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$\mathcal{L}_1$ Adaptive Control for Directional Drilling Systems

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Abstract: This paper considers downhole directional drilling systems in the presence of unexpected variations in steering force, input delays, measurement noise and measurement delays, and explores the application of $\mathcal{L}_1$ adaptive controller for the trajectory control problem. The Explicit Force, Finitely Sharp, Zero Mass (EFFSZM) model is used for the steering system, in which spatial delays, modeling inaccuracies, parametric uncertainties, and noise are considered. The $\mathcal{L}_1$ adaptive controller ensures that the centerline of the borehole follows a well path planned according to *a priori* available geologic conditions and local residential information. Path tracking results are demonstrated by simulations.

Keywords: adaptive control, directional drilling, time-delay systems

1. INTRODUCTION

This paper extends the results in Sun et al. (2011), where $\mathcal{L}_1$ adaptive controller was presented for directional drilling systems. Specifically, an $\mathcal{L}_1$ adaptive controller was designed to address the tracking problem of a linear system with internal delays, which was representative of a rotary steerable system. In this paper, in addition to internal spatial delay and steering force saturation, we consider scenarios with actuator delays, measurement noise, measurement delays and unexpected variations in the steering force. The $\mathcal{L}_1$ adaptive controller is shown to have reliable performance in the presence of these uncertainties as well. With sufficiently fast adaptation, the output of the directional drilling system can follow the desired reference path sufficiently closely.

The control of the drilling direction is of great importance to the oil and gas industry. The directional control facilitates drilling into the reservoir where vertical access is difficult or not possible, and allows more wellheads to be grouped together on the surface. With the directional control technique more of the oil and gas can be drained than purely vertical wells. Because of the formations of oil-bearing and gas-bearing layers, wells that intersect a producing formation at an angle or horizontally can often drain more of the oil and gas. There are numerous studies showing that directionally drilled wells have been able to extract 2 to 25 times more oil or gas than vertical wells drilled in the same oil or gas field (Molvar (2003); Aalund and Rappold (1993); Deskins et al. (1995)). From an environmental perspective, it provides the ability to access oil or gas by drilling a well that is miles away from specific property or site, residences or other areas that should not be disturbed. Locating the well sites this way helps to avoid or minimize surface disturbance in sensitive or special areas.

In directional drilling systems, the dynamic vibration response of the drillstring to the steering force is captured in different models in previous works (Millheim et al. (1978); Downton (2007); Dareing and Livesay (1968); Dunsyevsky et al. (1993); Aldred and Sheppard (1992); Downton and Ignova (2011)). Due to the imprecision in modeling and measurement, the controller design that is robust and handles large uncertainties becomes an important factor in the directional drilling technology. The $\mathcal{L}_1$ adaptive controller has guaranteed transient and steady-state performance bounds without introducing persistency of excitation or gain-scheduling in the controller parameters. The architecture of $\mathcal{L}_1$ adaptive controller introduces separation between estimation and control loops, which allows for arbitrary increase of the rate of adaptation (Cao and Hovakimyan (2008); Hovakimyan and Cao (2010)) without hurting robustness. It is guaranteed to have a time-delay margin bounded away from zero (Cao and Hovakimyan (2010); Hovakimyan and Cao (2010)). This paper applies the $\mathcal{L}_1$ adaptive controller to directional drilling systems. The uniform performance bounds guarantee that the centerline of the drilling hole follows the previously designed well path in the presence of internal delays, steering force saturation and variation, measurement noise, etc.

The paper is organized as follows. Section 2 introduces Directional Drilling System, and Section 3 formulates the control problem. Section 4 presents the design of the controller, followed by the performance bounds in Section 5. The closed-loop performance is illustrated by a test case in Section 6. Finally, Section 7 concludes the paper.

Throughout the paper, $\|\cdot\|_1$ denotes the 1-norm of a vector and $\|\cdot\|_2$ denotes the 2-norm of a vector. Notations $\|\xi_t\|_{\mathcal{L}_\infty}$ and $\|\xi\|_{\mathcal{L}_\infty}$ denote, respectively, the truncated (to $[0,t]$) $\mathcal{L}_\infty$-norm and the (untruncated) $\mathcal{L}_\infty$-norm of the time-varying signal $\xi(t)$. For a stable proper transfer matrix $G(s)$, $\|G(s)\|_{\mathcal{L}_1}$ denotes its $\mathcal{L}_1$-norm.

