A Moving Horizon Observer for Estimation of Bottomhole Pressure during Drilling *

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Abstract: To ensure safe and stable drilling operation, bottom hole pressure (BHP) should be kept within some region. However measurement of the BHP is sometimes not available or reliable, especially when the circulation is low, e.g., during pipe connection procedures. This paper presents the application of a moving horizon estimation (MHE) method for online estimation of the BHP during petroleum drilling. In the proposed MHE formulation the states are estimated by a forward simulation with a pre-estimating observer. Moreover, it considers the constraints of states/outputs in the MHE problem. Application of the observer to a real data set from a North Sea oil well illustrates potential benefits.

1. INTRODUCTION

Under some sufficient pump pressure the drilling fluid downwardly circulates through the drill pipe, through the drill collars, through small holes in the drill bit, up the annulus between the borehole and the drill pipe to the surface for reconditioning so as to be circulated. To ensure safe and stable drilling operation, bottom hole pressure (BHP) should be kept within some margin between pore and fracture pressure.Exceeding the fracture pressure will fracture the rock formation, and there is a high risk of an underground blowout. If the pressure in the well is lower than the pore pressure, it may not be an effective barrier agsinst a kick.

During drilling, the BHP can be measured, but its measurement is usually communicated with slow mud pulse telemetry. Several uncertain factors, for instance, movement of the drill pipe and reservoir influx, have impact on its measurement, which leads to high uncertainty. Moreover, its measurement is sometimes not available when the circulation is low or during pipe connection. Therefore, considering the unreliable and partial unavailability of the BHP measurement, accurate estimation is a challenging problem (Zhou et al. [2008a], Paasche et al. [2011], Stamnes et al. [2008], Zhou et al. [2009, 2008b], Nygaard et al. [2007c]).

To present date there are a few publications on the estimation of the BHP. For example, the use of low order models for estimation and control of the BHP can be founded in Nygaard et al. [2007a,b]. More recently, a third-order managed pressure drilling (MPD) model developed by Kaasa [2007] is widely used to estimate the BHP, see Zhou et al. [2008a], Paasche et al. [2011], Stamnes et al. [2008], Zhou et al. [2009, 2008b], Sui et al. [2011]. In Zhou et al. [2008a], Stamnes et al. [2008] a nonlinear model-based adaptive observer to estimate the BHP with estimation of other parameters is employed. In Paasche et al. [2011], the regularized moving horizon estimation (MHE) method proposed by Sui and Johansen [2011] is used to estimate the BHP. In Sui et al. [2011], an powerful ensemble method for estimating the BHP is presented.

In this paper, an MHE method proposed by Sui et al. [2010], Sui and Johansen [2012] for online estimation of the BHP is employed. The reason of choosing the MHE observer is that it can provide a high degree of robustness in the presence of modeling uncertainties since it is based on a batch of the most recent information/measurements. This is in contrast to nonlinear observers and nonlinear Kalman filters that update the next estimate based on the most recent measurement only, which is known to be optimal under white noise conditions that are rarely met in practical applications. Moreover, the constraints of states and parameters are considered in the MHE problem, which may lead to the more accurate esmation of the BHP. Sui and Johansen [2012] propose a novel MHE observer where the states are estimated by a forward simulation with a pre-estimating observer. Compared with standard MHE approaches, it has additional degrees of freedom to optimize the noise and disturbance filtering through the pre-estimator. Testing on the data from a North Sea well, the results show that such MHE observer can provide a promising behavior of the estimation of the BHP.

2. MODEL DESCRIPTION

The MPD system we consider is modelled by a simplified model developed by Stamnes [2007]. The drill string and annulus are treated as two separate control volumes that are connected through the drill bit's check valve. The model is based on a mass balance for the two separate control volumes, and a momentum balance at the drill bit. The parameters used in the paper are given in Table 1.

The pressure dynamics are thus given by

$$\dot{p}_p = \frac{\beta_d}{V_d} (q_{pump} - q_b), \tag{1a}$$

$$\dot{p}_c = \frac{\beta_a}{V_a} (q_b - q_c + q_{back} + q_{res} + \dot{V}_a). \tag{1b}$$

