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# Robust Dynamic Positioning of Offshore Vessels using Mixed-$\mu$ Synthesis Part I: A Control System Design Methodology ⋆

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**Abstract:** This paper describes a procedure to design robust controllers for Dynamic Positioning (DP) of ships and offshore rigs subjected to the influence of sea waves, currents, and wind loads using $\mathcal{H}_\infty$ and mixed-$\mu$ techniques. To this effect, practical assumptions are exploited in order to obtain a linear design model with parametric uncertainties describing the dynamics of the vessel. Appropriate frequency weighting functions are selected to capture the required performance specifications at the controller design phase. The proposed model and weighting functions are then used to design robust controllers. The problem of wave filtering is also addressed during the process of modeling and controller design. The key contribution of the paper is twofold: i) it affords system designers a new method to efficiently obtain linearized design models that fit naturally in the framework of $\mathcal{H}_\infty$ control theory, and ii) it describes, in a systematic manner, the different steps involved in the controller design process. Part II in a companion paper contains the details of simulations and results of experimental model tests in a towing tank equipped with a hydraulic wave maker.

*Keywords:* Robust Control, Dynamic Positioning, Wave Filtering.

## 1. INTRODUCTION

The advent of offshore exploration and exploitation at an unprecedented scale has brought about increasing interest in the development of Dynamic Positioning (DP) systems for surface vessels. Currently, there are more than 2000 DP vessels of various kinds operating worldwide, see Sørensen (2011a). DP systems are used with a wide range of vessel types and in different marine operations; in particular, in the offshore, oil, and gas industries many applications are only possible with the use of DP systems for service vessels, drilling rigs and drilling ships, shuttle tankers, cable and pipe layers, floating production off-loading and storage units (FPSOs), crane and heavy lift vessels, geological survey vessels, and multi-purpose vessels. Cable and pipe laying are typical operations that also need tracking functionality. The main purpose of DP systems is to keep the position and heading of marine structures within pre-specified excursion limits under expected weather windows. As such, they play a key role in many offshore operations aimed at improving the efficiency and safety of oil exploitation techniques.

DP systems came to existence in the 1960s for offshore drilling applications for the first time, due to the need to drill in deep waters and the realization that Jack-up barges and anchoring systems could not be used economically at such depths. Early DP systems were implemented using PID controllers, and in order to restrain thruster trembling caused by the wave-induced motion components, notch filters in cascade with low pass filters were used with the controllers. However, notch filters restrict the performance of closed- loop systems because they introduce some phase lag around the crossover frequency, which in turn tends to decrease the phase margin. An improvement in performance was achieved by exploiting more advanced control techniques based on optimal control and Kalman filter theory, see Balchen et al. (1976). All these techniques were later modified and extended in Balchen et al. (1980); Grimble et al. (1980) and Fossen and Strand (1999). Applying the LQG to the problem of DP requires the linearization of the dynamics and kinematics of the plant over different operating points; besides, for each operating point a set of variables (such as covariances of disturbances and weighting matrices) needs to be computed which makes the procedure of tuning the controllers costly and burdensome. Moreover, Doyle (1978) showed that LQG has no guaranteed phase and gain margins and the resulting closed-loop regulator may have arbitrary small stability margins. This led to the development of a simpler setup using passive observers and nonlinear multivariate PID controllers; see Fossen and Strand (1999). The literature on ship DP is vast and defies a simple summary. See for example Sørensen (2011b) and the references therein for a short presentation of the subject and its historical evolution.

Different sources of uncertainty in the DP problem led to the application of robust control techniques to DP, see Katebi et al. (1997, 2001); Donha and Katebi (2007). The $\mathcal{H}_\infty$ and mixed-$\mu$ are model based techniques and design of a DP controller based on these methodologies requires a linear model of the plant (computed by linearization of

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the plant about an operating point). The computation of the latter for different operating points is cumbersome, requires intensive computations, and may be very costly. For these reasons, and in spite of the potential benefits of using robust DP controllers, the assessment of their performance has, to be best of our knowledge, been carried out using only simulations or by performing experimental tank tests; see Katebi et al. (2001).
DP systems have generally been designed for low-speed and low Froude number applications, where the basic DP functionality is either to keep a fixed position and heading of a ship, or to move it slowly from one location to another. In this work, using the low speed assumption, a linear model with parametric uncertainties is developed based on which, by assigning appropriate frequency weighting functions and using $\mathcal{H}_\infty$ and mixed-$\mu$ techniques, robust controllers for different sea conditions (calm, moderate, high, and extreme seas) are designed.
The structure of the paper is as follows. A brief introduction to important issues that arise in DP are presented in section 2. Section 3 proposes a linear representative vessel model with parametric uncertainties. Section 4 summarizes the main ideas behind the DP robust controller designing process in calm to high sea conditions. Section 5 explains the DP controller design procedure in extreme sea conditions. Conclusions and suggestions for future research are summarized in section 6.

