2 + x_3 - \rho_o L_r A_r}{L_w A_w}, \tag{6}$$

respectively, where R is the universal constant of the ideal gases, T_a is the temperature in the annulus, V_a the volume of the annulus space, M is the molar mass of the gas, L_a the length of the annulus, g is the acceleration of gravity, A_w is the cross section area (assumed circular) of the column above the injection point, L_w is the length of the column above the injection point, A_r is the cross section area (assumed circular) of the column below the injection point and L_r is the length of the column below the injection point.

According to the modifications in the Eikrem's model proposed by Ribeiro (2012), the pressure p_{ai} (the annulus pressure at the injection point of the column), p_{wh} (well-head pressure), p_{wi} (column pressure at the injection point of the column) and p_{wb} (downhole pressure) are given by:

$$p_{ai} = \left(\frac{RT_a}{V_a M} + \frac{gL_a}{V_a}\right)x_1\,, \tag{7}$$
$$p_{wh} = \frac{RT_w}{M}\frac{x_2}{L_w A_w + L_r A_r - v_o x_3}\,, \tag{8}$$
$$p_{wi} = p_{wh} + \frac{g}{A_w}(x_2 + x_3 - \rho_o L_r A_r) + \rho_0 g h_f(L_w)\,, \tag{9}$$
$$p_{wb} = p_{wi} + \rho_o g(L_r + h_f(L_r))\,, \tag{10}$$

respectively, where T_w is the temperature in the column, $h_f(\cdot)$ is the head loss described in Ribeiro (2012). The flows w_{iv}, w_{pg}, w_{po} and w_{ro} are described by:

$$w_{iv} = C_{iv}\sqrt{\rho_{ai} max\{0, p_{ai} - p_{wi}\}}, \tag{11}$$
$$w_{pg} = \frac{x_2}{x_2 + x_3}w_{pc},\; w_{po} = \frac{x_3}{x_2 + x_3}w_{pc}\,, \tag{12}$$
$$w_{ro} = C_r\sqrt{\rho_0(p_r - p_{wb})}, \tag{13}$$

where,

$$w_{pc} = C_{pc}\sqrt{\rho_m max\{0, p_{wh} - p_s\}}, \tag{14}$$

C_{iv}, C_{pc} and C_r are positive constants, p_s is pressure in manifold downstream of the well where it is assumed that there is a control to maintain this pressure at a constant value, and p_r is the reservoir pressure far from the well, which is also considered constant.

Note that, from (12), one has that $w_{pg} = (x_2/x_3)w_{po}$. Therefore, the system (1)–(4) can be rewritten as follows:

$$\begin{aligned} \dot{x}_1 &= u - \varphi_1(x_1, x_2, x_3), \\ \dot{x}_2 &= \varphi_1(x_1, x_2, x_3) - \frac{x_2}{x_3}\varphi_3(x_2, x_3), \\ \dot{x}_3 &= \varphi_2(x_2, x_3) - \varphi_3(x_2, x_3), \\ y &= \varphi_3(x_2, x_3), \end{aligned} \tag{15}$$

where $\varphi_1 = w_{iv}$ is obtained from (11), (5), (7), (9) and (8), $\varphi_2 = w_{ro}$ is obtained from (13), (10), (9) and (8) and $\varphi_3 = w_{po}$ is obtained from (12), (14), (6) and (8).

3. MODIFIED EIKREM'S MODEL NUMERICAL EVALUATION

Aiming to obtain a simpler model that satisfactorily represent the system (15) being more suitable for the controller design, the system response to constant and sinusoidal inputs was evaluated via numerical simulations. The simulator EMSO (*Environment for Modeling Simulation and Optimization*), developed by the laboratory LADES/UFRJ (Soares and Secchi, 2003), was used in a friendly interface with *SIMULINK/MATLAB*.

The modified Eikrem's model parameters are: $M = 0.0289 kg/mol$, $R = 8.314 J/kmol$, $g = 9.81 m/s^2$, $T_a = 293K$, $L_a = 230.87m$, $V_a = 29.012m^3$, $\rho_o = 923.9kg/m^3$, $p_s = 3.704669 \times 10^6 Pa$, $p_r = 2.5497295 \times 10^7 Pa$, $T_w = 293K$, $L_w = 1217m$, $L_r = 132m$, $A_w = 0.203m^2$, $A_r = 0.203m^2$, $C_{iv} = 15 \times 10^{-5}m^2$, $C_{pc} = 1.655 \times 10^{-3}m^2$ and $C_r = 2.623 \times 10^{-4}m^2$.

