Bottomhole Pressure Estimation and $\mathcal{L}_1$ Adaptive Control in Managed Pressure Drilling System

Zhiyuan Li * **Naira Hovakimyan** ** **Glenn-Ole Kaasa** ***

* *Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign (e-mail: li64@illinois.edu)*
** *Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign (e-mail: nhovakim@illinois.edu)*
*** *Statoil Research Center, Porsgrunn, Norway (e-mail: gkaa@statoil.com)*

Abstract: This paper designs an integrated estimator and $\mathcal{L}_1$ adaptive control scheme to address the two main challenges involved in the Managed Pressure Drilling (MPD) system: first, the bottomhole states are updated at a low rate, which can be viewed as unmeasured and thus need to be estimated in real time; and second, the drilling process is subject to uncertainties including unknown parameters (e.g., frictions, densities), unmodeled actuator dynamics and noise, which require a robust adaptive controller for control of the bottomhole pressure. The estimator provides fast estimation of the bottomhole pressure and flow rate, based on the available measurements from the top-side. The $\mathcal{L}_1$ adaptive controller drives the bottomhole pressure to the desired value following a reference model. We also provide a solution to handle the input delay. The design is based on a recently developed nonlinear drilling model. The results demonstrate that the $\mathcal{L}_1$ adaptive controller has guaranteed performance bounds for both the input and the output signals of the system while using the estimation of the regulated outputs. Simulations that include different operational conditions verify the theoretical findings.

1. INTRODUCTION

During well drilling, a fluid circulation system is used to maintain the pressure profile along the well. The drill fluid (usually called mud) is pumped into the drill string, which is a structure of a series of connected pipes. The fluid then flows down to the drill bit, sprays out through the bit, circulates back up the annulus, and finally exits through a choke valve.

The pressure balance between the well bottom hole and the reservoir is critical to the well drilling system (Stamnes, 2007). It is desired to keep the bottomhole pressure in some safety margin: if the bottomhole pressure is too low (under-balanced), a kick incident could happen, which can lead to an influx oil/gas while drilling; on the other hand, if the pressure is too high (over-balanced), the well could be fractured and the mud will be lost. The safety margin is narrow especially in deep water and some matured well. The managed pressure drilling (MPD) is a technology to control the bottom hole pressure precisely, the advantage of which includes the capability to drill the otherwise undrillable well, reduced non-productive-time (NPT), fluid loss and influx, more productivity of the well, and reduced drilling hazards, etc. The basic principle of MPD is to seal the annulus top and use the chock opening and an additional back-pressure to control the bottom hole pressure and compensate for annular pressure fluctuations. A simplified schematic diagram of an automated MPD system is shown in Fig.1. A description of the standard setup of an automated MPD system can be found e.g. in (Riet et al., 2003).

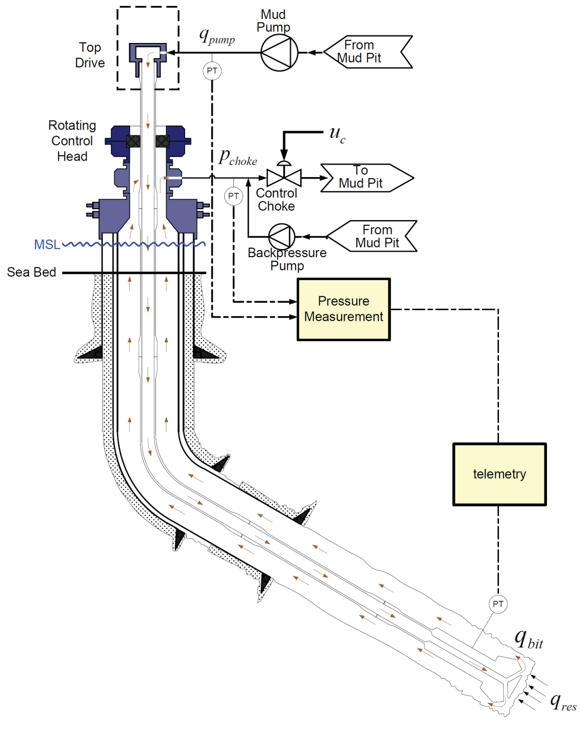

Fig. 1. MPD drilling process

One of the main challenges of MPD control is that the measurements from the bottom hole (flow rate and pressure, etc.), if possible, are transmitted by telemetry and updated at a low rate. For the purpose of control, these bottom hole states need to be estimated in real-time. Another challenge is the uncertainty in the model for the bottomhole, due to uncertainties in the friction, density and mud compressibility parameters, as well as the unmodeled dynamics in the actuator. Moreover, the model parameters are subject to significant changes during the drilling process.