2. DIRECTIONAL DRILLING SYSTEM

The directionally steered drilling system models the relationship between the centerline of the drilling hole and the actuator stimuli. From the geometry and the action of the actuators, the bit force can be computed and approximate models can be derived. In this paper, we use the EFFSZM (Explicit Force, Finitely Sharp, Zero Mass) model given in Downton (2007), where the directional drilling system is assumed to have a force actuator on the lower collar; the bit is assumed to be finitely sharp; and the pipe work is assumed to be infinitely stiff with zero mass.

2.1 EFFSZM Model

The system dynamics of the EFFSZM model is given by

$$\begin{aligned}\frac{dH(m)}{dm} =& \left(\frac{1+C_f}{b} - \frac{C_f}{d}\right) H(m) \\ &+ \frac{1+C_f}{b}(V(m) - H(m-b)) + \frac{C_f}{d} H(m-d) \\ &+ \frac{b-a}{b\, WOB\ K_{anis}} F_{pad}(m)\,, \\ \Psi(m) =& \frac{dH(m)}{dm}\,,\end{aligned} \tag{1}$$

where $C_f = \left(\frac{c-b}{K_{anis}} - \frac{d-b}{d-c}\frac{K_{flex}}{WOB}\right)\frac{d}{b(d-c)}$; m is the distance drilled along the direction of drilling; $H(m)$ is the lateral displacement of the borehole; a is the distance between the force actuator and the bit; b is the distance between the lower stabilizer and the bit; c is the relative position of the flex-joint to the bit; d is the position of the upper stabilizer to the bit; $V(m)$ is the actuator displacement at the lower stabilizer; WOB (Weight on Bit) is the applied drilling load; K_{anis} is the ratio of rates of penetration along and across the bit; K_{flex} is the angular spring rate of the flex joint; $F_{pad}(m)$ is the force actuator output; and $\Psi(m)$ is the angle of borehole-propagation with respect to the m-axis. The structure of the drilling system is shown in Figure 1.

Note that in the system described in (1), the independent variable is the drilled distance m instead of time t. The model can also be written in transfer function form as

$$\begin{aligned}H(s) =& \frac{\frac{1+C_f}{b}V(s) + \frac{b-a}{bW_{OB}K_{anis}}F_{pad}(s)}{s + \frac{C_f}{d}(1-e^{-sd}) - \frac{1+C_f}{b}(1-e^{-sb})}, \\ \Psi(s) =& sH(s).\end{aligned} \tag{2}$$

2.2 Open-loop Performance

For different parameter settings, the response of the aforementioned system to the same input varies. In the drilling process in different rock layers, some parameters determined by the actuator and stabilizer positions and spring rates are known, such as a, b, c, d, K_{flex}, while some

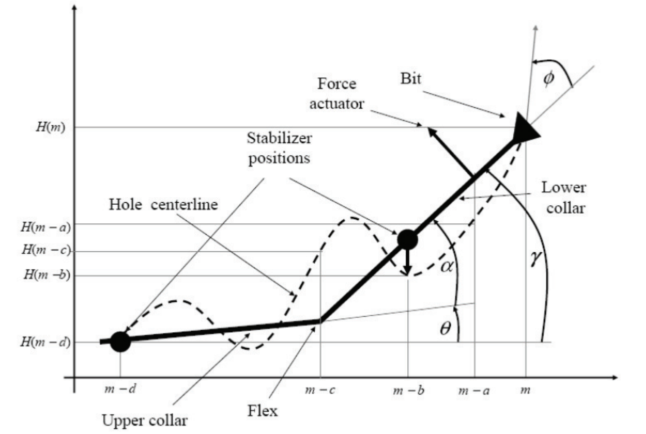
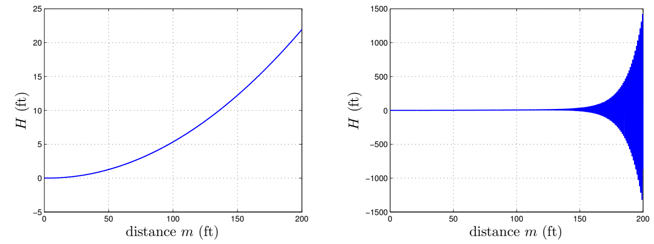


Fig. 1. Flex-hinge directional drilling system (Downton (2007)).



(a) $WOB = 8.9 \times 10^4$, $K_{anis} = 10$ (b) $WOB = 5 \times 10^4, K_{anis} = 1$

Fig. 2. Open-loop performance for different parameter values

others such as WOB and K_{anis} are unknown, and may take different values in a compact set.

