Integrated Capacitance-Resistance Model for Characterizing Waterflooded Reservoirs

Jong S. Kim*, Larry W. Lake, Thomas F. Edgar*****

**Department of Chemical Engineering, University of Texas at Austin, Austin, TX 78712 USA (Tel: 512-471-5150; e-mail: jongkim@che.utexas.edu).*
*** Department of Petroleum and Geosystems Engineering, University of Texas at Austin, Austin, TX 78712 USA (e-mail:larry_lake@mail.utexas.edu)*
**** Department of Chemical Engineering, University of Texas at Austin, Austin, TX 78712 USA (e-mail: tfedgar@austin.utexas.edu)*

Abstract: Capacitance-resistance modeling of petroleum reservoirs has been used successfully to analyze transient behavior of petroleum reservoirs both onshore and offshore. This paper presents a linear reservoir model that provides advantages over the nonlinear capacitance-resistance model: convex objective function, efficient solution, and direct estimation of confidence limits on model parameters. The proposed procedure uses a constrained linear multivariate regression to infer preferential permeability trends and fractures in a waterflooded reservoir. The relationship between interwell-connectivities and interwell-distance between injector-producer well-pairs was also investigated.

Keywords: capacitance-resistance model (CRM), confidence interval, gain, history-matching, integrated capacitance-resistance model (ICRM), linear regression, time constant, unique solution

1. INTRODUCTION

The capacitance-resistance model (CRM) is an input-output model that characterizes the properties of an oil reservoir using only historical production data. In the CRM, the input signal (injection rates) is converted to an output signal (total production rates) in a similar manner as electronic potential is converted to voltage or current in a resistance-capacitor (RC) circuit (Thompson, 2006), hence the name capacitance-resistance model. In chemical engineering, the CRM is analogous to a single (or a series of) first-order tank storage model(s), where the flow rate into the tank is used to predict the level of the incompressible fluid inside and the outflow rate (Seborg et al., 2010). Fig. 1 shows a schematic of how the total production of slightly compressible fluids (oil and water) responds to a step-change made on an injection rate in the CRM. The shape of the output response caused by a step-change in injection rate depends on the time lag and attenuation between a producer and an injector. Yousef et al. (2006) introduced the capacitance model (CM) that can quantify interwell connectivity and the degree of fluid storage between well-pairs. Two different approaches, the balanced capacitance model (BCM) and the unbalanced capacitance model (UCM), were proposed to study interwell connectivities depending on whether the waterflood is balanced or not. Sayarpour et al. (2007) introduced analytical solutions for fundamental differential equation of the CM based on superposition in time and presented these solutions for three different reservoir control volumes: 1) volume of the entire field or tank model (CRMT), 2) drainage volume of each producer (CRMP), and 3) drainage volume between each injector-producer pair (CRMIP). Weber et al. (2009) discretized the CRM and used the CRM to optimize injection allocation and well location in waterfloods with many variables and constraints. Compared to traditional reservoir simulators, both the CM and CRM offer a rapid evaluation of reservoir behavior between injectors and producers because both models only require water injection, total liquid production rates, which are typically already measured and collected, and producer bottom hole pressures (BHPs) to solve for model parameters. Neither method requires a prior geologic model. However, both the CM and CRM use nonlinear multivariate regression to estimate model parameters. For a typical large waterflood, hundreds of producers and injectors may be present in a reservoir, resulting tens of thousands of model parameters in a field to be determined to completely define either the CM or CRM. In this case, obtaining a unique solution in history matching large reservoirs by nonlinear multivariate regression can be

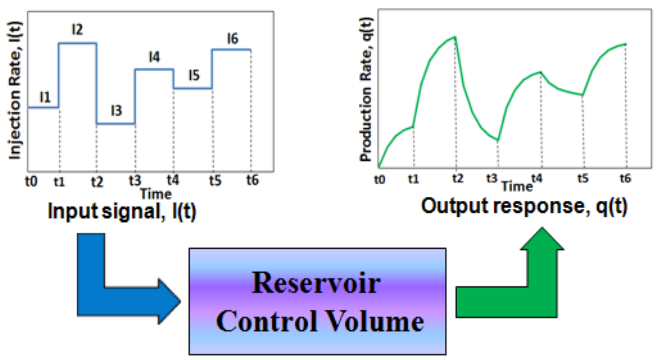

Fig. 1. Schematic representation of the impact of an injection rate signal on total production response for an arbitrary reservoir control volume in the CRM (Sayarpour, 2008).