3.1 Static Input-Output Mapping

In order to assure the existence of a static input-output mapping corresponding to (15), we assume that:

(A0) For fixed values of $u(t) = \theta_u \in \mathbb{R}$, the system (15) has an unique and constant steady state solution ($\dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0$), i.e., when $t \to +\infty$, denoted by $x_1(t) = \theta_1$, $x_2(t) = \theta_2$ and $x_3(t) = \theta_3$ (equilibrium point).

It can be verified that this assumption **(A0)** is not restrictive, by solving the following algebraic system (numerically) [1]

$$\theta_u - \varphi_1(\theta_1, \theta_2, \theta_3) = 0\,, \tag{16}$$

$$\varphi_1(\theta_1, \theta_2, \theta_3) - \frac{\theta_2}{\theta_3}\varphi_3(\theta_2, \theta_3) = 0\,, \tag{17}$$

$$\varphi_2(\theta_2, \theta_3) - \varphi_3(\theta_2, \theta_3) = 0\,, \tag{18}$$

for some values of θ_u in the region of interest $\theta_u \in (0, 10]$. For each θ_u, the system (16), (17) and (18) exhibited a single solution $(\theta_1, \theta_2, \theta_3)$ and, consequently, $y(t)$ converges to a single value in steady state, named $y(t) = \theta_y = \varphi_3(\theta_2, \theta_3)$ when $t \to +\infty$. Knowing the steady state values θ_2 and θ_3, it is possible to plot the curve $\theta_u \times \theta_y$, the WPC curve in Figure 1, which represents the oil production in steady state for each value of the constant flow of gas injection [2]. As it is well known, from Figure 1, it is clear that there exists an optimum *gas lift* flow ($2.68\ kg/s$) which maximizes the oil production.

3.2 Static Input-Output Model

By examining the solutions (16), (17) and (18) for each value of θ_u, one can verify that, as the gas injection increases (θ_u), the mass of gas in the column (θ_2) and in the annulus (θ_1) increase, while the mass of the oil in the column (θ_3) decreases. Indeed, via least squares fitting one can approximate the relationships $\theta_1(\theta_u)$ and $\theta_2(\theta_u)$ (increasing functions) and the relationship $\theta_3(\theta_u)$ (decreasing function). Inspired by this numerical evaluation, the following assumption is now made:

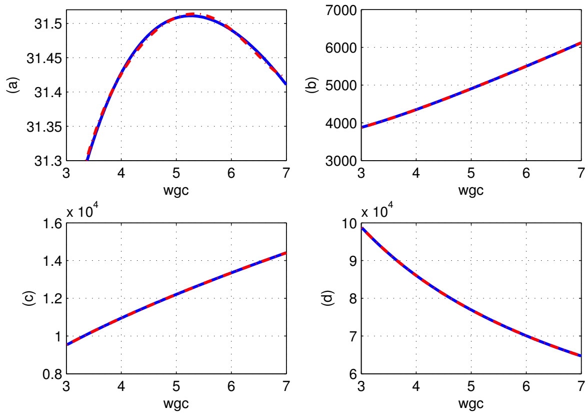

Figure 1. Steady state mapping for each value of the gas injection flow $w_{gc} = \theta_u$ in the region of interest $\theta_u \in (0, 7]$: (a) the WPC curve. i.e., the oil production $y = w_{po} = \theta_y$; (b) the mass of gas in the column (θ_2), (c) the mass of gas in the annulus (θ_1) and (d) the mass of the oil in the column (θ_3).

(A1) There are functions $\alpha_i \in \mathcal{K}_\infty$ $(i = 1, 2)$ and $\beta_3 \in \mathcal{L}$ and constants k_i $(i = 1, 2, 3)$ such that $\theta_1 = \alpha_1(\theta_u) + k_1$, $\theta_2 = \alpha_2(\theta_u) + k_2$ and $\theta_3 = \beta_3(\theta_u) + k_3$, where $x_1(t) = \theta_1$, $x_2(t) = \theta_2$ and $x_3(t) = \theta_3$ is the unique constant steady state solution of (15) corresponding to each fixed value of $u(t) = \theta_u$, according to (A0).