These challenges motivate the design of an integrated estimator and adaptive controller scheme. We first design an estimator based on the idea proposed in (Ma et al., 2010). Then we apply the $\mathcal{L}_1$ adaptive controller to regulate the bottomhole pressure to some desired value specified by an operator. The guaranteed performance bounds and robustness of $\mathcal{L}_1$ adaptive controller make it an ideal candidate for pressure control under uncertain parameters and unmodeled dynamics. The model is taken from (Kaasa, 2007), which is a simplified hydraulic model given by a set of nonlinear ODEs that capture the dominating hydraulics of the MPD system.

The main contribution of our work is that we address both the estimation of unmeasured states and the control in the presence of uncertainty, and show the stability and performance of the whole integrated estimator/controller, while the work by some other authors (Stamnes et al., 2008) consider only estimation of the bottomhole pressure, assuming that it is well regulated by some controller. Compared to our previous work on this problem (Li et al., 2011), this paper proposes a solution to handle an input delay, which is demonstrated by simulations.

The paper is organized as follows. Section 2 introduces the problem formulation and the available measurements. Section 3 designs an estimator for the bottomhole pressure and flow rate based on the top-side measurements. Section 4 designs an $\mathcal{L}_1$ adaptive controller to control the bottomhole pressure in the presence of uncertainties, which use the estimated bottomhole pressure and flow rate as feedback. Section 5 presents extensive simulations to test the performance of the integrated estimator and controller. Section 6 concludes the paper.

We use $\|\cdot\|_{\mathcal{L}_\infty}$ to denote the $\mathcal{L}_\infty$-norm of a signal, and $\|\cdot\|_{\mathcal{L}_1}$ to denote the $\mathcal{L}_1$ norm of a transfer function matrix (Khalil, 2002).

2. PROBLEM FORMULATION

We consider a recently developed simplified nonlinear model from (Kaasa, 2007) and (Kaasa et al., 2011) for the dynamics of the well drilling system. The model has been shown by experiments to have acceptable fidelity level for calculating the non-measured states and for parameter estimation (Stamnes et al., 2008). The model is given by the following set of nonlinear ODEs:

$$\frac{V_d}{\beta_d}\dot{p}_{pump}(t) = q_{pump} - q_{bit}(t)\,, \tag{1a}$$

$$\frac{V_a}{\beta_a}\dot{p}_{choke}(t) = -q_{choke}(t) - \dot{V}_a + q_{bit}(t) + q_{res} + q_{back}\,, \tag{1b}$$

$$M_a\dot{q}_{bit}(t) = p_{bit}(t) - p_{choke}(t) - F_a(q_{bit}(t)+q_{res})^2 - \rho_a g h_{bit}\,, \tag{1c}$$

$$M_d\dot{q}_{bit}(t) = p_{pump}(t) - p_{bit}(t) - F_d q_{bit}^2(t) + \rho_d g h_{bit}\,, \tag{1d}$$

$$q_{choke}(t) = K_c z_c(t)\,, \tag{1e}$$

$$z_c(s) = F(s)u_c(s)\,, \tag{1f}$$

$$p_{pump}(0) = p_{p0}\,,\ q_{pump}(0) = q_{p0}\,,\ q_{bit}(0) = q_{b0}\,, \tag{1g}$$

where p_{pump} and p_{chock} are the pressures at the pump and the chock, respectively; p_{bit} is the bottomhole pressure; q_{pump}, q_{chock}, q_{back}, q_{bit} and q_{res} are the flow rates through the pump, chock, back pipe, drilling bit and from the reservoir, respectively. The control signal is the chock opening command u_c; z_c is the actual chock opening; $F(s)$ is an unknown stable transfer function representing the unmodeled actuator dynamics of the chock valve. Note that (1c) and (1d) are two independent equations of the dynamics of q_{bit}, derived with respect to the drilling string and the annulus, respectively.

The topside pressures p_{pump} and p_{chock}, and flow rates q_{pump}, q_{chock} and q_{back}, are measured continuously and reliably, where q_{pump} and q_{back} are constants. Due to the measurement constraints, $q_{bit}(t)$ and $p_{bit}(t)$ are updated at a low rate, so they are viewed as unmeasured signals and need to be estimated. Other variables are described as follows:

- V_d, β_d: volume and the bulk modulus of the drill string;
- V_a, β_a: volume and the bulk modulus of the annulus;
- ρ_a, ρ_d: density in the annulus and the drill string;
- M_a, M_d: the density per meter of the annulus and the drill string;
- F_a, F_d: friction factor of the drill string and the annulus;
- K_c: valve flow constant;
- h_{bit}: vertical depth of the bit;
- $\dot{V}_a$: rate of change of the annulus volume.