Note that $s = 0$ is one of the infinitely many poles of the transfer function (2). The system can be at most marginally stable. Fig. 2 shows the response $H(m)$ of the open-loop system to a constant force $F_{pad}(m) \equiv 8.896 \times 10^3$ N. The applied drilling load, and the rate ratio are $WOB = 8.8964 \times 10^4$ N, $K_{anis} = 10$ and $WOB = 1 \times 10^4$ N, $K_{anis} = 1$ for Figs. 2a and 2b, respectively, and detailed settings for other parameters are introduced in Section 6. In the first case, the system is marginally stable. The response is like the step response of a double integrator. In the second case, the output diverges, and the system is unstable.

3. PROBLEM FORMULATION

From the dynamics in (1), the system can be rewritten in the form

$$\begin{aligned}\dot{x}(m) =& A_m x(m) + b_0\big(\omega u(m) + \theta_0(m)^\top x(m) \\ &+ \theta_1(m)^\top x(m-\tau_1) + \theta_2(m)^\top x(m-\tau_2) + \sigma(m)\big), \\ x(m) =& 0\ \forall m \in [-\tau_2, 0]\,, \\ y(m) =& c_0^\top x(m)\,,\end{aligned} \tag{3}$$

where A_m is a $n \times n$ Hurwitz matrix by choice, $b_0, c_0 \in \mathbb{R}^n$ are known constant vectors, (A_m, b_0) is controllable, ω is an unknown constant with known sign, $\theta_0(m), \theta_1(m), \theta_2(m) \in \mathbb{R}^n$ are unknown vectors, $\tau_1, \tau_2 \in \mathbb{R}^+ (\tau_1 < \tau_2)$ are known internal delays, $\sigma(m) \in \mathbb{R}$ models the input disturbance, $x(m) \in \mathbb{R}^n$ is the system state vector (measured), $u(m) \in \mathbb{R}^l$ is the control input, and $y(m) \in \mathbb{R}$ is the regulated output.

Assumption 1. The unknown parameters $\theta_0(m)$, $\theta_1(m)$, $\theta_2(m)$ belong to given compact convex sets Θ_0, Θ_1, Θ_2, respectively,

$$\theta_0(m) \in \Theta_0,\ \theta_1(m) \in \Theta_1,\ \theta_2(m) \in \Theta_2,\ \forall m \geq 0.$$

Let $\theta_{\max 0} \triangleq \max_{\theta \in \Theta_0} \|\theta\|_1$, $\theta_{\max 1} \triangleq \max_{\theta \in \Theta_1} \|\theta\|_1$, $\theta_{\max 2} \triangleq \max_{\theta \in \Theta_2} \|\theta\|_1$.

The input disturbance $\sigma(m)$ is upper bounded by

$$|\sigma(m)| \leq \Delta\,,\ \forall m \geq 0\,,$$

where $\Delta \in \mathbb{R}^+$ is a known conservative bound.

Assumption 2. Let $\theta_0(m), \theta_1(m), \theta_2(m)$ and $\sigma(m)$ be continuously differentiable with uniformly bounded derivatives

$$\|\dot{\theta}_0(m)\| \leq d_{\theta 0}\,,\ \|\dot{\theta}_1(m)\| \leq d_{\theta 1}\,,$$
$$\|\dot{\theta}_2(m)\| \leq d_{\theta 2}\,,\ |\dot{\sigma}(m)| \leq d_{\sigma}\,.$$

Assumption 3. Let $\omega \in \Omega_0 = [\omega_l, \omega_u]$, where $0 < \omega_l < \omega_u$ are given lower and upper bounds on ω.

The objective is to design a feedback controller that ensures that $H(m)$ follows a pre-determined wellhole curve. Note that as introduced in Section 2, the EFFSZM model uses the drilled distance m instead of time t as the independent variable. The spacial curvature response of the directional drilling system is characterized by the lateral displacement $H(m)$. The wellhole curve is given by a reference signal $r(m)$ describing the commanded displacement at the drilling distance m. This reference signal serves as input to a stable reference system which defines the desired curvature response. The $\mathcal{L}_1$ adaptive controller presented in the following section compensates for the uncertainties and disturbances in the system and ensures that the system output $H(m)$ follows the response of the desired reference system to the given signal $r(m)$.

4. $\mathcal{L}_1$ ADAPTIVE CONTROLLER

In this section we present the $\mathcal{L}_1$ adaptive controller for the system in (3). The $\mathcal{L}_1$ adaptive controller consists of a state predictor, an estimation law and a control law.