## 2. DYNAMIC POSITIONING AND WAVE FILTERING

In DP systems, the key objective is to maintain the ship's heading and position within desired limits by means of active thrusters. In order to design a robust DP controller a linear model of plant must be derived first. Here, we should stress that in the marine control literature different mathematical models with different complexity levels are used for different purposes. Two important models (see Sørensen (2011a)) are formulated as the control plant model (or design plant model) and the process plant model (or simulation model). The first is a simplified mathematical description containing only the main physical properties of the process or plant and is used for the purpose of controller design and stability analysis, using for example Lyapunov stability and passivity tools. The second is a comprehensive description of the actual process whose main purpose is to simulate the real plant dynamics and is used in numerical performance and robustness analysis and testing of the control systems designed.
In this section we formulate the problem of modeling a DP system using a low speed assumption. A linear plant model with parametric uncertainty is obtained and used for DP controller system design. Later on, in order to evaluate the performance of the designed controllers, a nonlinear high fidelity model, the Marine Cybernetics Simulator (MCSim), is used; For details on the MCSim see Hassani et al. (2012a).
In DP applications in open waters, waves produce a pressure change on the hull surface of the vessel. This change of pressure induces different forces and torques on the vessel. Usually, only first and second order effects of these pressure-induced forces are studied in DP applications. The first order effect of the waves has an oscillatory nature that depends linearly on the wave elevation. Hence, these forces have the same frequency as that of the waves and are therefore referred to as wave-frequency forces. The second order effect of the waves depends nonlinearly on the wave elevation, see Faltinsen (1990). The nonlinear component of wave forces are due to the quadratic dependence of the pressure on the fluid-particle velocity induced by the passing of the waves. They have a wider frequency range and they excite the vessel not only in the in the wave frequency range but also in lower and higher frequency ranges. While the mean wave forces make the vessel drift, the oscillatory components of the wave forces can lead to resonance in the horizontal motion of vessel under positioning control. Hence, the motions of marine vessels can often be divided into a low-frequency (LF) part and a wave-frequency (WF) part. For most positioning applications (usually for calm, moderate and high sea), only the slowly-varying wave disturbances and mean wave loads (in addition to wind and current loads) should be counterbalanced by the propulsion system, whereas the oscillatory motion induced by the waves (1st-order wave effect) should not enter the feedback control loop. The reasons for this could be that either the WF motion does not matter for the particular operation, or the vessel does not have enough power and thrust capacity for doing any noticeable compensation at all. The latter reason is of great importance, for there is no point in wasting fuel and cause additional wear and tear of the propulsion equipment. To this effect, the DP control systems should be designed so as to react to the low frequency forces on the vessel only. In the literature, this task is accomplished by using so-called wave filtering techniques, which separate the position and heading measurements into low-frequency (LF) and wave-frequency (WF) position and heading estimates (Fossen (2011)). Wave filtering observers provide an estimate of the (low frequency) velocities computed from corrupted measurements of position and heading. Later, these estimates are used for control purposes. In designing the robust DP controllers, this task is accomplished by introducing appropriate frequency weighting functions and performance signals. The latter will be addressed in details in next section.
In extreme seas with high wave heights and/or long wave lengths or in swell dominated seas, the assumption of producing control action from the LF motion signals only, may not be so evident, as the WF motions (due to long wave lengths and thereby low frequency) will enter the control bandwidth of the DP system. Furthermore, in high sea states limitation of power and loss of thrust due to ventilation, cavitation, and thruster-hull interactions will give reduced performance. See Sørensen et al. (2002), and Sørensen (2011b) for details on DP in extreme sea condition. In extreme sea condition the wave filtering is turned off and all the components of motion are compensated in the DP controller to the extend that the propulsion system allows.
In the following sections, the design of a robust DP controller will be addressed separately for normal (calm to high) seas and extreme seas.