In the above parameters, the parameters related to the drill string, V_d, β_d, M_d, ρ_d, F_d and h_{bit}, are measured reliably. Other parameters, especially those related to the annulus have some uncertainties and are considered as unknown constants with some known conservative bounds.

The objective is to regulate the bottomhole pressure p_{bit} to some desired value r set by an operator and provide estimation of p_{bit} and q_{bit}, based on the measurements from the top-side.

3. BOTTOMHOLE PRESSURE AND FLOW ESTIMATION

To estimate q_{bit} and p_{bit}, we rewrite equations (1a) and (1d) as

$$\frac{V_d}{\beta_d}\dot{p}_{pump}(t) = -q_{bit}(t) + q_{pump}\,,$$

$$M_d\dot{q}_{bit}(t) = -\zeta(t) + p_{pump}(t) + \rho_d g h_{bit}\,,$$

where $\zeta \triangleq p_{bit} + F_d q_{bit}^2$. We design the following estimator based on the fast estimation scheme proposed in (Ma et al., 2010), which consists of 3 components: the state predictor, the estimation update laws, and the low-pass filtering.

- State Predictor:

$$\frac{V_d}{\beta_d}\dot{\hat{\xi}}_1(t) = -k_1(\hat{\xi}_1(t) - p_{pump}) - \hat{q}_{bit}(t) + q_{pump}\,, \tag{2a}$$

$$M_d\dot{\hat{\xi}}_2(t) = -k_2(\hat{\xi}_2(t) - \hat{q}_{bit}(t)) - \hat{\zeta}(t) + p_{pump}(t) + \rho_d g h_{bit}, \tag{2b}$$

  where$k_1, k_2 > 0$. Note that (2a) and (2b) have the same structure as (1a) and (1d), respectively, except that q_{bit} and ζ are replaced by their estimations, $\hat{q}_{bit}$ and $\hat{\zeta}$, respectively.
- Estimation Update Laws:
  The estimations $\hat{q}_{bit}$ and $\hat{\zeta}$ are updated by

$$\dot{\hat{q}}_{bit} = \Gamma_1 \mathrm{Proj}(\hat{q}_{bit}(t), \tilde{\xi}_1(t))\,, \tag{3a}$$

$$\dot{\hat{\zeta}} = \Gamma_2 \mathrm{Proj}(\hat{\zeta}(t), \hat{\xi}_2 - \hat{q}_{bit})\,, \tag{3b}$$

  where Γ_i, $i = 1, 2$ are the updating gains, and $\mathrm{Proj}(\cdot,\cdot)$ is the projection operator that keeps $\hat{q}_{bit}$ and $\hat{\zeta}$ in their prespecified bounds (Pomet and Praly, 1992).
- Low-Pass Filtering:

$$\bar{q}_{bit} = C_1(s)\hat{q}_{bit}(s)\,, \tag{4a}$$
$$\bar{\zeta} = C_2(s)\hat{\zeta}(s)\,, \tag{4b}$$

where $C_i(s) = \frac{c_i}{s+c_i}$, $c_i > 0$, $i = 1, 2$ are low-pass filters.

- The final estimation of q_{bit} and p_{bit} are given by $\bar{q}_{bit}$ and
$$\bar{p}_{bit} \triangleq \bar{\zeta} - F_d\bar{q}_{bit}^2\,, \tag{5}$$
respectively.

3.1 Estimation Performance

It can be shown that if the signals to be estimated, p_{bit} and q_{bit}, are bounded and with bounded derivatives, then the estimation errors $\|\bar{q}_{bit} - q_{bit}\|_{\mathcal{L}_\infty}$ and $\|\bar{p}_{bit} - p_{bit}\|_{\mathcal{L}_\infty}$ can be rendered arbitrarily small by increasing the updating gains Γ_i and the bandwidth of the low-pass filters $C_i(s)$, $i = 1, 2$, upon an exponentially decaying transient phase due to the initialization errors. Refer to (Li et al., 2010; Ma et al., 2010) for detailed proof.

Remark 1. The reason we estimate p_{bit} based on the dynamics (1d) instead of (1c) is that in practice the drill string parameters M_d, ρ_d and F_d are accurately measured, while the annulus parameters M_a, ρ_a and F_a are subject to uncertainties.