difficult, and this approach can produce parameters that are statistically insignificant (Weber et al., 2009). Furthermore, establishing confidence intervals of the model parameters would also be difficult because of the nonlinear nature of both models. Nguyen et al. (2011) developed an integrated capacitance-resistance model (ICRM) that uses cumulative water injection and cumulative total production instead of water injection rate and total production rate. The ICRM performs linear regression to obtain the model estimates. Therefore, with ICRM, confidence intervals of model parameters can easily be established. ICRM guarantees a unique solution regardless of the number of parameters as long as the number of data points is greater than the number of unknowns (parameters). The main objectives of this work are to apply the ICRM to waterfloods and to evaluate the uncertainty on model parameters. Also, the power of the ICRM permits describing the interactions between newly introduced injectors and existing producers and predicting the future total liquid production based on interwell-distance dependent well-connectivities.

2. PROCEDURE

Both CRM and ICRM estimate two types of model parameters if producer bottom hole pressure (BHP) is constant: (1) well connectivities (or gains) that represent the degree of communication between injector-producer well-pairs and (2) time constants that represent the degree of fluid storage (compressibility) between well-pairs.

2.1 Capacitance-Resistance Model

Weber et al. (2009) developed the producer-based representation of CRM (CRMP):

$$q_{jk} = q_{j(k-1)} e^{-\Delta t / \tau_j} + \left(1 - e^{-\Delta t / \tau_j}\right)\left(\sum_{i=1}^{N_i} f_{ij} i_{ik} - J_j \tau_j \frac{p_{wf,j}^{k} - p_{wf,j}^{k-1}}{\Delta t}\right) \quad (1)$$

where q_{jk} is the total liquid production (oil and water) rate from a producer j at time step k, i_{ik} is the water injection rate of injector i at time step k, N_i is the total number of injectors, Δt is a discrete time period, p_{wf} is bottom hole pressure, J_j is productivity index of producer j, and f_{ij} is a *gain*. Physically, f_{ij} represents the fraction of water rate from injector i flowing towards producer j at steady state. The *time constant*, τ, is defined as

$$\tau = \frac{c_t V_P}{J} \quad (2)$$

where c_t is total compressibility, V_P is pore volume, and J is productivity index of a producer. Model parameters such as gains and time constants are estimated by nonlinear multivariate regression that minimizes the following objective function:

$$\min z = \sum_{k=1}^{n_t} \sum_{j=1}^{n_p} \left(\left(q_{jk}\right)_{obs} - \left(q_{jk}\right)_{cal} \right)^2 \quad (3)$$

where $(q_{jk})_{obs}$ is the observed total production rate, $(q_{jk})_{cal}$ is the calculated total production rate by the model, n_p is the total number of producers, and n_t is the total number of historic time periods selected in a fitting window. Equation (3) is solved with (1) as well as additional constraints:

$$\sum_{j=1}^{n_p} f_{ij} \le 1 \text{ for all } i \quad (4)$$

$$f_{ij}, \tau_j \ge 0 \text{ for all } j \quad (5)$$

Equation (4) is a total material balance (continuity equation) allowing for a loss of water injected within the control volume when the sum of gains is less than one (Weber et al., 2009). Constraint (5) ensures that injected water does not adversely affect the reservoir production.