The volume flow dynamics is derived from the momentum balance and is given by

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Para.	Description	Unit
V_a	Annulus volume	m^3
V_d	Drill string volume	m^3
β_a	Bulk modulus of fluid in annulus	bar
β_d	Bulk modulus of fluid in drill string	bar
p_c	Choke pressure	bar
p_p	Pump pressure	bar
q_b	Flow rate of the bit	m^3/s
q_c	Flow rate of the choke	m^3/s
q_{back}	Flow rate of the backpressure pump	m^3/s
q_{res}	Flow rate of influx from the reservoir	m^3/s
q_{pump}	Flow rate of the pump	m^3/s
λ_a	Friction parameter of annulus	bar s^2/m^6
λ_d	Friction parameter of drill string	bar s^2/m^6
ρ_a	Density mud in annulus	$10^{5} kg/m^{3}$
$ ho_d$	Density mud in drill string	$10^{5} kg/m^{3}$
g	Acceleration of gravity	m/s^2
h	Vertical depth of the bit	m
ℓ_a	Length of annulus	m
ℓ_d	Length of drill string	m
A_a	Cross sectional area of annulus	m^2
A_d	Cross sectional area of drill string	m^2
<i>p</i> _{bit}	Bottom hole pressure	bar

Table 1. Model parameters.



Fig. 1. A simplified drawing of the MPD drilling system.

$$\dot{q}_{b} = \frac{1}{M} (p_{p} - p_{c} - \lambda_{a} q_{b}^{2} - \lambda_{d} q_{b}^{2} + (\rho_{d} - \rho_{a}) gh), \quad (2)$$

where the parameter $M = M_a + M_d$ with

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$$M_a =
ho_a \int_o^{\ell_a} rac{1}{A_a(x)} dx,$$

 $M_d =
ho_d \int_o^{\ell_d} rac{1}{A_d(x)} dx.$

To simplify the model, it is assumed that $q_{res} = 0$. The bottom hole pressure, p_{bit} , depends on the choke pressure, pump pressure, friction pressure and hydrostatic pressure, and given as

$$p_{bit} = \frac{1}{M} (M_a p_p + M_d p_c + M_d \lambda_a q_b^2 - M_a \lambda_d q_b^2 + (M_d \rho_a + M_a \rho_d) gh).$$
(3)

In summary, the drilling system dynamics can be formulated in the state space representation

$$\dot{x} = f(x, u, \alpha), \tag{4}$$

$$y = h(x), \tag{5}$$

where the state x, input u, output y, time varying parameter α vectors are given as

$$x = \begin{bmatrix} p_p \\ p_c \\ q_b \end{bmatrix}, \quad u = \begin{bmatrix} q_{pump} \\ q_{back} - q_c + \dot{V}_a \end{bmatrix},$$
$$y = \begin{bmatrix} p_p \\ p_c \\ p_{bit} \end{bmatrix}, \quad \alpha = \begin{bmatrix} V_a \\ V_d \\ \ell_a \\ \ell_d \\ h \end{bmatrix}$$
(6)

Remark 1 Note that if the measurement of the BHP is not available at some time, the output should be considered as $y = [p_p, p_c]^T$.

In this paper, a linear model is considered. The nonlinear MPD model can be linearized around a solution (x_t^0, u_t^0) which satisfies

$$\dot{x}_t^0 = f(x_t^0, u_t^0, \alpha_t).$$
 (7)

The perturbations in x, u and y can be defined as

$$x_t = x_t^0 + \Delta x_t, \tag{8a}$$

$$u_t = u_t^0 + \Delta u_t, \tag{8b}$$

$$y_t = y_t^0 + \Delta y_t = h(x_t^0) + \Delta y_t.$$
(8c)

Such a linearized model, developed by Stamnes [2007], is shown below

$$\Delta \dot{x} = A(x_t^0, \alpha_t) \Delta x + B(x_t^0, \alpha_t) \Delta u, \qquad (9a)$$

$$\Delta y = C(x_t^0, \alpha_t) \Delta x, \tag{9b}$$

where A, B, C can be expressed as

$$A(x_{t}^{0}, \alpha_{t}) = \begin{bmatrix} 0 & 0 & -\frac{\beta_{d}}{V_{d}} \\ 0 & 0 & \frac{\beta_{a}}{V_{a}} \\ \frac{1}{M} & -\frac{1}{M} & -\frac{2(\lambda_{a} + \lambda_{d})|x_{3,t}^{0}|}{M} \end{bmatrix}, \quad (10a)$$

$$B(x_{t}^{0}, \alpha_{t}) = \begin{bmatrix} \frac{\beta_{d}}{V_{d}} & 0 \\ 0 & \frac{\beta_{a}}{V_{a}} \\ 0 & 0 \end{bmatrix}, \quad (10b)$$

$$C(x_t^0, \alpha_t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ \frac{M_a}{M} & \frac{M_d}{M} & 2(\frac{M_d}{M}\lambda_a - \frac{M_a}{M}\lambda_d)|x_{3,t}^0| \end{bmatrix}, \quad (10c)$$

and $x_t^0 = (x_{1,t}^0, x_{2,t}^0, x_{3,t}^0)^T.$

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3. LINEAR MOVING HORIZON ESTIMATOR