From (17), the steady state value of the plant output $y = \varphi_3(x_2, x_3)$ is given by

$$\theta_y = \varphi_3(\theta_2, \theta_3) = \frac{\theta_3}{\theta_2}\varphi_1(\theta_1, \theta_2, \theta_3)\,, \tag{19}$$

where $\varphi_1(\theta_1, \theta_2, \theta_3)$ is the steady state value of the flow gas $w_{iv} = \varphi_1(x_1, x_2, x_3)$. Moreover, from (16), one has that $\varphi_1(\theta_1, \theta_2, \theta_3) = \theta_u$, thus one can write $\theta_y = \frac{\theta_3}{\theta_2}\theta_u$. Finally, from **(A1)**, the following input-output relationship at steady state can be obtained:

$$\theta_y = \frac{\theta_3}{\theta_2}\theta_u = \frac{\beta_3(\theta_u) + k_3}{\alpha_2(\theta_u) + k_2}\theta_u = \beta(\theta_u)\theta_u\,, \tag{20}$$

where $\beta(\theta_u) := \frac{\beta_3(\theta_u)+k_3}{\alpha_2(\theta_u)+k_2} \in \mathcal{L}$. By using the least squares method, we obtain $\alpha_1(\theta_u) = 8\theta_u^4 - 20.7\theta_u^3 + 209.2\theta_u^2 - 366.1\theta_u$, $k_1 = 3583.3$, $\alpha_2(\theta_u) = -1.3\theta_u^4 + 34.3\theta_u^3 - 373.8\theta_u^2 + 3009.6\theta_u$, $k_2 = 3033.3$, $\beta_3(\theta_u) = 0.3\theta_u^4 - 7.8\theta_u^3 + 83.2\theta_u^2 - 475.2\theta_u$, $k_3 = 1851.5$.

4. PROPOSED MODEL FOR CONTROL DESIGN

From (1)–(3) or (15), one can observe that the system has three main time scales:

- Fastest – the x_1-dynamics (see also (1)): the time interval required for the equalization of the gas injection flow in the tubing ($w_{iv} = \varphi_1$) and the gas injection flow $u = w_{gc} = \theta_u$, for a constant θ_u.
- Medium – the x_2-dynamics (see also (2)): the time interval required for the equalization of the gas pro-

[1] The existence and uniqueness of solution of (16)–(18) have not been rigorously demonstrated. However, it can be verified that θ_2 and θ_3 can be obtained from $\theta_u - \frac{\theta_2}{\theta_3}\varphi_3(\theta_2, \theta_3) = 0$ and $\varphi_2(\theta_2, \theta_3) - \varphi_3(\theta_2, \theta_3) = 0$ while θ_1 can be obtained from $\theta_u - \varphi_1(\theta_1, \theta_2, \theta_3) = 0$.
[2] The static input-output relationship was assessed via numerical solution of the algebraic nonlinear equations (16)–(18) and via numerical simulation using the EMSO package.

duction flow ($w_{pg} = x_2\varphi_3/x_3$) and the gas injection flow in the tubing ($w_{iv} = \varphi_1$), corresponding to a constant $u = w_{gc} = \theta_u$.

- Slow – the x_3-dynamics (see also (3)): the time interval required for the equalization of the oil production flow ($w_{po} = \varphi_3$) and the oil flow from the reservoir ($w_{ro} = \varphi_2$), corresponding to a constant $u = w_{gc} = \theta_u$.

Inspired in the static input-output relationship (20), it is proposed a reduced order (first order) model that takes into account the difference between the fast and the slow/medium system time constants. Since the time intervals for x_2 and x_3 reach their steady state values θ_2 and θ_3, respectively, are significantly greater than the time interval for x_1 to reach its steady state value θ_1, the following dynamic model is proposed:

$$y = \beta(u_f)u\,, \quad u_f = \frac{1}{\tau_f s + 1}u\,. \tag{21}$$

This simplification is acceptable since the dynamics of the mass of gas in the annulus is, of course, the faster dynamics of the system and $\varphi_1 = w_{iv}$ is weakly dependent from x_2, x_3. Notice that this model does not belong to a HW class of systems. Furthermore, from (15), it is apparent that the system has relative degree two with respect to the input u. Although the order and the relative degree are reduced in comparison to (15), the behaviour of the modified Eikrem's model (15) is quite similar to the proposed model (21), as will be seen via numerical simulations.