4. $\mathcal{L}_1$ ADAPTIVE CONTROLLER DESIGN

Equations (1c) and (1d) provide an algebraic representation of p_{bit}:

$$\begin{aligned} p_{bit} =& \frac{1}{M}(M_d p_{choke} - (F_d M_a - F_a M_d)q_{bit}^2 \\ & (M_d\rho_a + M_a\rho_d)gh_{bit} + M_a p_{pump})\,. \end{aligned} \tag{6}$$

From this we can obtain the dynamics of p_{bit} by taking the time derivative of (6) and plugging in (1a)-(1d), which yields

$$\begin{aligned} \dot{p}_{bit}(t) =& \frac{1}{M}\Big(M_d\dot{p}_{choke}(t) + M_a\dot{p}_{pump} \\ & - 2(F_d M_a - F_a M_d)q_{bit}(t)\dot{q}_{bit}(t)\Big) \\ =& \frac{M_a}{M}\frac{\beta_d}{V_d}\Big(q_{pump} - q_{bit}(t)\Big) + \frac{M_d}{M}\frac{\beta_a}{V_a}\Big(q_{bit}(t) \\ & - \dot{V}_a + q_{res} + q_{back} - q_{choke}(t)\Big) \\ & + \frac{2}{MM_d}(M_d F_a - M_a F_d)q_{bit}(t)(p_{pump}(t) \\ & - p_{bit}(t) - F_d q_{bit}^2 + \rho_d g h_{bit})\,. \end{aligned} \tag{7}$$

Rewriting the resulting dynamics of p_{bit} in a concise form, we have

$$\dot{p}_{bit}(t) = a_m p_{bit}(t) + \phi q_{choke}(t) + \theta p_{bit}(t) + \sigma(t)\,, \tag{8a}$$
$$q_{choke}(s) = K_c F(s)u_c(s)\,, \tag{8b}$$

where $a_m < 0$ is a design parameter that defines the control specification, i.e., the time constant of the desired system behavior, $\phi = -\frac{M_d\beta_a}{MV_a}$, $\theta = \frac{-2(M_dF_a - M_aF_d)}{MM_d} - a_m$, and σ presents all the rest of the terms on the right hand side of (7).

Although most of the parameters are unknown, we always have some conservative knowledge of the physical parameters and variables, summarized in the following assumption.

Assumption 1. In (8), the unknown parameters θ, $\sigma(t)$ are subject to the following conservative bounds: $|\theta| < \Theta$, $|\sigma| < \Delta$, $|\dot{\sigma}(t)| \le d_\sigma$, where Θ, Δ and d_σ are known. There exists $L_F > 0$ verifying $\|\phi K_c F(s)\|_{\mathcal{L}_1} < L_F$ and $0 < \omega_l < \phi K_c F(0) \le \omega_u$.

The design of the $\mathcal{L}_1$ adaptive controller involves a strictly proper stable transfer function $D(s)$ and a gain $k > 0$, satisfying the following $\mathcal{L}_1$-norm condition:

$$\|G(s)\|_{\mathcal{L}_1} L < 1\,, \tag{9}$$

where $G(s) = (1 - C(s))(s\mathbb{I} - a_m)^{-1}$, $L = \theta$ and $C(s) = kF(s)D(s)/(1 + kF(s)D(s))$ is a strictly proper transfer function with DC gain $C(0) = 1$.

The $\mathcal{L}_1$ adaptive controller consists of three components: the state predictor, the adaptive laws, and the control law.

- State Predictor:
$$\begin{aligned} \dot{\hat{p}}_{bit}(t) =& a_m\hat{x}(t) + \hat{\omega}(t)u_c(t) + \hat{\theta}^\top(t)\bar{p}_{bit}(t) + \hat{\sigma}(t)\,, \quad (10) \\ \hat{p}_{bit}(0) =& p_{bit}(0)\,. \end{aligned}$$
- Adaptive Law:
  The parameter estimations $\hat{\omega}$, $\hat{\theta}$ and $\hat{\sigma}$ in the state predictor are updated by the following adaptive laws:
$$\dot{\hat{\omega}}(t) = \Gamma_c\text{Proj}(\hat{\omega}(t), -(\hat{p}_{bit}(t) - \bar{p}_{bit}(t))u_c(t))\,, \tag{11a}$$
$$\dot{\hat{\theta}}(t) = \Gamma_c\text{Proj}(\hat{\theta}(t), -(\hat{p}_{bit}(t) - \bar{p}_{bit}(t))p_{bit}(t))\,, \tag{11b}$$
$$\dot{\hat{\sigma}}(t) = \Gamma_c\text{Proj}(\hat{\sigma}(t), -(\hat{p}_{bit}(t) - \bar{p}_{bit}(t))). \tag{11c}$$
- Control Law:
  The choke opening command u is generated as the output of the following system
$$u_c(s) = -kD(s)(\hat{\eta}(s) - k_g r(s))\,, \tag{12}$$
  where $r(s)$ and $\hat{\eta}(s)$ are the Laplace transforms of $r(t)$ and $\hat{\eta}(t) \triangleq \hat{\omega}(t)u_c(t) + \hat{\theta}^\top(t)\bar{p}_{bit}(t) + \hat{\sigma}(t)$.