In this section, we introduce a linear MHE observer to estimate the BHP by considering system (9). The following discretization of the model is used considered,

$$x_{t+1} = Ax_t + Bu_t + \xi_t, \qquad (11)$$

$$y_t = Cx_t + \eta_t, \tag{12}$$

where $x_t \in X \subseteq R^{n_x}$, $u_t \in U \subseteq R^{n_u}$ and $y_t \in R^{n_y}$ are the state, input and the measurement, respectively. $\xi_t \in R^{n_x}$ is an unknown state disturbance, $\eta_t \in R^{n_y}$ is a measurement noise vector, and ξ_t , η_t are known only to the extent that they lie, respectively, in the polyhedral sets Ξ and Σ . It is assumed that:

(A1) the pair (A, C) is observable.

(A2) X is a polyheral set, and contains the origin in its interior. (A3) $x \in X$ for all t > 0

(A3) $x_t \in X$ for all $t \ge 0$.

The idea of the MHE is to estimate the current states by solving a least squares optimization problem, which penalizes the deviation between the measurements and predicted outputs and possibly the distance from the estimated state and an a priori information state. The basic strategy is to estimate the state using a moving window of data, such that the size of the data set used for estimation is fixed by looking at a subset of the available information. At time t, the information vector is defined as

$$I_t = \operatorname{col}(y_{t-N}, \dots, y_t, u_{t-N}, \dots, u_{t-1}),$$
(13)

where N + 1 is the window length or horizon. The problem consists in estimating, at any time t = N, N + 1, ..., the state vectors $x_{t-N}, ..., x_t$, on the basis of the a priori estimate $\bar{x}_{t-N,t}$ and I_t .

The MHE problem proposed by Sui and Johansen [2012] is formulated, as follows,

$$J(\hat{x}_{t-N,t}; \bar{x}_{t-N,t}, I_t) = \|y_{t-N}^t - \hat{y}_{t-N,t}^{t,t}\|_W^2 + \|\hat{x}_{t-N,t} - \bar{x}_{t-N,t}\|_M^2$$
(14a)

subject to

$$\hat{x}_{i+1,t} = A\hat{x}_{i,t} + Bu_i + L(y_i - \hat{y}_{i,t}), \quad i = t - N, \dots, t - 1, \quad (14b)$$

$$y_{i,t} = Cx_{i,t}, \ t = t - N, \dots, t,$$
 (14c)

$$x_{i,t} \in \mathbf{X}, \ t = t - N, \dots, t,$$
 (14d)

where W > 0, M > 0 are weight matrices and $L \in \mathbb{R}^{n_x \times n_y}$ is chosen to such that the eigenvalues of $\Phi := A - LC$) are contained in the unit disc, and

$$y_{t-N}^{t} = \begin{bmatrix} y_{t-N} \\ y_{t-N+1} \\ \dots \\ y_{t} \end{bmatrix}, \quad \hat{y}_{t-N,t}^{t,t} = \begin{bmatrix} \hat{y}_{t-N,t} \\ \hat{y}_{t-N+1,t} \\ \dots \\ \hat{y}_{t,t} \end{bmatrix}.$$
(15)

The purpose of the pre-estimating Luenberger observer with gain *L* is to reduce the effect of noise and disturbances before the MHE optimization is invoked. The optimal solution of (14) is defined by $\hat{x}_{t-N,t}^o$ and it yields the sequence of the state estimates $\hat{x}_{i,t}^o$, $i = t - N, \dots, t$ from (14b). It is assumed that the a priori estimate is determined from $\hat{x}_{t-N-1,t-1}^o$, that is

$$\bar{x}_{t-N,t} = A\hat{x}_{t-N-1,t-1}^{o} + Bu_{t-N-1} + L(y_{t-N-1} - \hat{y}_{t-N-1,t-1}^{o}),$$
(16a)

$$\hat{y}_{t-N-1,t-1}^{o} = C\hat{x}_{t-N-1,t-1}^{o}.$$
(16b)

The estimation error is defined as

$$e_{t-N} = x_{t-N} - \hat{x}_{t-N,t}^o.$$
(17)

Theorem 1. (Sui and Johansen [2012]) Suppose that assumptions (A1)-(A3) hold. There always exist the matrices W > 0

Copyright held by the International Federation of Automatic Control and M > 0 such that the estimation error dynamics e_t is inputto-state stable (ISS). Moreover, when $\xi_t = 0$ and $\eta_t = 0, t = 0, 1, ...,$ then e_t converges exponentially to zero.