4.1 Model Validation

The oil production response (in $\mathrm{kg\,s^{-1}}$) corresponding to a sequence of steps in the gas injection flow, ranging from 4 $\mathrm{kg\,s^{-1}}$ to 6 $\mathrm{kg\,s^{-1}}$, is illustrated in Figure 2(a). The initial conditions $x_1(0) = 4350.1$, $x_2(0) = 10951$ and $x_3(0) = 86038$ are such that the gas injection flow is 4 $\mathrm{kg\,s^{-1}}$. The sequence of steps occurs as follows: a step from 4 $\mathrm{kg\,s^{-1}}$ to 4.5 $\mathrm{kg\,s^{-1}}$ at $t = 0.5$, a step from 4.5 $\mathrm{kg\,s^{-1}}$ to 6 $\mathrm{kg\,s^{-1}}$ at $t = 1.5$, a step from 6 $\mathrm{kg\,s^{-1}}$ to 4.5 $\mathrm{kg\,s^{-1}}$ at $t = 2.5$ and a step from 4.5 $\mathrm{kg\,s^{-1}}$ to 4 $\mathrm{kg\,s^{-1}}$ at $t = 3.5$. In Figure 2(b), a slow transient response (7.2 h) is observed and after vanishing the transient, the output $y = w_{po}$ follows the WPC curve shown in Figure 1. The proposed model response captures the transient behavior and steady state values of the modified Eikrem's model (15).

In order to further explore the similarity between these two models, the frequency response was analyzed. A similar behaviour is observed for periods of the input ranging from 1 hour to 40 days. Figure 2(c) illustrates the response to a sinusoidal input with period equal to 1 hour and the average level of 4 kg/s. We note that only a small phase delay is observed.

The steady state oil production corresponding to a sinusoidal gas injection flow $u(t) = 0.1\sin(\omega t) + b$, with frequency $\omega = 2\pi/T$, period T and mean value b, was evaluated for 3 different values of the period: $T = 40$ days (very low frequency), $T = 1$ hour and $T = 1$ minute (high frequency). Moreover, for each period, two different mean values were considered: $b = 4$ and $b = 6$, below and above the WPC maximizer 5.3, respectively.

The results for $T = 10$ days are illustrated in Figure 2(d). The output was scaled and the average levels were removed to facilitate the illustration of the phase difference between input and output. It can be seen that the input and output are in phase for mean value ($b = 4$) below the WPC maximizer (black line) and are in counter-phase for mean value ($b = 6$) above the WPC maximizer (blue line). By increasing the frequency the phase difference is unobservable. Indeed, input and output in steady state are in counter-phase, independently of the mean value b. So, there exists a clear phase difference between input and output in steady state only for a very low frequency.

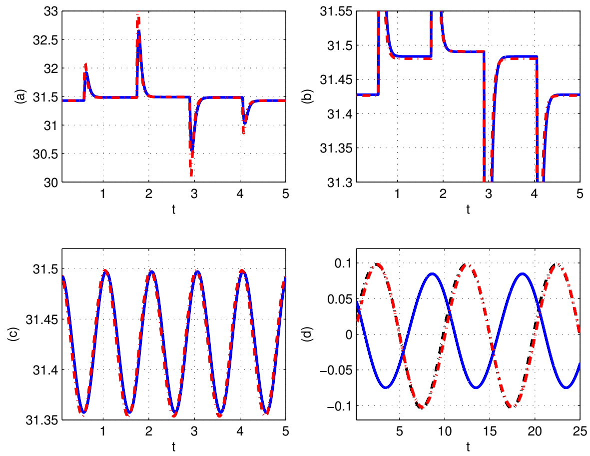

Figure 2. Comparison between the proposed model (21), solid line, and the modified Eikrem's model (15), dash-dot line, with the oil production flow in $\mathrm{kg\,s^{-1}}$ and t in days: (a) step response; (b) zoom in the step response; (c) steady state response to a sinusoidal input and (d) steady state response to a sinusoidal input in phase (black line) and in counter-phase (blue line).