## 3. MODELING DP SYSTEMS

In what follows, the vessel model, that is by now standard [1], is presented. See Fossen and Strand (1999); Sørensen (2011a). The model admits the realization

$$\dot{\xi}_W = A_W(\omega_0)\xi_W + E_W w_W \quad (1)$$
$$\eta_W = R(\psi_L)C_W\xi_W \quad (2)$$
$$\dot{b} = -T^{-1}b + E_b w_b \quad (3)$$
$$\dot{\eta}_L = R(\psi_L)\nu \quad (4)$$
$$M\dot{\nu} + D\nu = \tau + R^T(\psi_{tot})b \quad (5)$$
$$\eta_{tot} = \eta_L + \eta_W \quad (6)$$
$$\eta_y = \eta_{tot} + v \quad (7)$$

[1] The model described by (1)-(6) has minor differences with respect to the ones normally described in the literature. While in most of the references the WF components of motion are modeled in a fixed-earth frame, in this paper the WF motion is modeled in body-frame. The reader is referred to Hassani et al. (2012c,b) for details and improvements of the present model.

where (1) and (2) capture the 1st-order wave induced motion in surge, sway, and yaw; equation (3) represents the 1st-order Markov process approximating the unmodelled dynamics and the slowly varying bias forces (in surge and sway) and torques (in yaw) due to waves (2nd order wave induced loads), wind, and currents. The latter are given in earth fixed coordinates but expressed in body-axis. In the above, $\eta_W \in \mathbb{R}^3$ is the vessel's WF motion due to 1st-order wave-induced disturbances, consisting of WF position $(x_W, y_W)$ and WF heading $\psi_W$ of the vessel; $w_W \in \mathbb{R}^3$ and $w_b \in \mathbb{R}^3$ are zero mean Gaussian white noise vectors, and

$$A_W = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ -\Omega_{3\times3} & -\Lambda_{3\times3} \end{bmatrix}, \qquad E_W = \begin{bmatrix} 0_{3\times1} \\ I_{3\times1} \end{bmatrix},$$
$$C_W = [0_{3\times3} \; I_{3\times3}],$$

with

$$\Omega = \text{diag}\{\omega_{01}^2, \omega_{02}^2, \omega_{03}^2\},$$
$$\Lambda = \text{diag}\{2\zeta_1\omega_{01}, 2\zeta_2\omega_{02}, 2\zeta_3\omega_{03}\},$$

where $\omega_{0i}$ and $\zeta_i$ are the Dominant Wave Frequency (DWF) and relative damping ratio, respectively. Matrix $T = \text{diag}(T_x, T_y, T_\psi)$ is a diagonal matrix of positive bias time constants and $E_b \in \mathbb{R}^{3\times3}$ is a diagonal scaling matrix. Vector $\eta_L \in \mathbb{R}^3$ consists of low frequency (LF), earth-fixed position $(x_L, y_L)$ and LF heading $\psi_L$ of the vessel relative to an earth-fixed frame, $\nu \in \mathbb{R}^3$ represents the velocities decomposed in a vessel-fixed reference, and $R(\psi_L)$ is the standard orthonormal yaw angle rotation matrix (see Fossen (2011) for details). Equation (5) describes the vessels's LF motion at low speed (see Fossen (2011)), where $M \in \mathbb{R}^{3\times3}$ is the generalized system inertia matrix including zero frequency added mass components, $D \in \mathbb{R}^{3\times3}$ is the linear damping matrix, and $\tau \in \mathbb{R}^3$ is a control vector of generalized forces generated by the propulsion system, that is, the main propellers aft of the ship and thrusters which can produce surge and sway forces as well as a yaw moment. Vector $\eta_{tot} \in \mathbb{R}^3$ describes the vessel's total motion, consisting of total position $(x_{tot}, y_{tot})$ and total heading $\psi_{tot}$ of the vessel. Finally, (7) represents the position and heading measurement equation, with $v \in \mathbb{R}^3$ a zero-mean Gaussian white measurement noise.