We consider the following state predictor

$$\begin{aligned}\dot{\hat{x}}(m) =& A_m\hat{x}(m) + b_0\big(\hat{\omega}(m)u(m) + \hat{\theta}_0(m)^\top x(m)\\ &+ \hat{\theta}_1(m)^\top x(m-\tau_1) + \hat{\theta}_2(m)^\top x(m-\tau_2) + \hat{\sigma}(m)\big)\,,\\ &\hat{x}(m) = 0\ \forall m \in [-\tau_2, 0]\,, \qquad (4)\\ \hat{y}(m) =& c_0^\top \hat{x}(m)\,,\end{aligned}$$

where $\hat{x}(m) \in \mathbb{R}^n$, $\hat{y}(m) \in \mathbb{R}$ are the state and the output of the state predictor, $\hat{\omega} \in \mathbb{R}$, $\hat{\theta}_0(m), \hat{\theta}_1(m), \hat{\theta}_2(m) \in \mathbb{R}^n$, $\hat{\sigma}(m) \in \mathbb{R}$ are estimates of the unknown parameters ω, $\theta_0(m)$, $\theta_1(m)$, $\theta_2(m)$, and $\sigma(m)$, respectively. The projection-type adaptive laws for the estimates are given by

$$\begin{aligned}\dot{\hat{\theta}}_0(m) =& \Gamma\text{Proj}(\hat{\theta}_0(m), -\tilde{x}^\top(m)Pb_0x(m)),\ \hat{\theta}_0(0) = \hat{\theta}_{00},\\ \dot{\hat{\theta}}_1(m) =& \Gamma\text{Proj}(\hat{\theta}_1(m), -\tilde{x}^\top(m)Pb_0x(m-\tau_1)),\\ \dot{\hat{\theta}}_2(m) =& \Gamma\text{Proj}(\hat{\theta}_2(m), -\tilde{x}^\top(m)Pb_0x(m-\tau_2)),\\ &\hat{\theta}_1(0) = \hat{\theta}_{10},\ \hat{\theta}_2(0) = \hat{\theta}_{20},\\ \dot{\hat{\sigma}}(m) =& \Gamma\text{Proj}(\hat{\sigma}(m), -\tilde{x}^\top(m)Pb_0),\ \hat{\sigma}(0) = \hat{\sigma}_0,\\ \dot{\hat{\omega}}(m) =& \Gamma\text{Proj}(\hat{\omega}(m), -\tilde{x}^\top(m)Pb_0u(m)),\ \hat{\omega}(0) = \hat{\omega}_0,\end{aligned} \quad (5)$$

where $\tilde{x}(m) \triangleq \hat{x}(m) - x(m)$, $\Gamma > 0$ is the adaptation rate, $P = P^\top > 0$ solves the algebraic Lyapunov equation $A_m^\top P + PA_m = -Q$ for some symmetric $Q > 0$, and $\text{Proj}(\cdot,\cdot)$ denotes the projection operator (Pomet and Praly (1992)). In the implementation of the projection operator, we use the compact sets Ω, Θ_0, Θ_1, Θ_2, and $[-\Delta, \Delta]$.

The control signal is defined by

$$u(s) = -kD(s)\left(\hat{\eta}(s) - k_g r(s)\right)\,, \qquad (6)$$

where $k_g \triangleq -\frac{1}{c_0^\top A_m^{-1} b_0}$, $r(s)$ and $\hat{\eta}(s)$ are the Laplace transforms of $r(m)$ and $\hat{\eta}(m) \triangleq \hat{\omega}(m)u(m) + \hat{\theta}_0^\top(m)x(m) + \hat{\theta}_1^\top(m)x(m-\tau_1) + \hat{\theta}_2^\top(m)x(m-\tau_2) + \hat{\sigma}(m)$, $k > 0$ is the feedback gain, and $D(s)$ is a strictly proper transfer function leading to a strictly proper stable

$$C(s) \triangleq \frac{\omega k D(s)}{1 + \omega k D(s)} \qquad (7)$$

with DC gain $C(0) = 1$. One simple choice is $D(s) = \frac{1}{s}$, which yields a first order strictly proper $C(s)$ of the following form

$$C(s) = \frac{\omega k}{s + \omega k}.$$

The $\mathcal{L}_1$ adaptive controller consists of (4), (5) and (6), subject to the following $\mathcal{L}_1$ norm condition

$$\|G(s)\|_{\mathcal{L}_1}(\theta_{\max 0} + \theta_{\max 1} + \theta_{\max 2}) < 1\,, \qquad (8)$$

where

$$H(s) \triangleq (s\mathbb{I} - A_m)^{-1}b_0\,, \quad G(s) \triangleq H(s)(C(s) - 1)\,. \qquad (9)$$

