Problems of Identification of Hydrodynamic Models in Reservoir Engineering ⋆

Atlas V. Akhmetzyanov * **Anton M. Salnikov** **

* *Institute of Control Sciences, Russian Academy of Sciences Moscow,Russia (e-mail: awa@ipu.ru).*
** *Institute of Control Sciences, Russian Academy of Sciences Moscow,Russia (e-mail: salnikov@ipu.ru).*

Abstract: The need for solving the problem of identification of basic parameters (permeability coefficients, initial and boundary conditions) for hydrodynamic models of reservoirs on retrospective data arises from incompleteness and observational errors in source and current information about controlled object at all stages of reservoir engineering. Iterative methods for simultaneous solving of direct and inverse problems for original model equations on retrospective data are proposed to solve the problems of identification and adaptation of initial and boundary conditions and filtration parameters. Parallel computing technologies with optimal hierarchical (multilevel) embedding of algorithms into the architecture of supercomputers are offered for creation an integrated model of complex technological system.

Keywords: Identification, Adaptation, Integration, Hydrodynamic Models, Parallel Computing, Reservoir Engineering

1. INTRODUCTION

One of the main problem in the process of development and creation of hydrodynamic models for control and forecasting of reservoir engineering is incompleteness and observational errors of initial and current information about large-scale controlled object with distributed parameters. Additional uncertainty arises in the process of oil and gas recovery, since basic parameters of filtration in porous media of reservoirs (e.g. absolute and relative permeability) vary continuously in space and time.

Full and partial uncertainty of source or initial data, as well as current information coming into the process of reservoir engineering, require the solution of basic problem of identification of filtration parameters, initial and boundary conditions of the model of controlled object.

Three-dimensional simulation of large-scale fields as controlled objects always leads to the set of large-scale (10^9 or more) approximating difference equations. Therefore, the use of basic (sequential) computational methods and algorithms for such sets is impossible due to the limited capacity of computer systems. The required set of parallel algorithms for high-performance multiprocessor systems is discussed in the next section.

It is known that filtration characteristics (coefficients of absolute and relative permeability) of reservoir vary (get monotonically worse) over time and space, making it impossible to forecast parameters of their development. Hence monitoring of the state of reservoir in time and space requires adaptation of hydrodynamic models with allowance for filter parameters, frontal or continuous character of filtration processes at all stages of reservoir engineering. Methods and algorithms for identification of such adaptation of hydrodynamic models are discussed in Section 3.

2. MULTILEVEL PARALLEL METHODS OF COMPUTATIONAL SIMULATION

2.1 Hierarchical principles and parallel technologies of simulation

Characteristic features of porous media of oil and gas reservoirs are large volume, spatial extent and heterogeneity, compressibility of fluids and porous media itself. Therefore there is a need for the development and creation of custom software systems using hierarchical multigrid variants of decomposition with splitting on physical processes, spatial and temporal coordinates. See Akhmetzyanov (2008a, b).

In this approach the reservoir is initially divided into relatively homogeneous geological bodies. Then each geological body is divided into two disjoint subsets of macroelements (blocks), i.e. each geological body should be decomposed. It is assumed that each block can contain no more than one production or injection well. Therefore the main elements of computational models of fluid filtration in porous media become macroelements and wells. See Akhmetzyanov et al. (2008).

Multilevel parallel computing technologies corresponding to the following hierarchy levels are used to achieve the greatest versatility of simulation methods and cost-effectiveness (by the criterion of minimum computational volume and time) of algorithms.

⋆ Supported by the Program of Russian Foundation for Basic Research 11-08-01111-a.

0. Upper (zero) level corresponds to controlled object in general. Grid operators at this level determine the balance of flows between neighboring geological bodies calculated from the results of next level of hierarchy.