Usually, in the design of controllers or observers for DP systems (especially for station keeping missions), the following assumptions are made. These assumptions are widely used in the literature, see Fossen and Strand (1999):
**Assumption 1** The position and heading sensor noises are neglected, that is, $v = 0$ because the measurement error induced by measurement noise is negligible compared to the wave-induced motion. [2]
**Assumption 2** The amplitude of the wave-induced yaw motion $\psi_W$ is assumed to be small, that is, less than 2-3 degrees during normal operation of the vessel and less than 5 degrees in extreme weather conditions. Hence, $R(\psi_L) \approx R(\psi_L + \psi_W)$. From Assumption 1 it follows that $R(\psi_L) \approx R(\psi_y)$, where $\psi_y \cong \psi_L + \psi_W$ denotes the measured heading.
**Assumption 3** Low speed assumption, implying that the time-derivative of the total heading $\dot{\psi}_{tot}$ is small, bounded, and close to zero.
We will also exploit the model property that the bias time constants in the x and y directions are equal, i.e. $T_x = T_y$. In what follows we will consider a reference frame consisting of *vessel parallel coordinates* as introduced in Fossen (2011); Sørensen (2011a). In sea keeping analysis (vessel motions in waves) the hydrodynamic frame is generally moving along the path of the vessel with the x-axis positive forwards, y-axis positive to the starboard, and z-axis positive downwards. The XY-plane (in hydrodynamic frame) is assumed fixed and parallel to the mean water surface. The vessel is assumed to oscillate with small amplitudes about this frame such that linear theory can be used to model perturbations. In station keeping operations (dynamic positioning) about desired coordinates $x_d$, $y_d$, and $\psi_d$, the hydrodynamic frame is Earth-fixed and denoted as the vessel parallel frame. It is defined in a reference frame fixed to the vessel, with axes parallel to the earth-fixed frame and the origin is translated to the desired $x_d$ and $y_d$ (in this study we assume that $x_d = y_d = 0$). Let $\eta_L^p \in \mathbb{R}^3$ denote the LF position $(x_L^p, y_L^p)$ and LF heading $\psi_L^p$ of the vessel, respectively expressed in body coordinates, defined as

$$\eta_L^p = R^T(\psi_{tot})\eta_L. \tag{8}$$

Computing its derivative with respect to time yields

$$\begin{aligned}\dot{\eta}_L^p &= \dot{R}^T(\psi_{tot})\eta_L + R^T(\psi_{tot})\dot{\eta}_L \\ &= \dot{R}^T(\psi_{tot})R(\psi_{tot})\eta_L^p + R^T(\psi_{tot})R(\psi_L)\nu \end{aligned} \tag{9}$$

Using a Taylor series to expand $R^T(\psi_{tot})$ about $\psi_L$ and neglecting higher order terms, it follows that

$$R^T(\psi_{tot})R(\psi_L) \cong I + \psi_W S, \tag{10}$$

where

$$S = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Using simple algebra we obtain

$$\dot{R}^T(\psi_{tot})R(\psi_{tot}) = \dot{\psi}_{tot}S. \tag{11}$$

From (9), (10) and (11) we conclude that

$$\dot{\eta}_L^p \approx \dot{\psi}_{tot}S\eta_L^p + \nu + \psi_W S\nu. \tag{12}$$

We now study the time evolution of the slowly varying bias forces, $b$, expressed in the in the vessel parallel coordinates, $b^p$, as follows:

$$b^p = R^T(\psi_{tot})b. \tag{13}$$

Clearly,

$$b = R(\psi_{tot})b^p, \tag{14}$$

and differentiating both sides yields

$$\dot{b} = \dot{R}(\psi_{tot})b^p + R(\psi_{tot})\dot{b}^p. \tag{15}$$

Using (3), (14) and (15) we obtain

$$\dot{R}(\psi_{tot})b^p + R(\psi_{tot})\dot{b}^p = -T^{-1}R(\psi_{tot})b^p + E_b w_b. \tag{16}$$

Reordering (16) and multiplying both sides by $R^T(\psi_{tot})$ gives

$$\begin{aligned}\dot{b}^p &= -R^T(\psi_{tot})T^{-1}R(\psi_{tot})b^p - R^T(\psi_{tot})\dot{R}(\psi_{tot})b^p \\ &\quad + R^T(\psi_{tot})E_b w_b. \end{aligned} \tag{17}$$