5. ANALYSIS OF $\mathcal{L}_1$ ADAPTIVE CONTROLLER

5.1 Stability of The Reference System

Consider the reference system

$$\begin{aligned}\dot{x}_{\text{ref}}(m) =& A_m x_{\text{ref}}(m) + b_0\big(\omega u_{\text{ref}}(m) + \theta_0(m)^\top x_{\text{ref}}(m)\\ &+ \theta_1(m)^\top x_{\text{ref}}(m-\tau_1) + \theta_2(m)^\top x_{\text{ref}}(m-\tau_2)\\ &+ \sigma(m)\big)\,, \quad x_{\text{ref}}(m) = 0\ \forall m \in [-\tau_2, 0]\,,\\ y_{\text{ref}}(m) =& c_0^\top x_{\text{ref}}(m)\,,\end{aligned} \qquad (10)$$

and the following reference control

$$u_{\text{ref}}(s) = \frac{C(s)}{\omega}\big(-\eta_{\text{ref}}(s) + k_g r(s)\big)\,, \qquad (11)$$

where $\eta_{\text{ref}}(s)$ is the Laplace transform of $\eta_{\text{ref}}(m) \triangleq \theta_0^\top(m)x_{\text{ref}}(m) + \theta_1^\top(m)x_{\text{ref}}(m-\tau_1) + \theta_2^\top(m)x_{\text{ref}}(m-\tau_2) + \sigma(m)$.

Lemma 1. (Sun et al. (2011)). If the condition in (8) holds, then the reference system in (10) and (11) is BIBO stable with respect to $r(m)$.

5.2 Prediction Error

As defined in (5), $\tilde{x}(t) = \hat{x}(t) - x(t)$ is the error between the state of the system and the state of the predictor. From (3) and (4), we have the prediction error dynamics

$$\begin{aligned}\dot{\tilde{x}}(m) =& A_m\tilde{x}(m) + b_0\big(\tilde{\omega}(m)u(m) + \tilde{\theta}_0(m)^\top x(m) \\ &+ \tilde{\theta}_1(m)^\top x(m-\tau_1) + \tilde{\theta}_2(m)^\top x(m-\tau_2) + \tilde{\sigma}(m)\big)\,, \\ &\tilde{x}(m) = 0 \;\forall m \in [-\tau_2, 0]\,, \end{aligned} \quad (12)$$

where $\tilde{\theta}_0(m) \triangleq \hat{\theta}_0(m) - \theta_0(m)$, $\tilde{\theta}_1(m) \triangleq \hat{\theta}_1(m) - \theta_1(m)$, $\tilde{\theta}_2(m) \triangleq \hat{\theta}_2(m) - \theta_2(m)$, $\tilde{\sigma}(m) \triangleq \hat{\sigma}(m) - \sigma(m)$, and $\tilde{\omega}(m) \triangleq \hat{\omega}(m) - \omega$. Let

$$\begin{aligned}\tilde{\eta}(m) \triangleq& \tilde{\omega}(m)u(m) + \tilde{\theta}_0(m)^\top x(m) + \tilde{\theta}_1(m)^\top x(m-\tau_1) \\ &+ \tilde{\theta}_2(m)^\top x(m-\tau_2) + \tilde{\sigma}(m)\,. \end{aligned} \quad (13)$$

Then the prediction error dynamics in (12) can be written as

$$\tilde{x}(s) = H(s)\tilde{\eta}(s)\,. \quad (14)$$

Lemma 2. (Hovakimyan and Cao (2010)). For the system in (3) and the controller defined by (6), we have the following bound

$$\|\tilde{x}\|_{\mathcal{L}_\infty} \leq \sqrt{\frac{\theta_m}{\lambda_{\min}(P)\Gamma}}\,, \quad (15)$$

where

$$\begin{aligned}\theta_m \triangleq& 4\sum_{i=0}^{2} \max_{\theta_i \in \Theta_i} \|\theta_i\|_2^2 + 4\Delta^2 + (\omega_u - \omega_l)^2 \\ &+ 4\frac{\lambda_{\max}(P)}{\lambda_{\min}(Q)}\Big(\sum_{i=0}^{2} d_{\theta_i} \max_{\theta_i \in \Theta_i} \|\theta_i\|_2 + d_\sigma\Delta\Big)\,,\end{aligned}$$

and $\lambda_{\min}(\cdot)$ is the smallest eigenvalue of a matrix.

The proof is similar to the one in Hovakimyan and Cao (2010) and is thus omitted.