Proposition 1. (Sui and Johansen [2012]) Suppose that assumptions (A1)-(A3) hold. If the weight matrices M, W satisfy

$$\Phi^T M \Phi - M \le -Q_1, \tag{18a}$$

$$M - F_N^T W F_N \le -Q_2, \tag{18b}$$

$$M = M^T > 0, \tag{18c}$$

$$W = W^T > 0, \tag{18d}$$

for some small $Q_1 > 0, Q_2 > 0$, where

$$F_N = \begin{bmatrix} C \\ C\Phi \\ \vdots \\ C\Phi^N \end{bmatrix}.$$

then the estimated error dynamics e_t is ISS.

In the paper, we choose $M = M^T$ such that

and

$$M > \Phi^T M \Phi. \tag{19}$$

The above inequality is a linear matrix inequality (LMI), see Boyd et al. [1998], which can be efficiently solved with some existing numerical methods.

Assuming all variables are reasonably scaled, we propose to choose the matrix W such that

$$W = W_1^T W_1, (20)$$

$$W_1 F_N = \sqrt{\bar{\alpha}} \sqrt{M},\tag{21}$$

where $\bar{\alpha} > 0$ is a scalar tuning parameter. Since the system is observable, it leads to

$$W_1 = \sqrt{\bar{\alpha}} \sqrt{M} F_N^+, \qquad (22)$$

where $F_N^+ = (F_N^T F_N)^{-1} F_N^T$ is the pseudo-inverse. In order to guarantee the stability, *W* is chosen such that (18b) holds. Combining with (20) and (21), $\bar{\alpha}$ should satisfy $\bar{\alpha} > 1$. Since the positive tuning parameter $\bar{\alpha}$ is scalar, acceptable performance may depend on appropriate scaling of the state and output variables and the associated model equations.

4. EXPERIMENTS AND RESULTS

Parameter	Value	Unit		
Va	50.8393	m^3		
V_d	16.6953	m^3		
ρ_a	0.0161	$10^5 \times kg/m^3$		
$ ho_d$	0.0161	$10^5 \times kg/m^3$		
g	9.81	m/s^2		
h	1652.4	т		
ℓ_a	1854.8	т		
ℓ_d	1680.5	m		
M_a	935.3021	$10^{-5} \times kg/m^4$		
M_d	3223	$10^{-5} \times kg/m^4$		
Table 2 Decemptor values				

 Table 2. Parameter values

For testing of the observer's capabilities and suitability for the drilling industry, the available data employed consists of a time series from an MPD drilling operation in the North Sea. The time series consists of around 2.77 hours of drilling and pipe connection events. The total number of samples in the time series is 10000. Some of the measurements are noisy and also

contain outright errors in some places. This is typical for data sets in the drilling industry and a model should be robust to such errors if it to be used in a real-time setting.



Fig. 2. Drilling Inputs.

In this section, the MHE algorithm is applied to the combined state and parameter estimation problem of estimating p_p, p_c and p_{bit} . The data is sampled at 1Hz for the measurements of p_c, p_p . However during this period, the measurement of p_{bit} is not available, and the output is then limited to $y = [p_p, p_c]^T$. The selected parameter values for model used is shown in Table 2.

The 10000 samples of inputs are shown in Figure 2. The equilibrium point (x_t^0, u_t^0) is obtained by (7): $x_t^0 = (204, 19, 0.033)^T$, $u_t^0 = (0, 0)^T$. To convert the continuous system (9) to a discrete-

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time system, the sampling interval is $T_s = 1 s$. The discrete-time MPD drilling system is obtained

$$A = \begin{bmatrix} 0.9667 & 0.0333 & -274.7311 \\ 0.01 & 0.99 & 82.4137 \\ 0.0001 & -0.0001 & 0.1899 \end{bmatrix},$$
$$B = \begin{bmatrix} \frac{\beta_d}{V_d} & 0 \\ 0 & \frac{\beta_a}{V_a} \\ 0 & 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0.2 & 0.8 & -1.5216 \end{bmatrix}.$$

The matrix L is chosen by pole placement as

$$L = \begin{bmatrix} 0.8477 & -0.0986 & 0.1811 \\ 0.0194 & 0.9456 & -0.0537 \\ 0.0001 & 0 & -0.0001 \end{bmatrix}.$$

The MHE window size has been chosen as N = 19. From (19), M is chosen as

$$M = \begin{bmatrix} 1.0828 & -0.0093 & 0\\ -0.0093 & 1.0979 & 0\\ 0 & 0 & 1 \end{bmatrix}$$

Furthermore, based on (20)-(22), we choose W with $\bar{\alpha} = 100$. All estimates are normalized before used in the solver cost function (14a). Scaling is a tool to prioritize outputs and states as deemed appropriate. If not estimated, the friction parameters λ_a , λ_d and the bulk modulus β_a , β_d are tuned off-line to steady state information as available in the data set.