Using the assumption that $T_x = T_y$, it can be checked that $R^T(\psi_{tot})T = TR^T(\psi_{tot})$; simple algebra also shows that $R^T(\psi_{tot})\dot{R}(\psi_{tot}) = -\dot{\psi}_{tot}S$.
Equation (17) can be expressed as

$$\dot{b}^p = -T^{-1}b^p + \dot{\psi}_{tot}Sb^p + R^T(\psi_{tot})E_b w_b. \tag{18}$$

Summarizing the equations above yields

$$\dot{\xi}_W = A_W(\omega_0)\xi_W + E_W w_W \tag{19}$$
$$\eta_W = R(\psi_L)C_W\xi_W \tag{20}$$
$$\dot{b}^p = -T^{-1}b^p + \dot{\psi}_{tot}Sb^p + R^T(\psi_{tot})E_b w_b \tag{21}$$
$$\dot{\eta}_L^p = \dot{\psi}_{tot}S\eta_L^p + \nu + \psi_W S\nu \tag{22}$$
$$M\dot{\nu} + D\nu = \tau + b^p \tag{23}$$

[2] At this point, we stress that the noise free assumption is only used to derive a control plant model but later on, in the design process and simulation and verification, the effect of the measurement noise will be considered.

Moreover, using assumptions 1, 2 and 3 a linear model with parametric uncertainty is obtained that is given by

$$\dot{\xi}_W = A_W(\theta_1)\xi_W + E_W w_W \quad (24)$$
$$\eta_W^b = C_W \xi_W \quad (25)$$
$$\dot{b}^p = -T^{-1}b^p + \theta_2 S b^p + w_b^f \quad (26)$$
$$\dot{\eta}_L^p = \theta_2 S \eta_L^p + \nu + \theta_3 S \nu \quad (27)$$
$$M\dot{\nu} + D\nu = \tau + b^p \quad (28)$$
$$\eta_y^f = \eta_L^p + \eta_W^b + n \quad (29)$$

where $\eta_W^b$ are WF components of motion on body-coordinate axis, and $w_b^f$ and $\eta_y^f$ are a new modified disturbance and a modified measurement defined by $w_b^f = R^T(\psi_y)E_b w_b$ and $\eta_y^f = R^T(\psi_y)\eta_y$, respectively,[3] $n \in \mathbb{R}^3$ is the measurement noise, and finally $\theta_1$, $\theta_2$, and $\theta_3$ are $\omega_0$, $\dot{\psi}_{tot}$, and $\psi_W$, respectively, which will be treated as parametric uncertainties.[4]

## 4. ROBUST DP CONTROLLER DESIGN IN NORMAL SEA CONDITIONS

This section describes the application of $\mathcal{H}_\infty$-based, $\mu$ synthesis controller design techniques to the solution of the DP problem. See Skogestad and Postlethwaite (2006); Francis (1987) for an introduction to these techniques and Balas (2009) for a mixed-$\mu$ design suite implemented in Matlab. In what follows, we adopt the general setup and nomenclature in the seminal work of Doyle et al. (1989). This leads to the standard feedback system of Fig. 1 (a), where $w$ is the input vector of exogenous signals, $z$ is the output vector of errors and performance signals to be reduced, $y$ is the vector of measurements that are available for feedback, and $u$ is the vector of actuator signals. Suppose that the feedback system is well-posed, and let $T_{wz}(s)$ denote the closed loop transfer matrix from $w$ to $z$. The $\mathcal{H}_\infty$, synthesis problem is to find, among all controllers that yield a stable closed loop system, a controller $K$ that minimizes the infinity norm $\|T_{wz}(s)\|_\infty$ of $T_{wz}(s)$. We remind the reader that $\|T_{wz}(s)\|_\infty$ equals $\sup\{\sigma_{max}(T_{wz}(j\omega)) : \omega \in \mathbb{R}\}$ where $\sigma_{max}(.)$ denotes the maximum singular value. Furthermore, $\|T_{wz}\|_\infty$ may be interpreted as the maximum energy gain of the closed loop operator $T_{wz}$. In mixed-$\mu$ synthesis, the structured singular value of a linear fractional transformation (LFT) of the plant and controller are used instead of the maximum singular values. The Structured Singular Values, denoted SSV or complex-$\mu$ (later modified to mixed-$\mu$), were introduced in Doyle (1982) and Packard and Doyle (1993). In order to obtain a good design for a controller $K$, accurate knowledge of the plant is required. In practice, obtaining an accurate process model of the plant is almost impossible. The model may be inaccurate and there may be unmodelled dynamics and parametric uncertainties in the plant. To deal with this problem, the concept of model uncertainty must be considered. The unknown plant $P$ is assumed to belong to a "legal" class of control plant models, $\mathcal{P}$, built around a nominal model $P_0$. The set of models $\mathcal{P}$ is characterized by a matrix $\Delta$, which can be either a full matrix or a block diagonal matrix, that includes all possible system structured uncertainties. We also use the weighting matrices (and incorporate them into $P$) to express the uncertainty in terms of normalized uncertainties in such a way that $\|\Delta(s)\|_\infty \leq 1$. The general control configuration in Fig. 1 (a) may be extended to include model uncertainty as shown in Fig. 1 (b).