2.2 Integrated Capacitance-Resistance Model (ICRM)

The integrated capacitance-resistance model for secondary recovery is developed from the CRMP governing differential equation (6) that represents the in-situ-material balance over the effective pore volume of a producer (Sayarpour et al., 2007).

$$q_j(t) = \sum_{i=1}^{N_i} f_{ij} i_i(t) - \left(\frac{c_t V_P}{J}\right)_j \frac{dq_j(t)}{dt} - \left(c_t V_P\right)_j \frac{dp_{wf,j}(t)}{dt} \quad (6)$$

Equation (6) can be integrated over the time interval from t_0 to t_k:

$$\int_{q_{j0}}^{q_{jk}} dq_{jk} + \frac{1}{\tau_j}\left(\int_{t_0}^{t_k} q_{jk} dt\right) = \frac{1}{\tau_j}\left[\sum_{i=1}^{N_i}\left\{ f_{ij}\left(\int_{t_0}^{t_k} i_{ik} dt\right)\right\}\right] - J_j\left(\int_{p_{wf,j}^{0}}^{p_{wf,j}^{k}} dp_{wf,j}^{k}\right) \quad (7)$$

After rearranging terms and integrating (7), ICRM for secondary recovery is obtained:

$$N_{p,j}^{k} = \left(q_{j0} - q_{jk}\right)\tau_j + \sum_{i=1}^{n_i}\left(f_{ij} CWI_i^{k}\right) + J_j \tau_j \left(p_{wf,j}^{0} - p_{wf,j}^{k}\right). \quad (8)$$

Here $N_{P,j}$ represents the cumulative total liquid production from a producer j. The parameter CWI_i represents the cumulative water injection into an injector i. If producer BHPs are constant, (8) is simplified:

$$N_{p,j}^{k} = \left(q_{j0} - q_{jk}\right)\tau_j + \sum_{i=1}^{n_i}\left(f_{ij} CWI_i^{k}\right) \quad (9)$$

Model parameters are estimated by linear multivariate regression that minimizes the following objective function:

$$\min z = \sum_{k=1}^{n_t} \sum_{j=1}^{n_p} \left(\left(N_{p,j}^{k}\right)_{obs} - \left(N_{p,j}^{k}\right)_{cal} \right)^2 \quad (10)$$

This objective function is solved with (8) or (9) and the same constraints as for the CRMP. As seen in (4), (5), and (9), the ICRM and the constraints associated with it are all linear, indicating any local minimum found in (10) is a global minimum (Nguyen et al., 2011). Therefore, a unique set of parameters that give a global minimum is obtained when (10) is minimized. When the total water injection is approximately

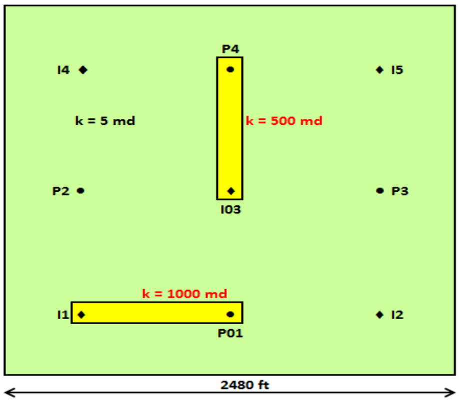

Fig. 2. Syfield-1 consists of two high-permeability streaks of 500 and 1,000 md (same example as in Sayarpour et al., 2007).

equal to the total liquid production, the waterflood is balanced (Sayarpour et al., 2007). In this case, (4) can be relaxed (ignored), and (11) can be solved for each producer j separately:

$$\min z = \sum_{k=1}^{n_t} \left(\left(N_{p,j}^k \right)_{obs} - \hat{N}_{p,j}^k \right)^2 \tag{11}$$

Furthermore, if all the constraints are ignored, then (11) can be solved analytically by matrix inversion.

3. RESULTS

The numerical simulator Eclipse was used to generate synthetic field data on which the CRMP and ICRM were fitted. The three synthetic fields used to validate the models are all undersaturated black-oil reservoirs.

3.1 Synfield-1: Streak Case

Synfield-1 is a synthetic field (streak case studied by Sayarpour et al., 2007) that consists of five vertical injectors and four vertical producers. Fig. 2 shows the well locations and permeability distributions of a synthetic field. This streak case is a homogeneous reservoir with the porosity of 18% and a permeability of 5 md except where two high-permeability streaks exist. The simulation ran for 100 months of simulated time, and both injectors and producers started operating at the same time (in the first month). The producer BHPs were kept at 250 psi. Injection history for each injector