The proposed model captures the following main features of the modified Eikrem's model: (i) the static input-output relationship described by the WPC, (ii) a large transient settling time and a large transient peak corresponding to a step input signal and (iii) steady state response to sinusoidal inputs in **phase** (**counter-phase**) with the input when the average input level is **below** (**above**) the maximizer (this phase difference decreases as the input frequency increases).

5. PROBLEM STATEMENT

Inspired by the proposed model in (21), consider the following class of nonlinear uncertain SISO plants

$$\tau\dot{x} = -x + u\,, \tag{22}$$
$$y = h(x,u) = \beta(x)u\,, \tag{23}$$

where $u \in \mathbb{R}$ is the control signal, $x \in \mathbb{R}$ is the plant state (not available), $y \in \mathbb{R}$ is measured output, $\beta : \mathbb{R} \to \mathbb{R}$ is a class $\mathcal{L}$ ($\beta \in \mathcal{L}$) function and the time constant $\tau > 0$ is considered uncertain. In order to assure existence and forward uniqueness of solutions, it is assumed that the nonlinear function β is locally Lipschitz continuous in x and sufficiently smooth (all required derivatives are continuous). For each solution of (22) there exists a maximal time interval of definition given by $[0, t_M)$, where t_M may be finite or infinite.

It is clear that the control law $u = \theta_u$ assures that $x = \theta_u$ is an equilibrium point globally exponential stable, where $\theta_u \in \mathbb{R}$ is a parameter. The corresponding plant output is given $\theta_y = \Phi(\theta_u)$, where $\Phi : \mathbb{R} \to \mathbb{R}$ is the *static input-output mapping*:

$$\Phi(z) := \beta(z)z\,. \tag{24}$$

Of course, for $\tau = 0$, we have a static input-output mapping given by $y = \Phi(u)$. Note that, due to the presence of the control signal u in (23), the system does not belong to the class of HW type systems [3], which is the most important class of nonlinear systems covered by ESC schemes founded in the literature, in particular, the perturbation-based ESC schemes (Ariyur and Krstic, 2003).

Here, we address the global real-time optimization control problem – problem of extremum seeking – considering only the maximum seeking problem, without lost of generality. We deal with the maximization of the static mapping (24) subjected to (23) and (22).Here we assume that the function $\Phi : \mathbb{R} \to \mathbb{R}$, in (24), has a unique maximizer $\theta^* \in \mathbb{R}$ such that $\Phi(\theta^*)$ is the maximum, i.e., $\forall\theta \in \mathbb{R}$ has $\Phi(\theta) \le \Phi(\theta^*)$. Moreover, we consider that θ^*, $\Phi(\cdot)$ and its gradient are unknown to the control designer and that all the uncertain parameters belong to some compact set.

6. EXTREMUM SEEKING CONTROL

Consider the perturbation-based extremum seeking control method described in (Krstić and Wang, 2000). For plants with $\tau = 0$ in (22), or with τ sufficiently small when compared with the period of the disturbance, the following input-output static relationship results: $y = \Phi(u)$.

In the ESC method the control signal is composed by an estimated $\hat{\theta}$ for maximizer θ^* added to a sinusoidal perturbation $v = asin(\omega t)$, i.e., $u = \hat{\theta} + v$. The estimate $\hat{\theta}$ is given by:

$$\dot{\hat{\theta}} = \frac{k}{a}\xi\,, \quad \hat{\theta}(0) = \hat{\theta}_0\,, \tag{25}$$