1. First level corresponds to the partition of reservoir into geological bodies $\Omega_i, i = \overline{1,m}$ that homogeneous in permeability. The numbering of geological bodies should be chosen so that $D_1 = \{\Omega_1, \Omega_3, \cdots\}$ and $D_2 = \{\Omega_2, \Omega_4, \cdots\}$ represent two subsets of elements that have only common vertices, which allow the organization of parallel solutions with decomposition methods for reservoir with splitting by spatial coordinates. To do this, at each iteration by half a time we independently and in parallel resolve subtasks with corresponding subsets D_1 and D_2. Grid operator equations on this level of hierarchy are formed according results of parallel computing on the next second level.

If some of geological bodies $\Omega_i, i = \overline{1,m}$ partially overlap along the borders, the construction of subsets $D_1, D_2, \cdots$ becomes somewhat complicated and the main iterations are divided into the larger number of fractional steps.

2. Second level of the hierarchy corresponds to the similar decomposition of each homogeneous geological body $\Omega_i, i = \overline{1,m}$ into disjoint subsets of macroelements $D_1^i = \{\Omega_1^i, \Omega_3^i, \cdots\}$ and $D_2^i = \{\Omega_2^i, \Omega_4^i, \cdots\}$. Parallel computations for each iteration by time at this level of hierarchy are also produced in two half steps for all macroelements pertaining to subsets D_1^i and $D_2^i, i = \overline{1,m}$.

3. Third level is the main one, which corresponds to the independent subtasks for each Ω_j^i, i.e. macroelements containing the face of production or injection well. In general, parallel solution of lower-level subtasks is performed using multigrid variants of finite-difference, finite-volume (balancing) or finite-element (variational) methods combined with splitting by physical processes and spatial coordinates to ensure efficiency, required accuracy and fast convergence.

The proposed parallel technologies are naturally and optimally incorporated into the architecture of multiprocessor computing systems (e.g. clusters) using MPI, OpenMP, OpenCL etc.

2.2 Integration of hydrodynamic models of reservoir-well-oil and gas collecting system as a whole

Multilevel problem of matching the gas dynamic and hydrodynamic processes in reservoir engineering becomes urgent: filtration in formations, lift strings, ground oil and gas collecting systems of pipelines and oil and gas treatment facilities. See Akhmetzyanov et al. (2012). This means that in the process of mathematical simulation of such interconnected system as a whole, except for the bottom-hole pressures for calculation of equations of filtering in formation, it is necessary to specify buffer pressure at the well mouths and distribution of flows in collecting networks and oil and gas treatment facilities. In general, the physical processes at these levels are determined by the laws of conservation of mass, momentum and energy, with taking into account the compressibility of fluids and porous media, and describes by the system of nonlinear parabolic equations grouped together through boundary conditions.

For the correct statement of the problem of gas flows distribution (pressure and rate) in these subsystems it is required to consistently (balanced) choose the technological parameters, initial and boundary conditions, as well as control actions. The system of nonlinear equations of integrated model is also solved using discretization (grid approximation) by time and space. Here space discretization, depending on heterogeneity of oil and gas reservoirs, should be carried out according to the structure of partition the reservoir on geological bodies.

3. PROBLEMS OF IDENTIFICATION

Traditional methods of statistical analysis of historical data are ineffective when solving the problem of identification and adaptation of hydrodynamic models of oil and gas fields. The proposed methods are based on the methods of the theory of inverse problems for quasilinear equations of mathematical physics. In particular, they based on coefficient, boundary and evolution inverse problems of optimal control, when statement and solution of direct problems are impossible due to the lack of sufficiently precise information about the values of nonlinear functions in the coefficients and right sides of operator equations, in boundary and initial conditions on internal and external borders.

We propose iterative methods for solving the inverse (usually incorrect) problems of field development control, which are based on (as for the direct usually well-posed problems) various options for constructing the approximating sequences for approximating solutions. In other words, we associate ill-posed inverse problems with a sequence of direct problems, consistent with error source (input) data. At the same time the dimension of approximating space becomes regularization parameter.