The $\mathcal{L}_1$ adaptive controller is defined via (10)–(12), subject to the $\mathcal{L}_1$-norm condition in (9).

Remark 2. When the plant input is subject to time delay τ, i.e., (8b) is replaced by $z_c(s) = F(s)e^{-\tau s}u_c(s)$, we can easily incorporate this delay information in the state-predictor by replacing (10) with

$$\dot{\hat{p}}_{bit}(t) = a_m\hat{x}(t) + \hat{\omega}(t)u_c(t - \tau_0) + \hat{\theta}^\top(t)\bar{p}_{bit}(t) + \hat{\sigma}(t)\,.$$

where τ_0 represents our knowledge of the delay. This highlights the structural benefit of the $\mathcal{L}_1$ adaptive controller to accomodate more realistic systems. The results is demonstrated in Section 5.

4.1 Performance of $\mathcal{L}_1$ Adaptive Controller

The key feature of the $\mathcal{L}_1$ adaptive controller is that both the state and the control signal of the closed-loop system can be rendered arbitrarily close to the corresponding signals of a closed-loop reference system in the sense of $\mathcal{L}_\infty$-norm. For the system in (8), the closed-loop reference system is defined by

$$\begin{aligned} \dot{p}_{ref}(t) =& a_m p_{ref}(t) + \mu_{ref}(t) + \theta^\top p_{ref}(t) \\ & + \sigma_0(t)\,, p_{ref}(0) = p_{bit0} \end{aligned} \tag{13a}$$
$$\mu_{ref}(s) = F(s)u_{ref}(s) \tag{13b}$$
$$u_{ref}(s) = C_u(s)(k_g r(s) - \eta_{ref}(s))\,, \tag{13c}$$

where $p_{ref}(t)$ is the reference system state, $\eta_{ref}(s)$ is the Laplace transform of $\eta_{ref}(t) \triangleq \theta^\top p_{ref}(t) + \sigma_0(t)$ and $C_u(s) = C(s)/F(s)$. The reference system assumes compensation of uncertainties within the bandwidth of the controller, and thus it is used only for analysis purposes. The following theorem from (Hovakimyan and Cao, 2010) states the performance bounds of the $\mathcal{L}_1$ adaptive controller.

Theorem 3. For the system in (8), subject to Assumption 1, if the real value p_{bit} is available for feedback, and the adaptive gain in (10)–(12) is selected according to

$$\Gamma \geq \frac{\theta_m(\rho_u, \rho_{\dot{u}})}{\gamma_0^2}, \tag{14}$$

where $\gamma_0 > 0$ is an arbitrary constant, then we have

$$\|\hat{p}_{bit} - p_{bit}\|_{\mathcal{L}_\infty} \leq \gamma_0, \tag{15a}$$
$$\|p_{ref} - p_{bit}\|_{\mathcal{L}_\infty} \leq \gamma_1, \tag{15b}$$
$$\|u_{ref} - z_c\|_{\mathcal{L}_\infty} \leq \gamma_2, \tag{15c}$$

where $\theta_m = (\omega_u - \omega_l)^2 + 4\Theta^2 + 4\Delta^2 + (\Theta d_\theta + \Delta d_\sigma)$, $\gamma_1 = \|C(s)\|_{\mathcal{L}_1} \gamma_0 / (1 - \|G(s)\|_{\mathcal{L}_1} L) + \beta$, $\gamma_2 = \|C_u(s)\|_{\mathcal{L}_1} L\gamma_1 + \|C_u(s)(s\mathbb{I} - A_m)\|_{\mathcal{L}_1} \gamma_0$, and $\beta > 0$ is arbitrarily small.

4.2 Integration of the Estimator and Adaptive Controller

Note that the analysis above assumes decoupled estimation and controller structure. In the performance of the estimator, it is assumed that p_{bit} and q_{bit} are bounded, an assumption that needs to be ensured by the controller design; in the performance of the controller, it is assumed that the real value of p_{bit} is used for feedback, which, in our case is not available. When the estimator and the controller are integrated, i.e., the estimated value $\bar{p}_{bit}$ is fed to the controller, neither of the assumptions hold any more.