Table 1. Inferred Model Parameters for Synfield-1

CRMP / ICRM		I1	I2	I3	I4	I5	τ_j (day)
P1	CRMP	**0.896**	0.580	0.224	0.217	0.184	16.0
	ICRM	**0.896**	0.593	0.198	0.252	0.163	5.16
P2	CRMP	0.029	0.033	0.051	0.201	0.039	28.2
	ICRM	0.036	0.035	0.040	0.205	0.033	13.6
P3	CRMP	0.012	0.181	0.087	0.035	0.170	24.2
	ICRM	0.020	0.181	0.086	0.040	0.166	12.3
P4	CRMP	0.067	0.199	**0.650**	0.557	0.586	21.6
	ICRM	0.059	0.199	**0.663**	0.551	0.593	10.6

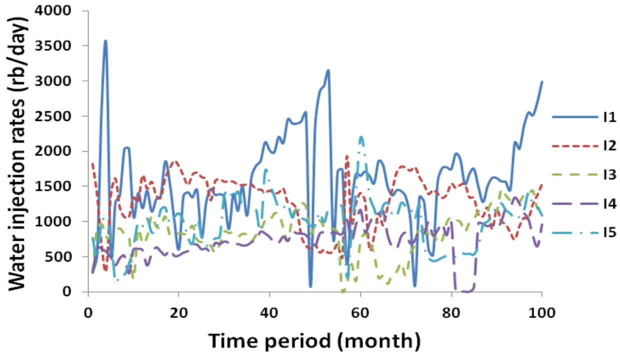

Fig. 3. Monthly water injection rates in Synfield-1 (Albertoni and Lake, 2003).

with water injected for 100 months is shown in Fig. 3. Both the CRMP and ICRM methodologies were applied to match simulated data starting from month 58 to month 100. In this example, the total water injection was approximately equal to the total liquid production; therefore, (4) was not used. With ICRM, (11) was solved analytically by matrix inversion for each producer. Table 1 shows the estimated parameters from CRMP and ICRM. Fig. 4 shows the comparison between the estimated gains from CRMP and those from ICRM presented in histograms. The results show the gains estimated from the both models are comparable to each other. Fig. 5 shows the total production match for all producers. The result shows both CRMP and ICRM fit the data well. We expect the majority of the water injected into I1 would flow towards P1 because there is a high-permeability (1000 md) streak between I1 and P1. The gain calculated by both models between I1 and P1 is 0.896, the highest gain obtained. The large gain indicates 90 % of the water injected into I1 travelled to P1 (at steady state), as expected. The gain calculated by both models for another high-permeability (500 md) streak between I3 and P4 also indicates 65% of water injected into I3 travelled towards P4. Therefore, the estimated gains obtained from both CRMP and ICRM are consistent with the imposed geology. Time constants estimated by CRMP are about twice those estimated by ICRM, but the two quantities are of the same order of magnitude. The cause of

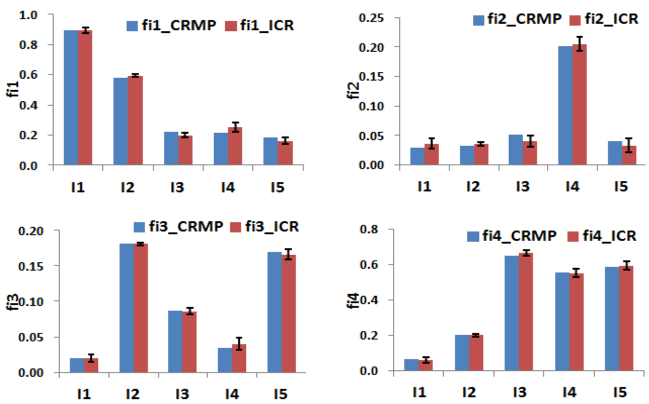

Fig. 4. Comparison between CRMP gains and ICRM gains in Synfield-1. 95% confidence intervals on gains (f_{ij}) estimated by ICRM. A subscript i is an injector index in the range 1 to 5.