5.3 Performance Bounds

Theorem 3. (Sun et al. (2011)). Consider the system in (3) and the controller in (4), (5) and (6). If the $\mathcal{L}_1$-norm condition in (8) holds, then the errors are upper bounded by

$$\|x - x_{\text{ref}}\|_{\mathcal{L}_\infty} \leq \gamma_x\,,\; \|u - u_{\text{ref}}\|_{\mathcal{L}_\infty} \leq \gamma_u\,, \quad (16)$$

where γ_x and γ_u are given by

$$\begin{aligned}\gamma_x \triangleq& \frac{\|C(s)\|_{\mathcal{L}_1}}{1 - \|G(s)\|_{\mathcal{L}_1}\big(\theta_{\max 0} + \theta_{\max 1} + \theta_{\max 2}\big)}\sqrt{\frac{\theta_m}{\lambda_{\min}(P)\Gamma}}, \\ \gamma_u \triangleq& \left\|\frac{C(s)}{\omega}\right\|_{\mathcal{L}_1}\big(\theta_{\max 0} + \theta_{\max 1} + \theta_{\max 2}\big)\gamma_x \\ &+ \left\|\frac{C(s)}{\omega c_o^\top H(s)} c_o^\top\right\|_{\mathcal{L}_1}\sqrt{\frac{\theta_m}{\lambda_{\min}(P)\Gamma}}\,.\end{aligned}$$

6. TEST CASE

This section shows the simulation results of a test case of directional drilling systems. The test case is representative of controlling the path at which a directional steering system drills with respect to the hole propagation direction. As introduced in Section 2, the EFFSZM model is used for the directional drilling system. In the system, the steering control commands produce the actuator force $F_{pad}(m)$ that pushes against the borehole wall. The magnitude of $F_{pad}(m)$ is limited, which cannot exceed 8.896×10^3N in either the positive or the negative direction. The unexpected variations in $F_{pad}(m)$ and collisions of the tools with the borehole (which can happen for severe up or down displacement of the hole) are captured by a sinusoidal disturbance signal.

The geometrical and structural parameters used in the EFFSZM model are fixed. The values are given by

$$\begin{aligned}&a = 0.305\text{m},\; b = 0.953\text{m},\; c = 1.407\text{m},\; d = 2\text{m}, \\ &K_{flex} = 8.577 \times 10^5\text{N} \cdot \text{m/rad}.\end{aligned}$$

The uncertain parameters WOB and K_{anis} have the following range of variation $WOB \in [10^4, 1.6 \times 10^5]$N, $K_{anis} \in [1, 100]$.

First we show the scenario where the lateral displacement $H(m)$ can be measured. The desired smooth centerline of the borehole is given in the dashed line in Figure 3. The control objective is to drive $H(m)$ to follow the desired borehole center path predetermined by geologists and engineers.

We use the $\mathcal{L}_1$ adaptive controller with the following parameters

$$\begin{aligned}&A_m = -0.5\,,\; \Gamma = 10^8\,,\; k = 10^4\,, \\ &C_f \in [-252.8380, -11.1425]\,,\; \Theta_0 = [-49.7445, -1.8306]\,, \\ &\Theta_1 = [3.0249, 75.1083]\,,\; \Theta_2 = [-25.2838, -1.1142]\,, \\ &\Omega \in [5.6815 \times 10^{-8}, 9.0904 \times 10^{-5}]\,.\end{aligned}$$

Figure 3 demonstrates the performance of the closed-loop system. The closeness of the dashed line and the solid line shows that the $\mathcal{L}_1$ adaptive controller drives the system output close to the desired well path in the presence of uncertain parameters, internal delays, and control saturation in the system.

Next we show that the $\mathcal{L}_1$ adaptive controller is robust to delays and disturbances. Figure 4 shows the case with unexpected variations of the actuator force $F_{pad}(m)$ characterized by a sinusoidal signal with an amplitude of $5\times10^2 N$ (about 20% of the average steering force value); Figure 5 presents the control and the output in the presence of an input delay of 3 ft; Figure 6 plots the control and the output in the presence of a measurement delay of 2 ft; Figure 7 has the measurement noise which is white Gaussian noise with a variance of $6 \times 10^4\text{rad}^2$ (about 2% of the average value of the designed lateral displacement $H_{\text{des}}(m)$).

We then simulate the case when the lateral displacement $H(m)$ cannot be measured and only $\Psi(m)$ (the angle of borehole-propagation with respect to the m-axis) is available. The model can be transformed to a suitable form by setting $x(m) \triangleq \Psi(m)$, $u(m) \triangleq \dot{F}_{pad}(m)$. Now the control objective is to servo $\Psi(m)$ to a user defined angle (assume that the system starts from 0 degrees). We assume that the driller wants to change inclination in 1 degree steps of random sign every 90 ft drilled.