Since the "certainty equivalence" argument does not apply to nonlinear systems, one needs to show the performance based on the whole closed-loop system involving the plant, the estimator and the controller. The analysis of the stability and performance of the integrated system can be done following the same steps as in (Li et al., 2009).

Remark 4. Reference (Li et al., 2009) considers the problem with less known parameters, i.e., the case in which only V_d and β_d are known, and does not consider the unmodeled actuator dynamics. In that case, equations (2b), (3b) and (4b) cannot be implemented to estimate p_{bit}. Thus (Li et al., 2009) applies the standard RLS algorithm to estimate p_{bit} using (6).

5. SIMULATION RESULTS

In this section, we apply the proposed estimation and adaptive control scheme to the nonlinear ODE model (1) under different conditions. In Section 5.1, we feed the real value of p_{bit} to the $\mathcal{L}_1$ adaptive controller, and thus we can see the performances of the decoupled estimator and controller. In Section 5.2, we integrate the estimator and the controller by feeding the estimated value $\bar{p}_{bit}$ to the controller, and test the system for different reference signals and parameter variations.

The parameters of the plant are given by: $\beta_a = \beta_d = 14000$, $V_d = 28.3$, $V_a = 96.1$, $M_a = 1700$, $M_d = 5700$, $F_a = 20800$, $F_d = 165000$, $\rho_a = \rho_d = 1250 \times 10^{-5}$, $h_{bit} = 2000$, $g = 9.8$, $p_0 = 1$, $K_c = 4.63 \times 10^{-3}$, $q_{res} = 0.001$, $\dot{V}_a = 0$, $q_{pump} = 0.01$, $q_{back} = 0.003$. The simulation starts from steady state, with $p_{bit}(0) = 320$ barg, and $q_{bit}(0) = 0.01\text{m}^3/\text{s}$. The unmodeled actuator dynamics are given by $F(s) = \frac{1}{s^2+1.4s+1}$, which is a stable and slightly underdamped system. The states $p_{pump}(t)$ and $p_{choke}(t)$ are measured in real-time.

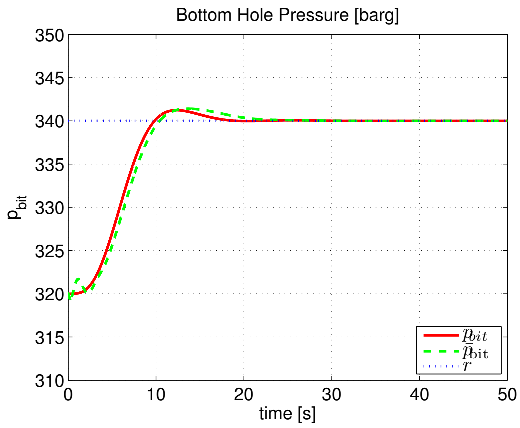

Fig. 2. p_{bit}, $\bar{p}_{bit}$, r for decoupled estimator and controller

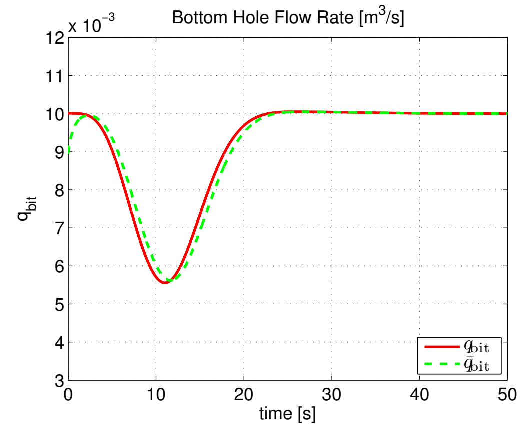

Fig. 3. q_{bit}, $\bar{q}_{bit}$ for decoupled estimator and controller

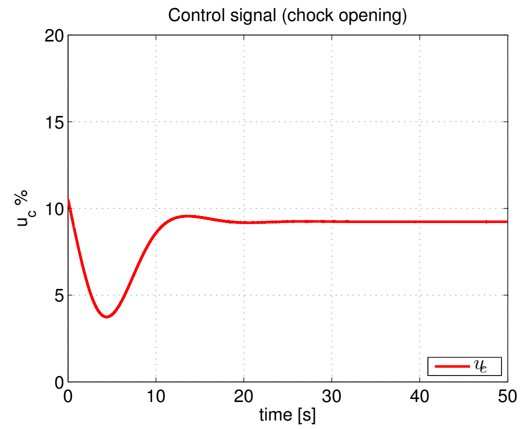

Fig. 4. u_c for decoupled estimator and controller

5.1 Testing the Decoupled Estimator and the $\mathcal{L}_1$ Adaptive Controller

To test the decoupled estimator and the $\mathcal{L}_1$ adaptive controller, we set the reference pressure $r = 340$ barg, and use the real value of p_{bit} in the controller.