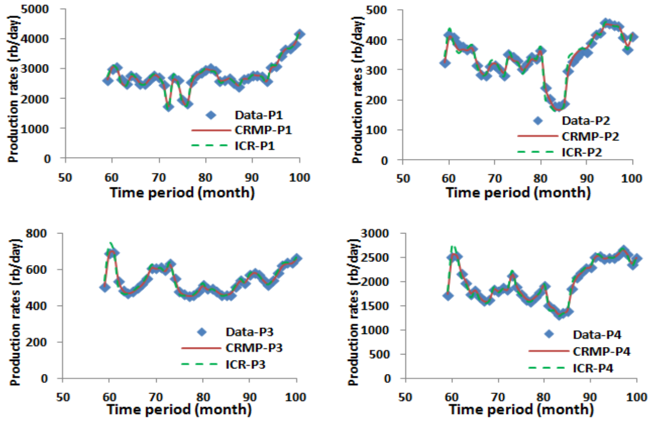

Fig. 5. Synfield-1 CRMP and ICRM match for total liquid production.

this difference is that the observed cumulative production, which was calculated numerically by using the trapezoidal rule, has been overestimated. The ICRM fit could be improved by decreasing the time constant when the model is fitted to the piecewise constant approximation to the response (cumulative production). The linearity of ICRM makes it easy to establish confidence intervals of the model parameters (Montgomery and Peck, 1982). Fig. 4 shows the 95% confidence intervals on the ICRM gains. The confidence intervals of the parameters are narrow enough to conclude regression coefficients are statistically significant. As CRMP is a nonlinear model, a straightforward statistical analysis of the variability of the estimates is not possible. For this reason, the confidence intervals on the CRMP gains were not established in this work.

3.2 Adding New Injectors

A method that can guide reservoir engineers to decide where to drill new injectors to increase future oil production without having to run additional reservoir simulations for each scenario was investigated using ICRM. First, the relationship between gains and interwell-distance between injector-producer well-pairs (d_{ij}) was studied by applying ICRM on Synfield-2. This synthetic reservoir is characterized as a homogeneous isotropic reservoir where wells (five water injectors and four producers) are located randomly (see Fig. 6). ICRM was applied to match simulated data starting from January, 2007 to November, 2010. If the relationship between the gain and the interwell-distance between injector-producer well-pair is assumed to be linear, one can estimate

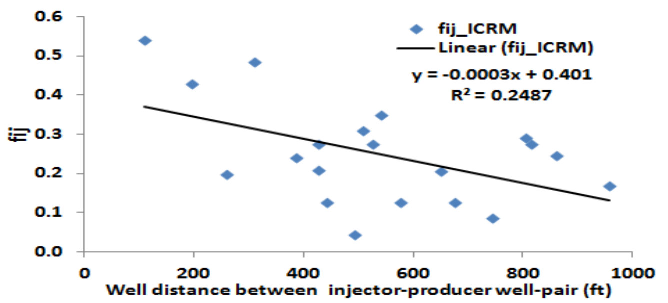

Fig. 7. ICRM gains vs interwell-distance in Synfield-2.

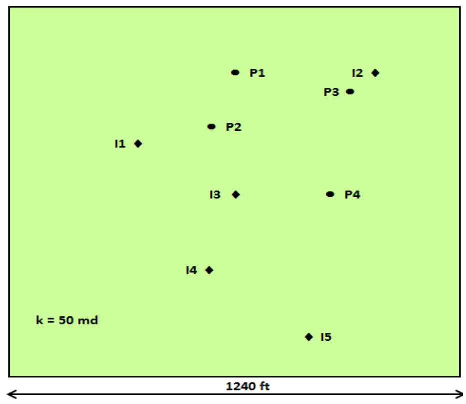

Fig. 6. Synfield-2 is a homogeneous reservoir (k=50 md and ϕ=0.2) and consists of five injectors and four producers.

an interwell-distance dependent gain (f_{ij}^d) at a given d_{ij} :