We show the angle of drilling direction $\Psi(m)$ and the actuator force $F_{pad}(m)$ for three different sets of parameters. The parameters are $K_{anis} = 5$ and $W_{OB} = 1.5 \times$

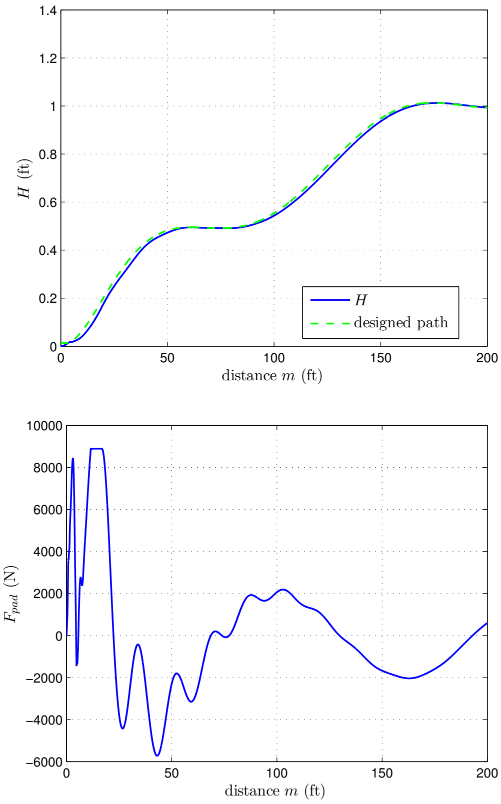


Fig. 3. Closed-loop performance with $K_{anis} = 10$ and $W_{OB} = 8.8964 \times 10^4$

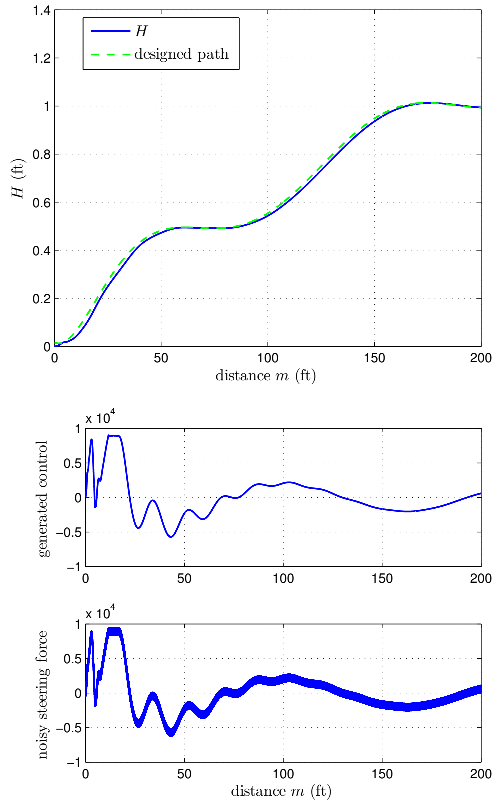


Fig. 4. The lateral displacement H and steering force F_{pad} in the presence of unexpected variations

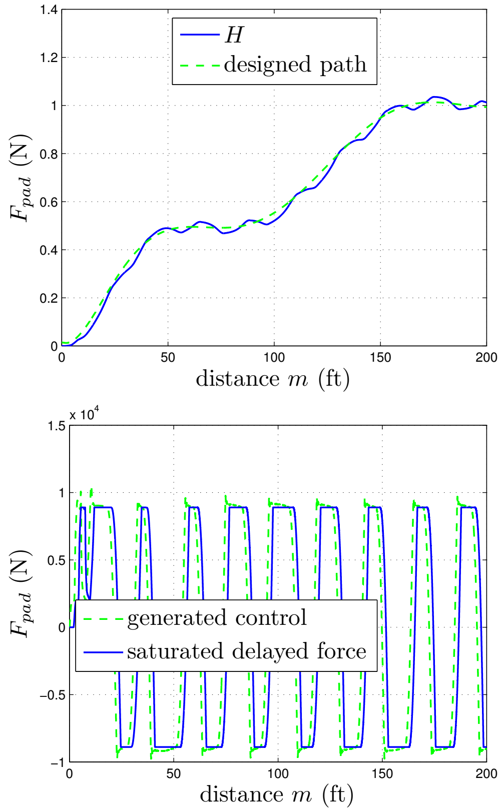


Fig. 5. The lateral displacement H and steering force F_{pad} in the presence of input delay

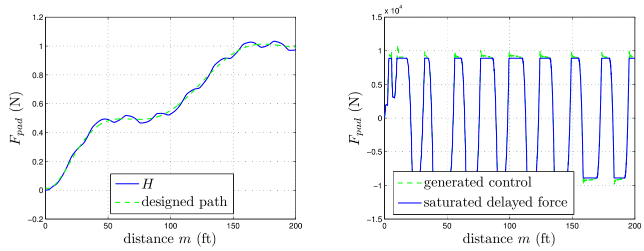


Fig. 6. The lateral displacement H and steering force F_{pad} in the presence of measurement delay