Iterative methods of this type are commonly referred to as *iterative regularization methods*. They provide the ability to build effective methods for a wide class of inverse problems of optimal oil and gas development control with perturbed retrospective input.

The choice of this regularization caused by several reasons. Typically, the optimal choice of regularization parameter is performed for given values of error of right sides, boundary and initial conditions, and given a priori information about the decision. See Tikhonov et al. (1977).

When solving applied problems such a priori information is usually partially or even totally absent. Therefore, to solve the problems of identification and adaptation of hydrodynamic models of oil and gas fields it is preferable to use iterative regularization methods, the use of which do no calls for an assessment of regularization parameter.

Computational technologies for different variants of the statement of inverse problems of considered type are focused on application of the theory of adjoint equations or disturbances after (local or not local) linearization of original system of operator equations of the model, when the Lagrange identity is true for direct and adjoint operators.

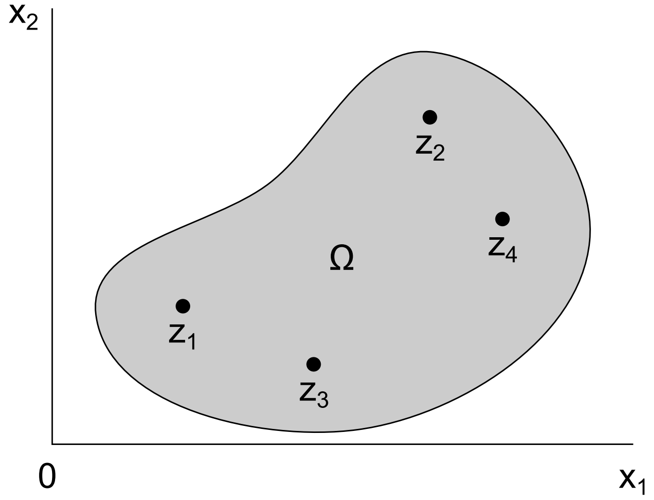

Fig. 1. Two-dimensional model

3.1 Restoring time-dependent fields (initial conditions) on retrospective data

For simplicity the inverse problem is formulated for two-dimensional model (Fig. 1), i.e. for time-dependent parabolic equation

$$\partial p/\partial t - \sum_{i=1}^{i=2} \partial/\partial x_i (k(x)\partial p/\partial x_i) = 0, x \in \Omega, \quad (1)$$

with unknown initial conditions $p(x, 0)$ and given retrospective data (measurements) for all wells $p(z_m, t) \approx \varphi(t), 0 < t < T, m = \overline{1, M}$, where p, z_m, T, M — pressure, vectors of well coordinates with number $m = \overline{1, M}$, planning period and number of wells respectively.

The solution to this problem, according to the method of regularization by A.N. Tikhonov, is formulated as optimal control problem with control input $u(x) \in H = \{u(x)|u(x) \in L_2(\Omega), k(x)(\partial u/\partial n) = 0, \forall x \in \partial\Omega\}$ and initial condition $p(x, 0; u) = u(x), \forall x \in \Omega$, i.e.:

$$J_\alpha(w) = \\ = \min_{\nu \in H} \left\{ \sum_{m=1}^{m=M} \int_0^T (p(z_m, t; u) - \varphi_m(t))^2 \partial t + \alpha ||u||^2 \right\}, \quad (2)$$

where $\alpha > 0$ — regularization parameter. It is assumed that the observation points (wells) in finite-element approximation coincide with some internal nodes Ω, and approximate solution of inverse problem $p(x, t) = p(x, t; w)$ is determined by iterative method for solving the Euler equation $B\psi_0 + \tau\alpha u = 0$, i.e. according to necessary and sufficient condition for optimality.