Figure 2 shows the time history of p_{bit} and its estimation $\bar{p}_{bit}$ together with the desired value r. Figure 3 shows the estimated value $\bar{q}_{bit}$ compared to the real value q_{bit}. Figure 4 shows the control signal u_c.

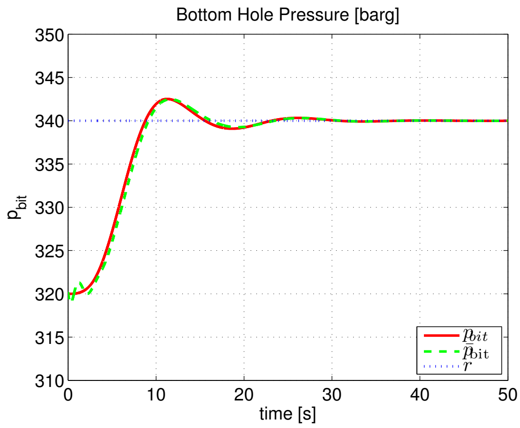

Fig. 5. p_{bit}, $\bar{p}_{bit}$, r for integrated estimator and controller

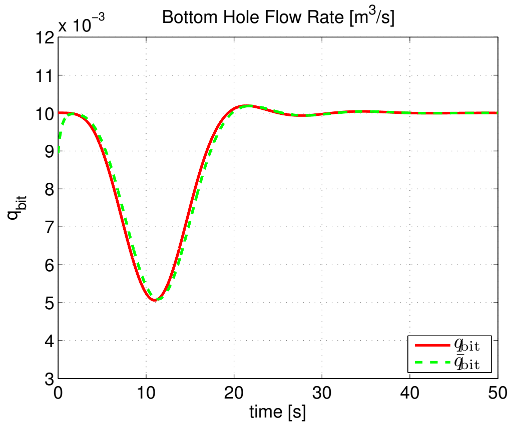

Fig. 6. q_{bit}, $\bar{q}_{bit}$ for integrated estimator and controller

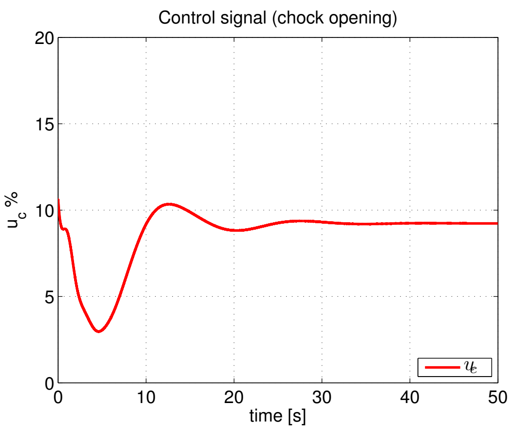

Fig. 7. u_c for integrated estimator and controller

We can see that the $\mathcal{L}_1$ adaptive controller regulates the bottomhole pressure p_{bit} to the desired value in the presence of unknown parameters and unmodeled dynamics. Note that although the signal σ depends on p_{pump}, p_{choke} and some unknown parameters, the $\mathcal{L}_1$ adaptive controller does not try to estimate these parameters individually, but estimate it as a whole by $\hat{\sigma}$. The speed of response can be adjusted by the parameter a_m.

From Figure 4 we see that although the adaptive rate Γ is set to be very large, the control signal does not have high frequency components due to the low-pass filter.