$$f_{ij}^d = Ad_{ij} + B. \tag{12}$$

In (12), the parameters, A and B, are linear regression coefficients. Equation (12) should only be used to interpolate gains; therefore, it is valid in the range of $f_{ij}lo$ to $f_{ij}hi$, where $f_{ij}lo$ is the lowest gain and $f_{ij}hi$ is the highest gain estimated for a given reservoir. In Fig. 7, ICRM gains estimated from Synfield-2 are plotted on the y-axis and corresponding well distance between each injector-producer well-pair is plotted on the x-axis. As expected, the well-connectivity (gain) tends to decrease as the interwell-distance increases. Equation (12) was regressed on ICRM gains in Synfield-2, and regression coefficients were found to be -0.0003 ft^{-1} for A and 0.401 ft for B. In Fig. 8, the ICRM fits are compared to the fits whose model parameters were calculated by (12). The results show that interwell-distance dependent gains could provide plausible solutions in predicting reservoir productions for a homogeneous reservoir. In Synfield-2, the allocation factor for water injection rate could be estimated reasonably from the interwell-distance between well-pairs. This allows prediction of well-connectivities between newly drilled injectors and existing producers in homogenous reservoirs. Thus, (12) was used to predict future total liquid production

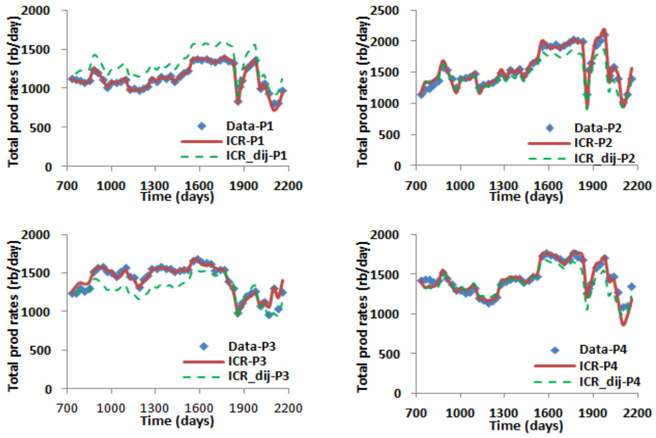

Fig. 8. ICRM fits (red solid line) and predicted total production rates based on f_{ij}^d (green dashed line) for Synfield-2.

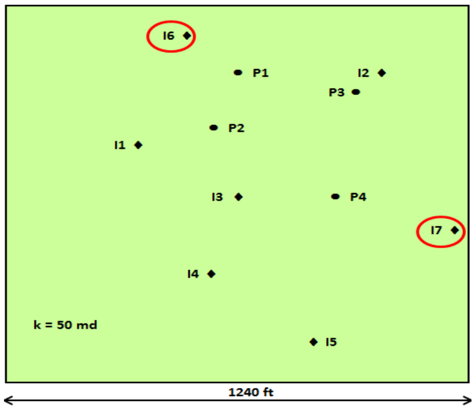

Fig. 9. Synfield-3 consists of five water injectors and four producers initially. After six years of oil production, two injectors (I6 and I7) were added in a reservoir.

rates after new injectors have been added in a homogeneous reservoir. Synfield-3 (Fig. 9) is identical to Synfield-2, but two injectors (I6 and I7) were added in a reservoir after 2200 days (six years) of oil production. After new injectors have been added in a reservoir, oil production has been carried out for an additional four years. In Fig. 10, adding two new injectors supported reservoir pressure substantially, causing a sudden increase in total production rates of all four producers after oil production has occurred for 2200 days. Predicting gains between newly added injectors and producers by using (12) would make sense only if it is assumed that the model parameters remain constant with the introduction of new injectors. In order to validate this assumption that model parameters are invariant to adding new injectors, ICRM was applied on Synfield-3 by selecting two different fitting windows (see Fig. 10). The results (see Fig. 11) show that the gains between existing injectors and producers before adding new wells do not vary significantly compared to the gains between the same well-pairs that were calculated after adding injectors. After validating that ICRM gains between existing injectors and producers are insensitive to adding new injectors, the gains between newly added injector (I6 and I7) and producers (P1-P4) were calculated by (12). ICRM gains were also estimated via linear regression on newly simulated data after adding two injectors. Fig. 12 compares f_{ij}^d to f_{ij} and the results show that they are similar to each other (less than