Step 1. For a given w^k (k — number of iterations), determine ground state

$B(y^k_{n+1} - y^k_n)/\tau + Ay^k_n = 0, n = \overline{0, N_0 - 1}, y^k_0(x) = w^k(x), x \in \omega.$

Step 2. Then define conjugate state in backward time

$B(\psi^k_n - \psi^k_{n+1})/\tau + A\psi^k_{n+1} = 2\chi_h(y^k_n - \varphi_h(x, t_n)), n = N_0, N_0 - 1, \cdots, 1; B(\psi^k_0 - \psi^k_1)/\tau + A\psi^k_1 = 0, \psi^k_{N_0+1} = 0, x \in \omega,$

where χ_h — sum of δ-functions from the space of generalized functions H at points with wells; B, A — grid operators; y, φ — basic and conjugate variables.

Step 3. Finally, specify the initial condition

$(w^{k+1} - w^k)/s^{k+1} + B\psi^k_0 + \alpha w^k = 0, x \in \omega.$

Thus the algorithm based on the solution of two time-dependent problems and refinement of initial condition at each iteration.

However the regularization parameter α should be determined by the level of error in initial data (usually unknown). In practice we can limit to method of minimizing residual functional (2), assuming that $\alpha = 0$, i.e. solution of equation $B\psi_0 = 0$ according to similar scheme. Consequently the main advantage of iterative methods over other methods is absence of necessity to determine regularization parameter.

3.2 Restoring boundary conditions

The direct problem is formulated for (1) in the rectangle $\Omega = \{x|x = (x_1, x_2), 0 < x_i < l_i, i = 1, 2\}$ with the following initial and boundary conditions $p(x, 0) = 0$, $k(x)(\partial p/\partial n) = 0, x \in \gamma_* \cup \Gamma$, $x \in \Omega$, $k(x)(\partial p/\partial n) = \mu(x_1, t)$, $x \in \gamma$, where γ, γ_* and Γ_1, Γ_2 — borders of area Ω parallel to axes ($\partial\Omega = \gamma \cup \gamma_* \cup \Gamma = \Gamma_1 + \Gamma_2$). Boundary condition identification problem on $\gamma \subset \partial\Omega$ $k(x)(\partial p/\partial n) = \mu(x_1, t)$ is formulated as inverse problem with replacement of this condition ($\gamma \subset \partial\Omega$) to additional condition on the boundary $\gamma_* \subset \partial\Omega, p(x, t) = \varphi(x_1, t)$. Inverse problem is represented as operator equation $A\mu = \varphi$, where linear non-symmetrical operator A converts functions defined on $\gamma \subset \partial\Omega$ into functions defined on $\gamma_* \subset \partial\Omega$. Then operator equation is transformed to symmetric form $A^*A\mu = A^*\varphi$, where A^* — adjoint operator, and its solution is given by explicitly iterative method of steepest descent.

$$\begin{aligned} &(\mu_{k+1} - \mu_k)/s_{k+1} - A^*A\mu_k = A^*\varphi, k = 0, 1, \cdots; \\ &s_{k+1} = ||r_k||^2/||Ar_k||^2_*, r_k = A^*A\mu_k - A^*\varphi. \end{aligned} \quad (3)$$

Here norms are defined in Hilbert spaces of functions defined on γ and γ_*. Adjoint operator A^* is defined by $A^*\nu = \psi(x, t), x \in \gamma$, where $\psi(x, t)$ — conjugate state determined by solving the boundary value problem

$$-\partial\psi/\partial t + L\psi = 0, \psi(x, T) = 0, x \in \Omega, 0 < t < T. \quad (4)$$

3.3 Parametric identification of permeability coefficients

For simplicity and clarity (without loss of generality) as a model problem we'll consider one-dimensional parabolic equation of pressure distribution

$$\begin{aligned} &\partial p/\partial t - \partial/\partial x(k(p)\partial p/\partial x) = 0, 0 < x, l; \\ &p(0, t) = 0, p(l, t) = g(t); p(x, 0) = 0, 0 \le x \le l; \\ &p(z_m, t) \approx \varphi_m(t), 0 < t \le T, m = \overline{1, M}. \end{aligned} \quad (5)$$