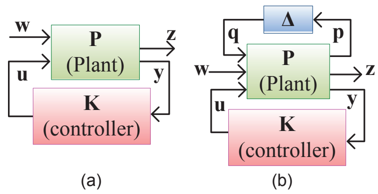


Fig. 1. Standard Feedback Configuration (with and without uncertainty).

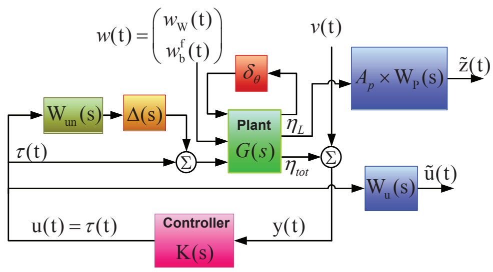


Fig. 2. Nominal Setup with Frequency Weighting Functions to Design a Robust Controller.

Fig. 2 shows the nominal setup for designing a robust controller. Later, this setup will be used to form the standard feedback system of Fig. 1 which will be used in the mixed-$\mu$ synthesis methodology; Balas (2009). To this effect, we design a robust DP controller which yields stability and performance robustness; using the mixed-$\mu$ software (see Balas (2009)), the performance parameter $A_p$ in Fig. 2 is increased as much as possible, until the upper-bound on the mixed-$\mu$, $\mu_{ub}(\omega)$, satisfies the inequality

$$\mu_{ub}(\omega) \leq 1 \qquad \forall\omega. \quad (30)$$

In what follows we explain the different blocks of the Fig. 2 in detail. Using the control model of the marine vessel given in (24)-(29), a state-space representation of the plant, including the disturbance and noise inputs, is given by

$$\dot{x}(t) = A(\theta)\,x(t) + B\,u(t) + L\,w(t),$$
$$y(t) = C_1 x(t) + v(t),$$
$$z(t) = C_2 x(t),$$

where $u(t) = \tau(t)$ is a control vector of generalized forces generated by the propulsion system, $w(t) = [w_W \; w_b^f]^T$ is a disturbance vector, $v(t)$ is measurement noise, $y(t)$ is the measured output (total motion in body-frame), $z(t)$ is the performance signal (LF component of motion in parallel-frame), the state vector is $x(t) = [{\xi_W}^T \; {\eta_L^p}^T \; \nu^T \; {b^p}^T]^T$, and the system matrices $(A(\theta), B, C)$ are defined in the obvious manner. Notice that the $A(\theta)$ matrix contains parametric uncertainties ($\theta_1$, $\theta_2$, and $\theta_3$) as defined before. We assume that the pairs $(A(\theta), B)$ and $(A(\theta), C)$ are controllable and observable, respectively, for all admissible parameter values.

Table 1 shows the definition of the sea conditions associated with the particular model of offshore supply vessel that is used in our study. In the study we will design a robust DP controller for four different scenarios: calm seas, moderate seas, high seas, and, extreme seas. The in-

[3] When designing observers for wave filtering in DP, since the controller regulates the heading of the vessel, the designer can assign a new intensity to $w_b^f$; however, assigning the intensity of the noise in practice requires considerable expertise.

[4] In this paper, during the controller design process $\theta_1$, $\theta_2$ and $\theta_3$ are treated as fixed parametric uncertainties. The methodology introduced can be extended to deal with time-varying parametric uncertainties with bounded rates of variation; see Rosa et al. (2009).