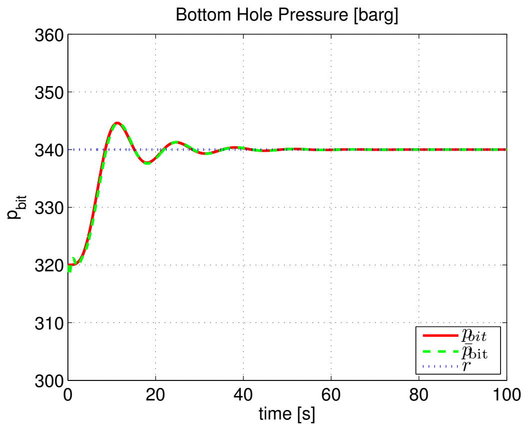

Fig. 8. With input delay and state predictor modification

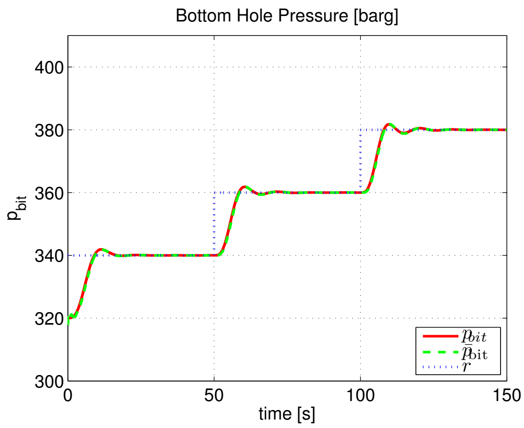

Fig. 9. Scaled response for p_{bit}

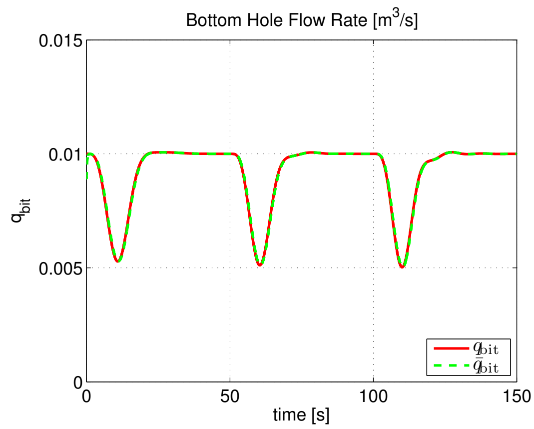

Fig. 10. Scaled response for q_{bit}

The estimated signals $\bar{p}_{bit}$ and $\bar{q}_{bit}$ track the real values, even though p_{bit} is rising very fast. The convergence rates are adjusted by Γ_1 and Γ_2.

5.2 Integrating the Estimator with the $\mathcal{L}_1$ Adaptive Controller

In this section we integrate the estimator and the $\mathcal{L}_1$ adaptive controller by feeding the estimated value $\bar{p}_{bit}$ instead of the real value to the controller.

First we present the simulation results for $r = 340$ barg in Figures 5–7. We can see that the integration only slightly affects the transient performance of the controller [1], due to the initial estimation error of $\bar{p}_{bit}$, the control signal is still smooth and the estimation performance is not affected.

Second, we inject an input delay of 1s, for which the controller (10)–(12) fails to stabilize the system. However, by applying the modification to the state predictor in Remark 2, with $\tau_0 = 0.9$ we regain the closed-loop stability and performance, as shown in Figure 8.

Next we set the reference signal to be a series of steps, from 340 barg to 380 barg, with an increment of 20 barg for every 50 seconds. The resulting signals are shown in Figures 9-10. Note that p_{bit} has scaled response to scaled reference signals, *which is the typical behavior of an LTI system, despite the fact that the plant, the controller and the estimator are all nonlinear.*

Finally, in Figure 11, we test the integrated estimation and control scheme in the case of a sudden drop of q_{pump} from 1000 l/min to 250 l/min, which simulates the extreme case of a power loss during drilling. We command a step reference $r = 340$ barg for the bottomhole pressure, and then inject a sudden drop of q_{pump}; as shown in Figure 11, the integrated estimator and controller drive p_{bit} to the commanded value in 15 seconds and then successfully hold this value when the drop of q_{pump} happens at 40s, with good transient and steady state performance.

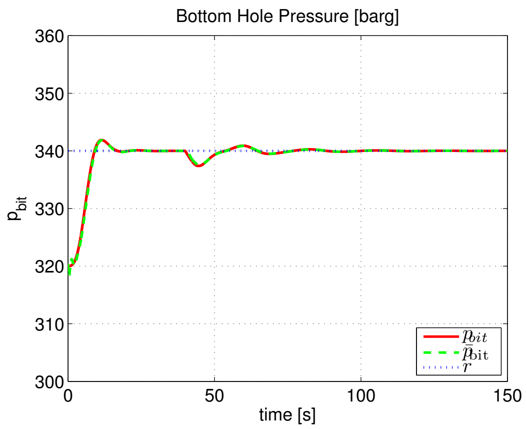

Fig. 11. p_{bit} during large parameter variation

6. CONCLUSION

This paper designs an $\mathcal{L}_1$ adaptive controller integrated with an estimator for an automated MPD system in the presence of unmeasured bottom hole variables, unmodeled dynamics, uncertain system parameters and input time delay. With the estimated parameters used in the feedback path, the $\mathcal{L}_1$ adaptive controller achieves uniform performance bounds for system's input and output signals.

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[1] In order to make the difference obvious, we use a small Γ_1 in the estimator to make the estimation slow.