In this statement of coefficient inverse problem the uniqueness of its solution is provided by assumption of sufficient smoothness of $k(p)$ and monotony of $g(t)$. Approximate solution of parametric identification of permeability coefficient is represented as linear combination of basis functions

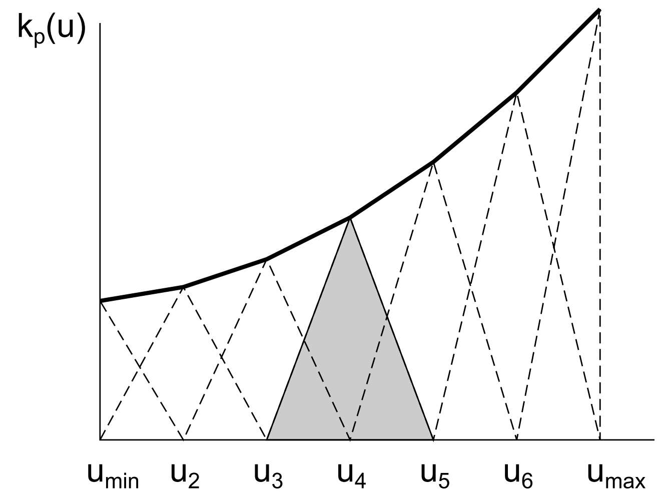

Fig. 2. Identification by piecewise linear finite functions

$\eta_r(p), r = \overline{1,R}$ of finite-dimensional subspace K_R in space of functions K, i.e.

$$k_R(p) = \sum_{r=1}^{r=R} a_r \theta_r(p), p_{min} \leq p \leq p_{max}. \quad (6)$$

To solve the problem of parametric identification again take an alternative approach, where subspace dimension K_R act as regularization parameter, i.e. step of uniform grid

$p_r = p_{min} + (r-1)(p_{max} - p_{min})/(R-1), r = \overline{1,R}$

by p, determining the number of basis elements in (6), i.e. piecewise linear finite functions with coefficients $a_r = k_R(p_r), r = \overline{1,R}$ (Fig. 2).

Parametric identification using traditional methods of regularization is associated with complicating factor associated with nonlinear dependence of $p(x,t)$ from $k(p) = k_R(p)$. In monotonicity conditions of boundary modes we can limit to sequential identification algorithms. In this case with each $t \leq t_* < T$ we can find function $k(p)$, where $p \leq g(t_*)$. These properties provide the simplicity and efficiency of identification by piecewise constant finite functions on uniform grid

$p_r = p_{min} + r(p_{max} - p_{min})/R, r = \overline{0,R}.$

With this parameterization in the interval $p_{r-1} \leq p \leq p_{max}$ one numerical parameter a_r is determined, as $a_q, q = \overline{1, r-1}$ is already defined in previous iterations. Such a procedure is possible with other types of approximation (6), for example, piecewise linear identification of the production coefficient.

If the permeability coefficient is independent of pressure, then the problem of identification and adaptation of $k(x,t)$ is greatly simplified. In this case, we can calculate the estimated values of the coefficient on retrospective data by moving average methods for monotonically decreasing functions (exponents).

3.4 Generalization of identification methods

Suggested parameter identification methods can be easily generalized to determine distribution coefficient values