Table 1. Definition of Sea States

| Sea States | DWF $\omega_0$ (rad) | Significant Wave Height $H_s$ (m) |
|---|---|---|
| Calm Seas | $> 1.11$ | $< 0.1$ |
| Moderate Seas | [0.74 1.11] | [0.1 1.69] |
| High Seas | [0.53 0.74] | [1.69 6.0] |
| Extreme Seas | $< 0.53$ | $> 6.0$ |

Table 2. Interval of Parametric Uncertainties

| Sea States | $\theta_1$ rad/s | $\theta_2$ rad/s | $\theta_3$ rad |
|---|---|---|---|
| Calm Seas | [1.11 1.8] | Int* | [−0.038 0.038] |
| Moderate Seas | [0.74 1.11] | Int | [−0.04 0.04] |
| High Seas | [0.53 0.74] | Int | [−0.042 0.042] |
| Extreme Seas | [0.39 0.53] | Int | [−0.04 0.04] |

* Int=$[-5 \times 10^{-4} \;\; 5 \times 10^{-4}]$

tervals of parametric uncertainty for $\theta$ in the four different scenarios are given in Table 2.

### 4.1 Frequency Weighting Functions

As is well known, given a plant with structured and unstructured uncertainty it is not possible in general to obtain (by proper controller design) robust stability and performance uniformly, across all frequencies, where the latter is measured with the help of properly chosen performance signals. For this reason, it is crucial (for $\mathcal{H}_\infty$ control systems design) that frequency-dependent performance weights be introduced so as to reflect desired performance objectives over different frequencies. Appropriate selection of these weights provides flexibility in the control design process. The DP control design methodology that we propose builds heavily on the new design model introduced here and exploits the difference in the frequency contents of $\eta_L$ and $\eta_W$. In the process, the choice of a weighting function for low frequency disturbance attenuation purposes is crucial.

During the process of DP robust controller design, controllers, we used the mixed-$\mu$ synthesis toolbox to maximize $A_p$ (in Fig. 2) while making sure that robust stability and performance are observed. Fig. 3 depicts graphically the magnitude of the frequency response of nominal performance weighting transfer function, $W_p(s)$. We remark that the performance weight $W_p(s)$ penalizes output $\eta_L^p$ in the low frequency range where the slowly varying disturbance $b^p$ has most of its effect. The gain parameter $A_p$ in $W_p(s)$ specifies our desired level of LF disturbance-rejection. The larger $A_p$, the greater the penalty on the effect of the disturbances on the LF motion. For superior disturbance-rejection in the LF range, $A_p$ should be as large as possible. Moreover, the performance weight $W_p(s)$ places a smaller penalty on performance output $\eta_L^p$ in the mid-range frequencies where WF motion has most of its effect. In particular, this selection dictates our wave filtering demands to the $\mathcal{H}_\infty$ controller. Such a $W_p(s)$ can be found by cascading a low-pass and a narrow band-pass filter together, see Fossen (2011); Sørensen (2011a) for details of DP wave filtering using cascaded low-pass and notch filtering.

To reduce the thruster modulation to the lowest possible level, an appropriate weighting function should be chosen to penalize the control action differently over different frequencies. The rational is that the weight should be selected so that the control energy is penalized in the high-frequency. This avoids saturation as well as excitation of the high-frequency dynamics. The magnitude of the frequency response of a nominal control weighting transfer function, $W_u(s)$, is presented graphically in Fig. 3. This

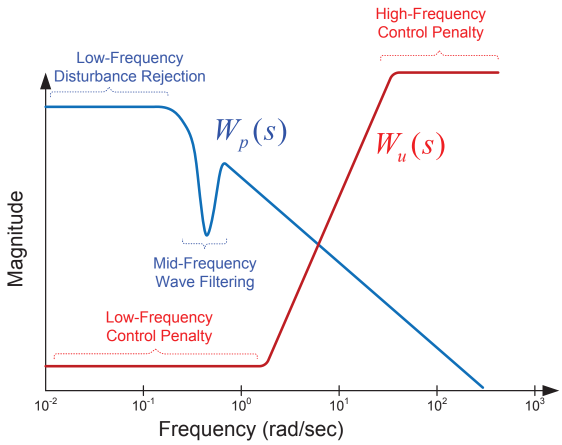


Fig. 3. Choice of Weighting Functions $W_p(s)$ and $W_u(s)$.

selection allows for larger control activity in lower frequencies and penalizes large controls at higher frequencies.