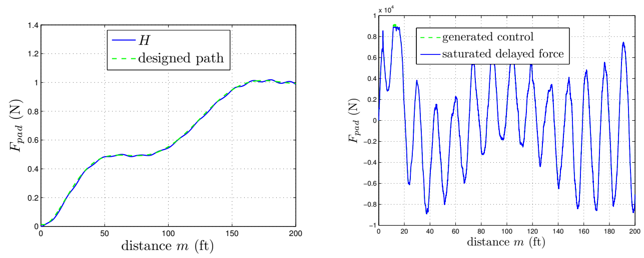


Fig. 7. The lateral displacement H and steering force F_{pad} in the presence of measurement noise

10^4; $K_{anis} = 1$ and $W_{OB} = 10^4$; and $K_{anis} = 5$ and $W_{OB} = 8.8964 \times 10^4$ in Figures 8, 9, and 10, respectively. The inclination angle Ψ and the control input F_{pad} are presented. In all cases, as shown in Figs 8, 9, and 10, the closed-loop system responds to a step command uniformly.

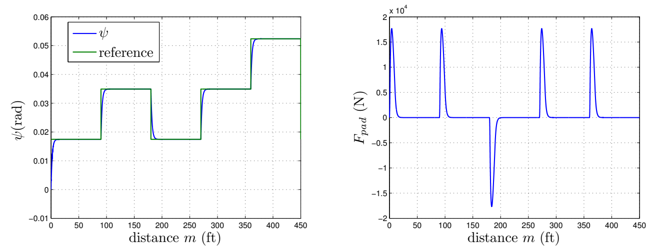


Fig. 8. Closed-loop system with $K_{anis} = 5$ and $W_{OB} = 1.5 \times 10^4$

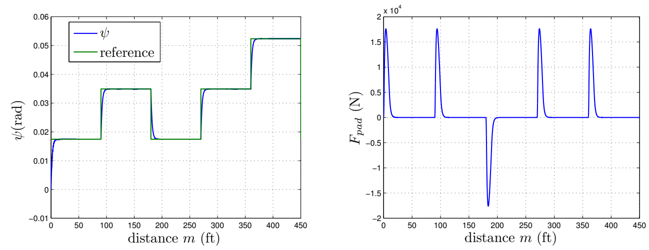


Fig. 9. Closed-loop system with $K_{anis} = 1$ and $W_{OB} = 1 \times 10^4$

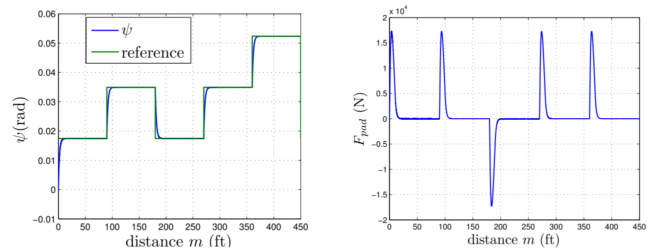


Fig. 10. Closed-loop system with $K_{anis} = 5$ and $W_{OB} = 8.8964 \times 10^4$

Further, we present results for the case where the two uncertain parameters K_{anis} and W_{OB} are "distance"-varying, representing the condition changes when the bit penetrates different rock layers, as in Figure 11. Note that in all cases the design of the adaptive controller is fixed (there is no retuning), and the controller adjusts to the new parameter settings and ensures that the closed-loop system maintains the desired output response.

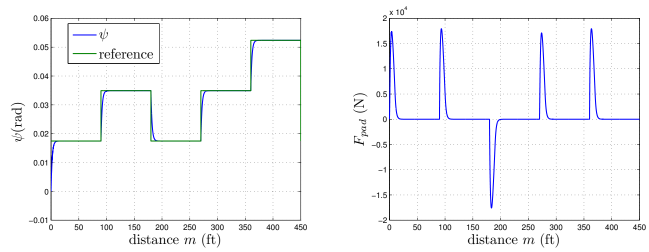


Fig. 11. Closed-loop system with K_{anis} varies every 80 ft between 1 and 100 and W_{OB} varies every 50 ft between 10^4 and 1.6×10^5 N.

7. CONCLUSION

In this paper, we presented the $\mathcal{L}_1$ adaptive controller for a class of uncertain systems with disturbances and internal delays, and studied the application of it to directional drilling systems described by the EFFSZM model. Future work will include applications to other models of drilling systems for different specifications and control of distributed drilling systems where multiple boreholes are drilled from one site.

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