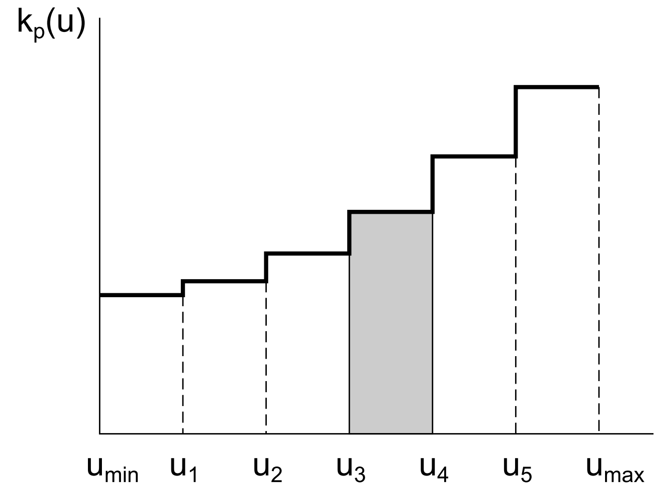

Fig. 3. Identification by piecewise constant finite functions

$k(x, p(x,t)), x \in \Omega \subset R^2$ for the entire set of nodes of two-dimensional grid approximation of oil and gas fields. For inhomogeneous multilayer fields with different coefficients of permeability we can perform parametric adaptation of coefficients for each layer, and thus we can build quasi three-dimensional reservoir model. In practice, such a spatial hydrodynamic model is quite sufficient for adequate representation of fluid filtration processes in heterogeneous porous media of reservoirs.

Therefore, the construction of three-dimensional hydrodynamic models in general form are now inappropriate because of the incompleteness and inaccuracy of the initial and current information.

Similarly we can generalize methods for solving the problems of restoration of initial and boundary conditions for multilayer oil and gas fields heterogeneous by permeability.

4. CONCLUSION

This paper provides the results of basic scientific research to ensure the creation and adaptation of hydrodynamic models of fluid filtration in porous media of natural reservoirs of oil and gas with complex geometry and geological structure at all stages of development. In particular, the identification of functional parameters of nonlinear quasi three-dimensional operator models of fluid filtration processes using functional and parametric optimization to recover the distribution of filtration parameters (absolute and relative permeabilities, coefficients of efficiency, initial and boundary conditions) on retrospective data of measurements in wells.

We investigated implementation features of proposed methods and algorithms for multiprocessor systems with multi-level parallelization of computations using different architectures and interfaces (MPI, OpenMP and OpenCL) focusing on specific applications of simulation of nonlinear problems of fluid filtration processes in porous media for optimal control of multi-layer oil and gas fields development. In particular we investigated and presented methods of coordinated embedding of multilevel structure of algorithms in architecture of supercomputers optimal by criteria of minimum complexity and computation time.

REFERENCES

Akhmetzyanov, A.V. (2008a). Computational Aspects in Controlling Filtration of Fluids and Gases in Porous Media. *Automation and Remote Control*, vol. 69 (No. 1), 1–12.

Akhmetzyanov, A.V. (2008b). Large-Scale Nonlinear Multivariable Systems (Decomposition, Modelling and Control Problems). *Proceeding 17th IFAC World Congress. Seoul, Korea*, ID: 2217.

Akhmetzyanov, A.V., Ermolaev A.I. (2008). Simulation of Hidrocarbon Filtration Processes in Porous Media Using the Decomposition of the Simulation Zone. *Proceedings of the 11th European Conference on the Mathematics of Oil Recovery. Bergen, Norway*, P22.

Akhmetzyanov, A.V., Ibragimov, I.I., Yaroshenko, Y.A. (2012). Integrated Hidrodynamic Models of Oil Field Development Processes. *Automation and Remote Control*, vol. 72 (No. 2), 345–354.

Jansen, J.D., Bosgra, O.H., Van den Hof, P.M.J. (2008). Model-based Control of Multiphase Flow in Subsurface Oil Reservoirs. *Journal of Process Control*, vol. 18 (9), 846–855.

Samarskii, A.A., Vabishchevich, P.N. (2007). *Numerical Methods for Solving Inverse Problems of Mathematical Physics*. Walter de Gruyter, Berlin.

Tikhonov, A.N., Arsenin, V.Y. (1977). *Solutions of Ill-posed Problems*. Winston & Sons, Washington, D.C.