where ξ is an estimate for the gradient of the function Φ in (24) evaluated at $\hat{\theta}$, i.e., $\Phi^{'}(\hat{\theta})$. Note that, the dynamics $\dot{\hat{\theta}} = \frac{k}{a}\Phi^{'}(\hat{\theta})$ has an asymptotically stable equilibrium point of given by the maximizer $\hat{\theta} = \theta^*$. Now, using the first two terms of the Taylor series to approximate the input-output relationship $\Phi(u)$ with $u = \hat{\theta} + asin(\omega t)$, it follows that $y \approx \Phi(\hat{\theta}) + \Phi^{'}(\hat{\theta})asin(\omega t)$. In general, the output y can be consider to present the form $y_{ss}(t) = \theta_1(t)\sin(\omega t) + \theta_2(t)$ in steady state [4]. Therefore, θ_1 and θ_2 can be estimated as follows. The amplitude θ_1 can be estimated by $\hat{\theta}_1 = \frac{2}{\tau_1 s+1}[\sin(\omega t)z(t)]$, where τ_1 is a positive design constant and z is the output of a high-pass filter, i.e., $z = \frac{s}{s+w_h}y = y - \frac{w_h}{s+w_h}y$. Indeed, the filter $\frac{w_h}{s+w_h}$, designed properly, attenuate the term $\theta_1(t)\sin(\omega t)$ in $y = \theta_1(t)\sin(\omega t) + \theta_2(t)$. Then, $z \approx \theta_1(t)\sin(\omega t)$. The signal is then demodulated, multiplying by $\sin(\omega t)$, resulting in $\theta_1(t)\sin^2(\omega t)$. Reminding that $2\sin^2(\omega t) = 1 - \cos(2\omega t)$, only the DC component is not filtered by $\frac{2}{\tau s+1}$. Hence, $\hat{\theta}_1 \approx \theta_1$. The average value (DC component) is estimated directly by $\hat{\theta}_2 = \frac{1}{\tau s+1}y$, i.e., $\hat{\theta}_2 \approx \theta_2$. Thus, $\hat{\theta}_1 \approx \Phi^{'}(\hat{\theta})a$, $\hat{\theta}_2 \approx \Phi(\hat{\theta})$ and an approximation for the gradient of the function Φ evaluated at $\hat{\theta}$ is given by $\xi = \hat{\theta}_1/a$. For the correct operation of the algorithm the cutoff frequencies w_h and $1/\tau$ must be less than ω. The amplitude a of the sinusoidal perturbation defines the size of the oscillations around the optimal point, while the integrator gain k defines how fast the output will reach this neighbourhood. These two constants should be small enough to assure the stability of the algorithm (Krstić and Wang, 2000).

6.1 Proposed Pre-Compensation

The main idea is to reduce the system to a HW system which has the same input-output static mapping and for which the ESC frequency can be increased.

Consider the system described in (22) and (23), with static mapping defined in (24) and control signal given by $u(t) = \hat{\theta}(t) + a\sin(\omega t)$, where $\hat{\theta}$ the estimate (25), i.e., the estimate $\dot{\hat{\theta}} = \frac{k}{a}\xi$, with ξ being an estimate for $\Phi^{'}(\hat{\theta})$, with Φ in (24). Differently from (25), where ξ is obtained from the plant output y, here we use the following available auxiliary output signal

$$\mathcal{Y} := \frac{y}{u}\hat{x}\,, \quad u \neq 0\,,$$

to generate ξ by considering the resulting plant

$$\tau\dot{x} = -x + u\,, \tag{26}$$

$$\mathcal{Y} = \beta(x)\hat{x}\,. \tag{27}$$

Note that, by choosing $\hat{x}$ such that $\hat{x} \approx x$, the system (26)–(27) has the same input-output static mapping $\Phi(z) = \beta(z)z$ as the original system (22)–(23) and now belongs to a class of HW systems. This class of plants has been extensively addressed in the ESC literature. Now, it is possible to detect phase difference between $\mathcal{Y}$ and u for higher ESC operating frequencies.

For the case where τ is a **known** parameter in (26), let

$$\hat{x} := \hat{\theta} + a\frac{1}{\sqrt{\omega^2\tau^2+1}}\sin(\omega t + \phi)\,,$$

where $\phi = \angle P(j\omega) = -\arctan\tau\omega$ and $P(j\omega) := \frac{1}{j\tau\omega+1}$. From (26) one can verify that this choice for $\hat{x}$ approximates the steady state value of the plant state x, if $\hat{\theta}(t)$ is in the passband of (26).