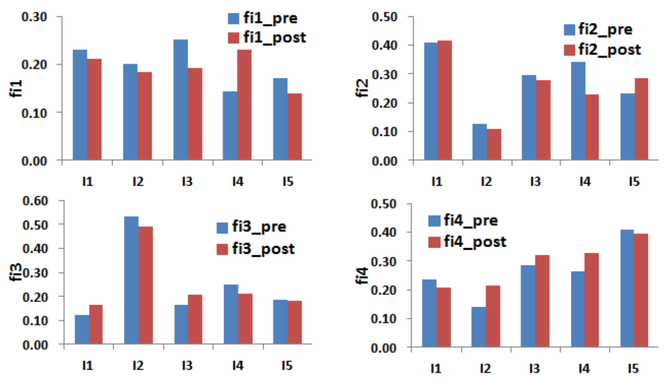

Fig. 11. ICRM gains before adding injectors (blue histograms) vs ICRM gains after adding injectors (red histograms) for Syfield-3.

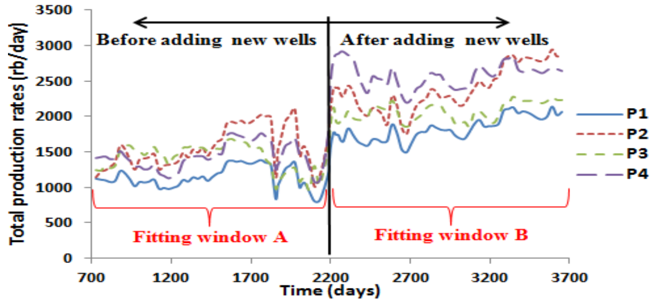

Fig. 10. Daily total liquid production rates in Synfield-3.

30% error). These predicted gains (f_{ij}^d) did not match exactly the ICRM gains (f_{ij}) but the general trend of the gains leads to productions if two injectors would have been added with known water injection scheme. In Fig. 13, green dashed line represents the future total liquid production rates that were predicted by f_{ij}^d for P1 if I6 and I7 have been added in Synfield-3 after 2200 days of oil production. In the same figure, red solid line represents the future liquid production rates for P1 without adding new injectors. It seems that the ICRM fits based on f_{ij}^d are able to predict the future liquid production rates quite well if new injectors would have been added in the reservoir. In Fig. 13, it is clearly shown that the total production rates would increase substantially by Δq after 2200 days of oil production due to the additional pressure support caused by adding two new injectors. One needs to be aware that (12) should be used to estimate gains between new injectors and existing producers only. It should not be used to estimate gains between existing injectors and new producers. If new producers are added in a reservoir, the gains between existing injectors and producer that are calculated by regression prior to adding wells will change significantly. In practice, the use of (12) is limited by strong assumptions, i.e., a homogeneous reservoir. One could run as many reservoir simulations as he or she desires to estimate gains between existing producers and newly introduced injectors at different locations for the particular non-homogeneous reservoir of interest. Estimated gains then could be fitted by empirical models as a function of injector location. However, this method would require too many reservoir simulations for each reservoir of interest and is not practical to use. The purpose of this section is not to identify the best injector location optimization method but to provide rapid predictions of the future production rates with the aid of ICRM.

4. CONCLUSIONS

CRMP and ICRM were applied to synthetic oilfields under waterflooding. In Synfield-1, ICRM fits in comparison to

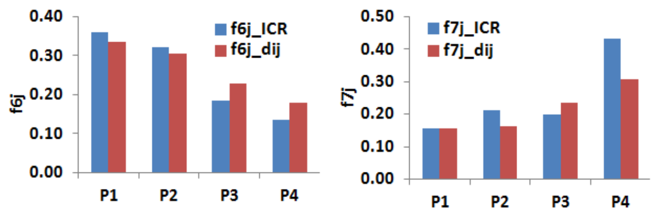

Fig. 12. f_{ij} (blue histogram) vs f_{ij}^d (red histogram) with new injectors for Syfield-3.

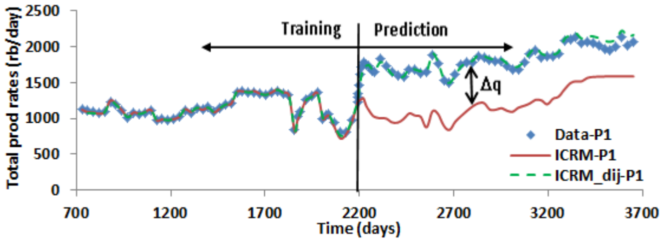

Fig. 13. Prediction of the future total liquid production rates for P1 without (red solid line) or with new injectors (green dashed line) in Synfield-3.