### 4.2 Unmodelled Dynamic and Unstructured Uncertainty

Robust controllers designed using mixed-$\mu$ synthesis can yield robust stability and performance in presence of both parametric uncertainty and unstructured uncertainties (or unmodelled dynamics). In the work of Katebi et al. (1997), a robust $\mathcal{H}_\infty$ controller is designed by minimizing the infinity norm of the transfer matrix from disturbances to a performance signal. That being done, the authors examined what amount of unmodelled dynamics could be tolerated in the feedback loop, using a small gain theorem. Clearly, such formulation may lead to conservative results. In this paper we aim for less conservative results with maximum performance and also simplicity in the design procedure. To capture the effect of unmodelled dynamics in our control plant model, it is also assumed that input forces and torque are provided through an actuator whose bandwidth is unknown but in some fixed known interval and its dc gain has 2 percent uncertainty; this amplifier can be described in the form of some nominal first order transfer function $G_0(s)$ and a multiplicative uncertainty described with some transfer function $W_{\text{unc}}(s)$. The computed frequency-domain upper-bound for the unstructured uncertainty, which serves in this example as a surrogate for unmodelled dynamics, $W_{\text{unc}}(s)$, captures some important practical features. This implies that the designed controller $K(s)$ provides robust-stability and-performance for the nominal vessel model with some percentage of model perturbation (one can easily compute its exact value) over different frequencies; see Hassani et al. (2012a) for details.

Summarizing our design process, Fig. 4 shows the appropriate augmented structure for DP $\mathcal{H}_\infty$ controller design.

## 5. ROBUST DP CONTROLLER DESIGN IN EXTREME SEA CONDITION

In extreme seas and extreme conditions the nonlinearities due to large motions will be more noticeable for the WF motions. Also, the coupling between the horizontal plane motions and the vertical motions will become more important. As the sea state builds, it is also a challenge to distinguish the LF motions from the WF motions. At higher sea states, the period of the waves gets longer, resulting in decreasing wave frequencies. Thus, the formulation of hydrodynamics models appropriate for controller designs is still a subject for research. In such conditions

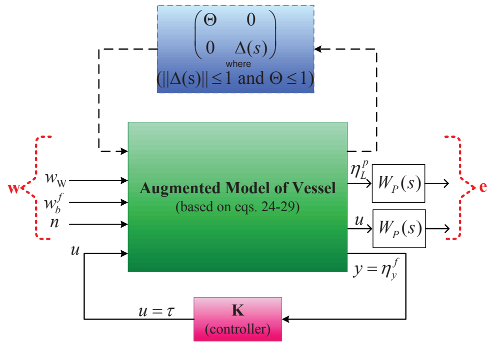


Fig. 4. Standard Feedback Configuration Developed for DP.

(extreme seas or swell with very long wave periods) wave filtering should be turned off, see Sørensen (2011b), and in particular Sørensen et al. (2002) for details on the effect of wave filtering in extreme seas. Based on Sørensen et al. (2002) the state space control plant model for DP in extreme sea can be described by

$$\dot{b} = -T^{-1}b + E_b w_b \tag{31}$$

$$\dot{\eta} = R(\psi)\nu \tag{32}$$

$$M\dot{\nu} + D\nu = \tau + R^T(\psi)b \tag{33}$$

$$\eta_y = \eta + v \tag{34}$$

which is similar to the one in (1)-(7), excluding the WF motion components.

To design a robust DP controller for extreme sea conditions, the methodology explained in the previous section can be used. However, the frequency weighting functions must be changed. We suggest a new frequency weighting function $W_p(s)$ as

$$W_p(s) = A_p W_L(s)$$

where $W_L(s)$ is some low-pass filter and $W_p(s)$ is applied to the total motion of the vessel, i.e. the controller should compensate for both LF and WF motions.

## 6. CONCLUSION

This paper proposed a new strategy for the design of robust DP controllers for marine vessels under different sea conditions using mixed-$\mu$ synthesis. The next part of this work, Hassani et al. (2012a), describes the outcome of the design process and contains details of simulations as well as results of experimental model tests in a water tank.

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