In the estimation, state constraints are added to the optimization problem:

$$x_t \ge 0 \tag{23}$$

or

 $\Delta x_t \ge -x_t^0. \tag{24}$

4.1 Estimation of p_c , p_p and p_{bit}

In this case, the parameters λ_a , λ_d and β_a , β_d are tuned offline as shown in Table 3. Figure 3 shows the estimated p_c , p_p

Parameter	Value	Unit	
β_a	1.0368×10^{4}	bar	
β_d	1.1478×10^4	bar	
λ_a	4.0432×10^{3}	bar s^2/m^6	
λ_d	1.1534×10^{5}	bar s^2/m^6	
Table 3. Parameter Values.			

and Figure 4 shows the estimated p_{bit} by the MHE. The Mean Average Error (MAE) between the BHP from the memory of the pressure sensor and estimation of p_{bit} is 3.1448 bar.

From Figure 3 and Figure 4, it is easy to see that the BHP can be well estimated when the inputs are persistently exciting. However when the inputs are not persistently exciting, due to the mismatch between the true system and the model and incorrect selections of drilling parameters, there exist significant estimation errors. During the period between 0.5 hours to 2 hours, since the pumps are off, the BHP should equal to the hydrostatic pressure. When the pumps are off, the drilling fluid is allowed to reach thermal equilibrium with the surrounding rock formation, which in this case resulted in a net temperature increase across the annulus. As the mud is not constrained by



Fig. 3. Estimates of p_p and p_c .



Fig. 4. Bottom hole pressure by MHE.

the choke, the hot mud is allowed to expand, leading to a lower mud density. Due to the selection of the constant mud density, the estimation error is slightly large during 0.5 hours to 2 hours. Therefore, the mud density estimat in the estimation model might change based on the drilling activity in order to further improve estimation accuracy.

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4.2 Estimation of p_c, p_p, p_{bit} and parameter ρ_a

In general, pure state estimation might be limited in its results. Combined with parameter estimation, a powerful tool is available to improve model accuracy, see more discussion in Paasche et al. [2011]. As what we discussed above, drilling parameters like the mud density tend to vary during the drilling activity. The poor selection of drilling parameters might lead to degraded estimation performance. In this case, the MHE algorithm is applied to the combined state and parameter estimation problem of estimating p_c , p_p , p_{bit} and model parameter ρ_a . It is assumed that

and

$$ho_l \leq
ho_a \leq
ho$$

 $\dot{\rho}_a = 0$

where the boundaries ρ_l and ρ_u are chosen as $\rho_l = 0.01585$ and $\rho_u = 0.0163$. The model is re-linearized due to the augmentation with parameters. The value of the annulus density directly impacts the estimation of p_{bit} . It should be estimated to consider the sensitivity of the model to changes in ρ_a . However, it should be noted that the parameter M also depends on ρ_a . In order to reduce the complexity of the observer we neglect such dependency. More discussions about it are given in Stamnes [2007]. Figure 5 shows the estimated p_c , p_p and ρ_a and Figure 6 shows the estimated p_{bit} by the MHE. The MAE between memories of p_{bit} and estimations of p_{bit} is 1.9711 bar. From Figure 5 and Figure 6, we see that there still exists some estimation error, but due to the consideration of parameter estimation, the performance is improved. During 0.5 hours to 2 hours, the estimation error becomes smaller since the mud density becomes lower.

5. CONCLUSION

In this paper, a MHE observer for estimation of the bottom hole pressure while drilling and pipe connect is applied. The proposed observer is parameterized to optimize the noise filtering and include constraints of states and parameters in the MHE problem. Application of the observer to a real data set from a North Sea oil well illustrates promising and good behavior.

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305 BHP by MHE • true BHP 300 а ²⁸⁵ На ₂₈₀ 280 275 270 265 260 0.5 2.5 1.5 2 3 hours

Fig. 6. Bottom hole pressure by MHE.

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Fig. 5. Estimates of p_p , p_c and ρ_a .

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