For the case where τ is a **unknown** parameter in (26) we can estimate $\angle P(j\omega)$ by using a phase lock loop estimator (PLL) applied to the available signal $y/u = \beta(x)$, when $u \neq 0$, resulting in the estimate $\hat{\phi}$. Indeed, since $\beta \in \mathcal{L}$ the signal $\beta(x)$ has, approximately, the same phase shift ($\angle P(j\omega)$) as x w.r.t. to the input u (in steady state). This can be verified by using the Taylor series for representing $\beta(x)$. Hence, an estimate for τ can be obtained from

[3] Note that, however, by adding a stable filter at the system input results in a augmented HW system.

[4] Note that, even for the case $\tau > 0$, the dynamic system is stable and it still reasonable to consider that the output y presents this form in steady state.

$\hat{\tau} = -\tan(\hat{\phi})/\omega$ and then the auxiliary signal $\hat{x}$ can be redefined as

$$\hat{x} := \hat{\theta} + a\frac{1}{\sqrt{\omega^2\hat{\tau}^2+1}}\sin(\omega t + \hat{\phi})\,.$$

The results, considering the proposed model (with $\tau = 2100$), are given in Figure 3. Before $t = 3.5$, a rough estimate of the phase shift ϕ were considered, see Figure 3 (c) (solid line). In this case, we note from Figure 3 (a) that the estimate $\hat{\theta}$ (solid line) of the WPC maximizer $\theta^* = 5.3$ (dash-dot line) approaches a wrong value (≈ 6). After $t = 3.5$ the correct phase shift were detected by the PLL estimator and after $t \approx 5.4$, the gain $\frac{1}{\sqrt{\omega^2\hat{\tau}^2+1}}$ was corrected, resulting the convergence of the estimate $\hat{\theta}$ to the maximizer $\theta^* = 5.3$. In Figure 3 (b), it is illustrated the estimate $\hat{\tau}$ (solid line) changing from the wrong initial value 2000 to the correct value $\tau = 2100$ (dash-dot line). The PLL phase estimate (black dashed-dot line) converges to the ideal value $-arctan(\omega\tau)$ (red dashed-dot line). After $t = 3.5$ the correct phase shift (solid line) changes from the wrong initial value (≈ -1279) to the correct value ≈ -1280 (dash-dot line). The ESC oscillation period is now reduced from 10 days to 10 hours.

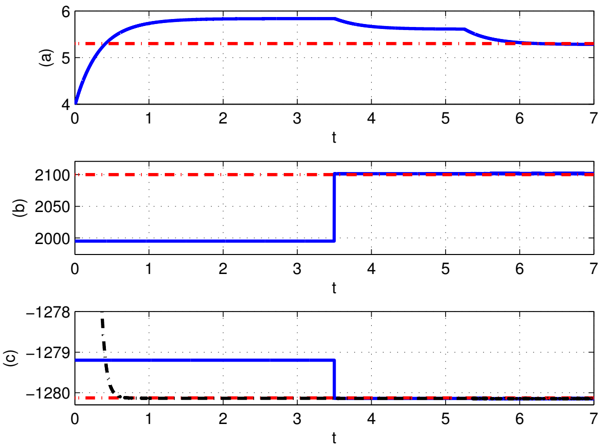

Figure 3. Simulation results with oscillation period equal to 10 hours: (a) maximizer θ^* (dashed-dot line) and its estimate $\hat{\theta}$ (solid line); (b) the time constant τ (dashed-dot line) and its estimate $\hat{\tau}$ and (c) the pause shift ϕ (dashed-dot line), its estimate $\hat{\phi}$ (solid line) and the PLL estimation (black dashed-dot line).

7. CONCLUSION

The modified Eikrem's model for gas lifted wells was presented and analyzed. In order to maintain the oil production around the optimum point of the well-performance curve, a perturbation-based extremum seeking control (ESC) scheme was evaluated. It was verified that only operating at a very low frequency of the perturbation, the ESC was capable to assure that the oil production reaches a small vicinity of the optimum point. This fact motivated us to develop a simple nonlinear dynamic model capturing the essential dynamics of the modified Eikrem's model (transient and steady state behaviour). Based on this model a pre-compensation was developed allowing the application of the ESC scheme without reducing excessively the frequency of operation. The key idea was to approximately reduce the original nonlinear system into a HW type system. The control performance was evaluated via numerical simulations. A tuning methodology for the ESC parameters and the complete closed loop stability analysis (including the pre-compensation) were left for future work.

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