CRMP were presented, and this study showed that ICRM is an attractive alternative to CRMP. In Synfield-2, the method to estimate the gains between well-pairs solely by interwell-distance between well-pairs for a homogeneous reservoir was validated. In Synfield-3, the gains between existing injectors and producers remain constant with the introduction of new injectors, and future liquid production after adding new injectors could be plausibly predicted by the approximate gains. This method can help decide where to drill new injectors to increase future oil recovery and provide rapid solutions without having to run additional reservoir simulations for each scenario.

NOMENCLATURE

L, F, t mean length, force, and time, respectively.

BHP = bottom hole pressure (F/L^2)
CM = capacitance model
CRM = capacitance-resistance model
CRMP = a producer-based representation of the capacitance-resistance model
CWI = cumulative water injection (L^3)
c_t = total reservoir compressibility (L^2/F)
d_{ij} = interwell-distance between injector-producer well-pair (L)
f_{ij} = fraction of injection from injector i flows to producer j, dimensionless
f_{ij}^d = interwell-distance dependent gain, dimensionless
ICRM = integrated capacitance-resistance model
i = water injection rate (L^3/t)
J = productivity index (L^5/F-t)
n_p = total number of producers
n_t = total number of historic time periods
N_i = total number of injectors
N_p = cumulative total production (L^3)
P = pressure (F/L^2)
P_{wf} = bottom-hole pressure (F/L^2)
q = total liquid production rate (L^3/t)
R^2 = correlation coefficient
t = time (t)
V_p = pore volume (L^3)

Greek alphabets
τ = time constant (t)

Subscripts and superscripts
cal = calculated quantity by the model
i = injector index
ij = injector-producer pair index
j = producer index
k = time step index
obs = observed quantity
0 = initial time step

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REFERENCES

Albertoni, A., and Lake, L.W. (2003). Inferring Connectivity Only From Well-Rate Fluctuations in Waterfloods. SPE *Reservoir Evaluation and Engineering Journal*, **6** (1): 6-16.

Montgomery, D.C. and Peck, E.A. (1982). *Introduction to Linear Regression Analysis*. John Wiley & Sons, Ch.4, p. 124-141.

Nguyen, A.N., Kim, J.S., Lake, L.W., Edgar, T.F., and Haynes, Byron (2011). Integrated Capacitance-Resistive Model for Reservoir Characterization in Primary and Secondary Recovery. Paper SPE 147344 presented at the SPE Annual Technical Conference and Exhibition, Denver, Colorado, 30 October-2 November.

Sayarpour, M., Zuluaga, E., Kabir, C.S., and Lake, L.W. (2007). The Use of Capacitance-Resistive Models for Rapid Estimation of Waterflood Performance and Optimization. Paper SPE 110081 presented at the SPE Annual Technical Conference and Exhibition, Anaheim, California, 11-14 November.

Sayarpour, M. (2008). Development and Application of Capacitance-Resistive Models in Water/CO_2 Floods, Ph.D. Dissertation, The University of Texas at Austin, Austin, Texas.

Seborg, D., Edgar, T.F., and Mellichamp, D. (2010). *Process Dynamics and Control*. 3rd Ed. John Wiley & Sons, Ch.2, p.25.

Thompson, M. (2006). *Intuitive Analog Circuit Design*. Elsevier e-Book, Ch. 2, p.16.

Weber, D., Edgar, T.F., Lake, L.W., Lasdon, L., Kawas, S., and Sayarpour, M. (2009). Improvements in Capacitance-Resistive Modeling and Optimization of Large Scale Reservoirs. Paper SPE 121299 presented at the SPE Western Regional Meeting, San Jose, California, 24-26 March.

Yousef, A.A., Gentil, P.H., Jensen, J.L. and Lake, L.W. (2006). A Capacitance Model to Infer Interwell Connectivity from Production and Injection Rate Fluctuations, SPE *Reservoir Evaluation & Engineering Journal*, **9** (5): 630-646. SPE-10.2118. DOI: 10.2118/95322-PA.